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MCT Project - 2

Exercise 1.

1. For lateral control:

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_\alpha}{m\dot{x}} & \frac{4C_\alpha}{m} & \frac{-2C_\alpha(l_f - l_r)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-2C_\alpha(l_f - l_r)}{I_z\dot{x}} & \frac{2C_\alpha(l_f - l_r)}{I_z} & \frac{-2C_\alpha(l_f^2 + l_r^2)}{I_z\dot{x}} \end{bmatrix}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \frac{2C_\alpha l_f}{I_z} & 0 \end{bmatrix}\begin{bmatrix} \delta \\ F \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} + [0]u$$

Now, for controllability

$$P = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix}$$

Using MATLAB to calculate,

a) at $\dot{x} = 2\,m/s$

$\text{rank}(P) = 4 \Rightarrow$ full rank

$\therefore$ controllable

b) at $\dot{x} = 5\,m/s$

$\text{rank}(P) = 4 \Rightarrow$ full rank

$\therefore$ controllable

c) at $\dot{x} = 8\,m/s$

$\text{rank}(P) = 4 \Rightarrow$ full rank

$\therefore$ controllable

Now, for observability

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}$$

Using MATLAB to calculate,

a) at $\dot{x} = 2\, m/s$

$rank(O) = 4 \Rightarrow$ full rank

$\therefore$ observable

b) at $\dot{x} = 5\, m/s$

$rank(O) = 4 \Rightarrow$ full rank

$\therefore$ observable

c) at $\dot{x} = 8\, m/s$

$rank(O) = 4 \Rightarrow$ full rank

$\therefore$ observable

For longitudinal control :-

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix}\begin{bmatrix} \delta \\ F \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} + [0]\,u$$

Now, for controllability

$$P = [B \quad AB]$$

$\text{rank}(P) = 2 \Rightarrow$ full rank

$\therefore$ controllable

And for observability

$$O = \begin{bmatrix} C \\ CA \end{bmatrix}$$

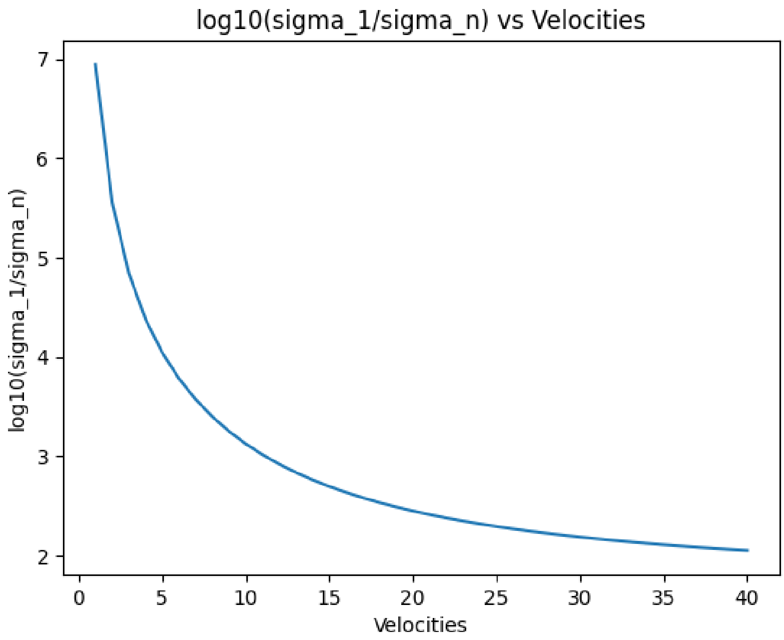
$\text{rank}(O) = 2 \Rightarrow$ full rank

$\therefore$ observable.

The longitudinal controller's controllability & observability don't depend on $\dot{x}$, as $\dot{x}$ is a state variable.

