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MCT Project - 1

Exercise 1.

Lateral dynamics :-

$$u = \begin{bmatrix} \delta \\ F \end{bmatrix}, \quad s_1 = \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix}$$

$$\dot{s}_1 = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix}$$

$$= \begin{bmatrix} \dot{y} \\ -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \\ \dot{\psi} \\ \frac{2l_f C_\alpha}{I_z}\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{2l_r C_\alpha}{I_z}\left(-\frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \end{bmatrix}$$

$$A_1 = \frac{\partial f}{\partial S_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{\partial f_2}{\partial \dot{y}} & 0 & \frac{\partial f_2}{\partial \dot{\psi}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{\partial f_4}{\partial \dot{y}} & 0 & \frac{\partial f_4}{\partial \dot{\psi}} \end{bmatrix}$$

where,

$$\frac{\partial f_2}{\partial \dot{y}} = \frac{2C_\alpha}{m}\left(\frac{-\cos\delta}{\dot{x}} - \frac{1}{\dot{x}}\right)$$

$$\frac{\partial f_2}{\partial \dot{\psi}} = -\dot{x} + \frac{2C_\alpha}{m}\left(\frac{-l_f\cos\delta}{\dot{x}} + \frac{l_r}{\dot{x}}\right)$$

$$\frac{\partial f_4}{\partial \dot{y}} = \frac{2l_f C_\alpha}{I_z}\left(-\frac{1}{\dot{x}}\right) - \frac{2\,l_r C_\alpha}{I_z}\left(-\frac{1}{\dot{x}}\right)$$

$$\frac{\partial f_4}{\partial \dot{\psi}} = \frac{2l_f C_\alpha}{I_z}\left(-\frac{l_f}{\dot{x}}\right) - \frac{2\,l_r C_\alpha}{I_z}\left(\frac{l_r}{\dot{x}}\right)$$

$$\therefore A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{2C_\alpha}{m}\left(\frac{-\cos\delta - 1}{\dot{x}}\right) & 0 & -\dot{x} + \frac{2C_\alpha}{m}\left(\frac{l_r - l_f\cos\delta}{\dot{x}}\right) \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-2C_\alpha}{\dot{x} I_z}(l_f - l_r) & 0 & -\frac{2C_\alpha}{\dot{x} I_z}(l_r^2 + l_f^2) \end{bmatrix}$$

$$B_1 = \left(\frac{\partial f}{\partial u}\right) \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m}\left(-\delta\sin\delta + \cos\delta + \sin\delta\left(\frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right)\right) & 0 \\ 0 & 0 \\ \frac{2l_f C_\alpha}{I_z} & 0 \end{bmatrix}$$

$$\therefore \dot{s}_1 = A_1 s_1 + B_1 u$$

Longitudinal dynamics :-

$$u = \begin{bmatrix} \delta \\ F \end{bmatrix}, \quad s_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$\dot{s}_2 = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}$$

$$\dot{s}_2 = \begin{bmatrix} \dot{x} \\ \dot{\varphi}\dot{y} + \frac{1}{m}(F - f mg) \end{bmatrix}$$

$$= \begin{bmatrix} \dot{x} \\ \frac{1}{m}(F - f mg) \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\varphi}\dot{y} \end{bmatrix}$$

Disturbance

$$A_2 = \frac{\partial f}{\partial s_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$B_2 = \frac{\partial f}{\partial u} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix}$$

$$\therefore \dot{s}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} s_2 + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix} u + \begin{bmatrix} 0 \\ \dot{\varphi}\dot{y} \end{bmatrix}$$

Distiorbance

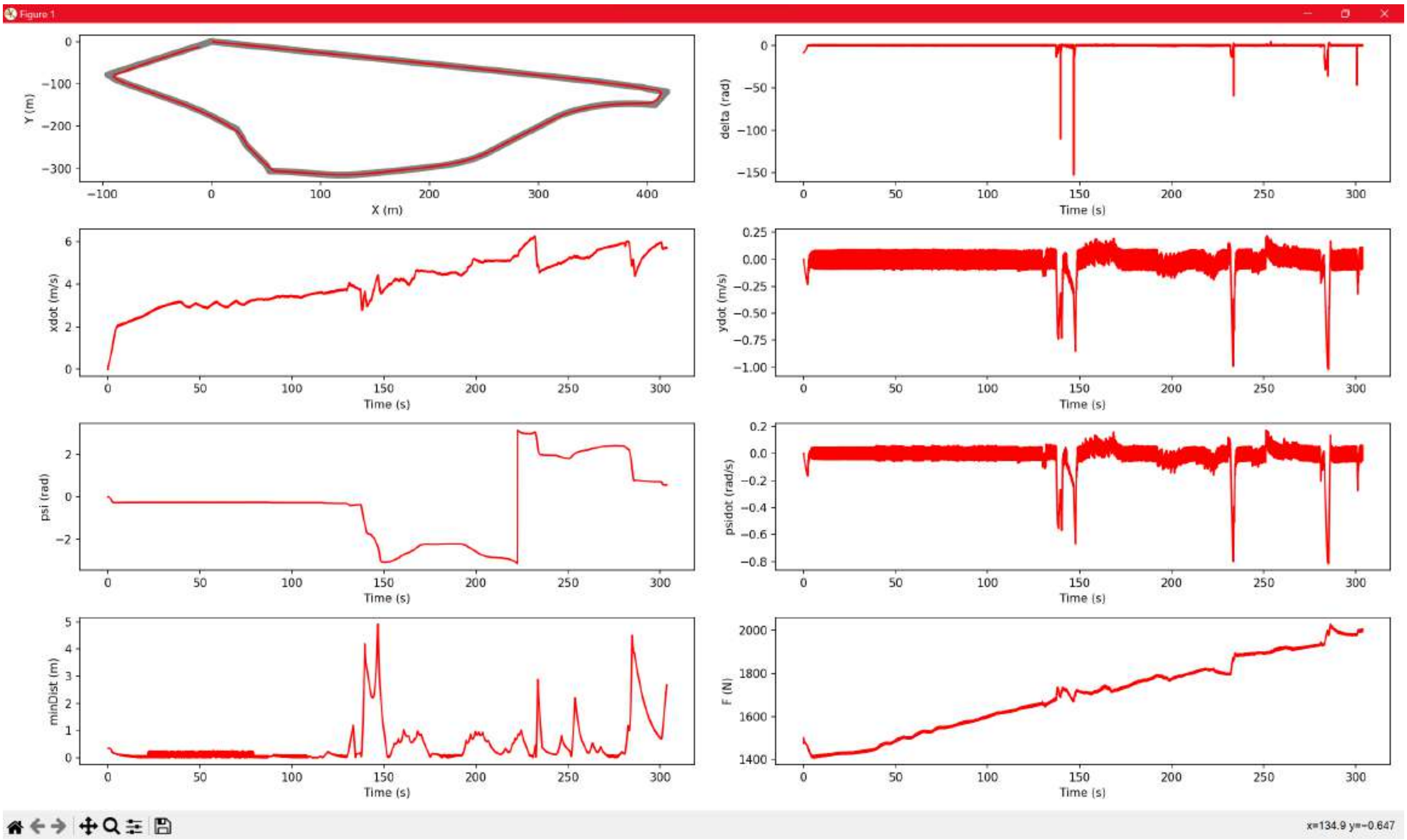

Figure 1
Y (m)
X (m)
delta (rad)
Time (s)
xdot (m/s)
ydot (m/s)
psi (rad)
psidot (rad/s)
minDist (m)
F (N)
x=134.9 y=-0.647