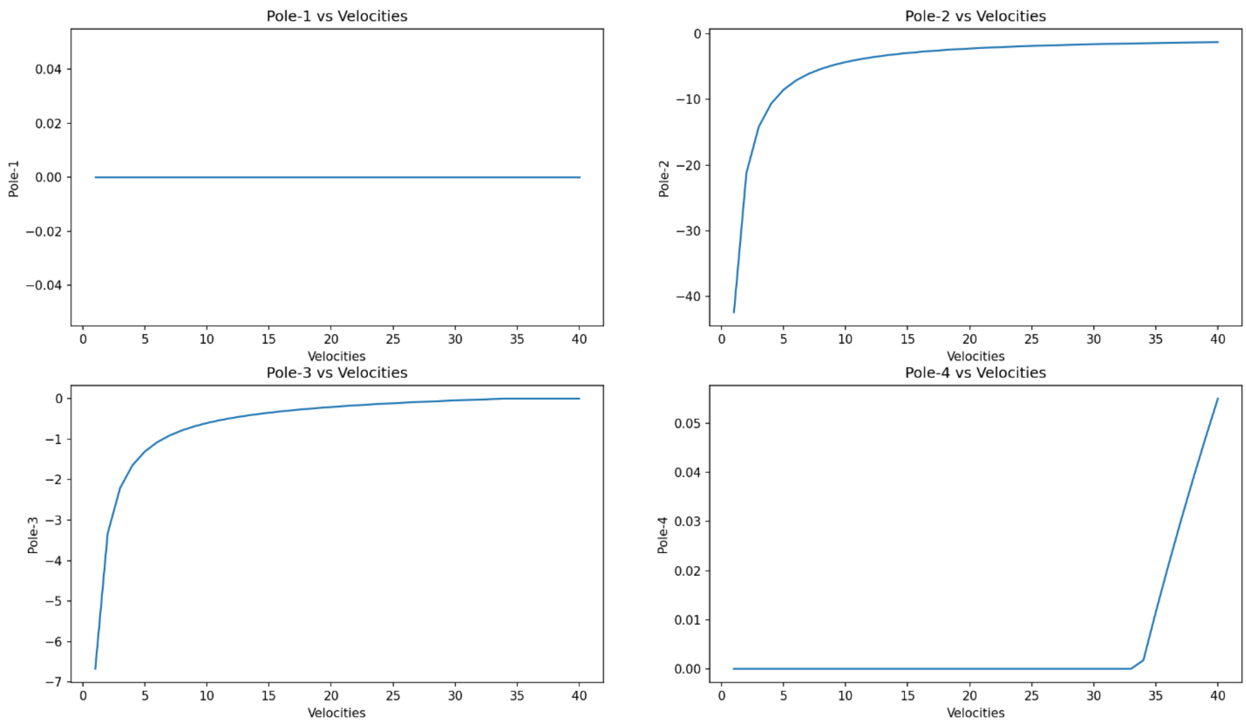
<u>Exercise 1:</u>

Q2. (a)



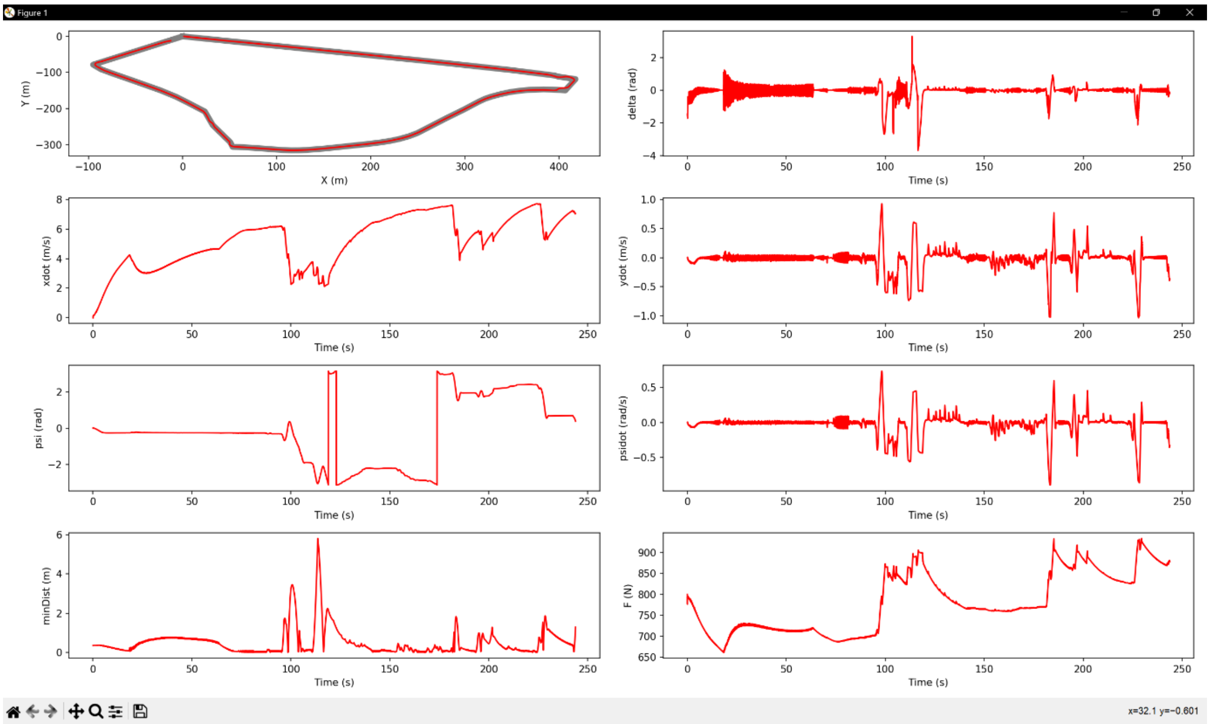
Since the ratio of sigma_1 and sigma_n is decreasing with increasing velocity, it implies that the singular value corresponding to the least controllable state is increasing. Hence, the controllability increases.

Q2. (b)



Apart from Pole-1 (which is constant at the origin), the other poles increase in value and go towards zero. Hence, the stability of the system decreases with increasing velocity.

Exercise 2:




```
Score for average distance: 30.0/30.0
Score for maximum distance: 30.0/30.0
Your time is  243.776
Your total score is : 100.0/100.0
total steps:  243776
maxMinDist:  5.804970845530392
avgMinDist:  0.5007847578327572
INFO: 'main' controller exited successfully.
```