classmate
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ASSIGNMENT - 2

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Q1. A large software development company employs 100 computer programmers. Of them, 45 are proficient in Java, 30 in C#, 20 in Python, six in C# and Java, one in Java and Python, five in C# and python, and just one programmer is proficient in all three languages above.

Determine the number of computer programmers that are not proficient in any of these three languages.

Ans:-

Let U, J, C & P be the sets of all computer programmers, those who are proficient in Java, proficient in C# and proficient in python, respectively.

Expressed mathematically

$|U| = 100 \qquad |C| = 30$

$|J| = 45 \qquad |P| = 20$

Then, $|C \cap J| = 6$, $|J \cap P| = 1$, $|C \cap P| = 5$, $|C \cap J \cap P| = 1$

According to the principle of inclusion, exclusion.

$|(C \cap J \cap P)^c| = |U| - ((|C| + |J| + |P|) + (|C \cap J|) + (|C \cap P|) + (|C \cap P|$
$- (|C \cap J \cap P|))$

$\Rightarrow 100 - (45 + 30 + 20) + (6 + 1 + 5) - 1$

$\Rightarrow 100 - 95 + 12 - 1$

$= 16$

$\therefore$ This 16 out of 100 computer programmers employed by company are not proficient in any of these 3 programming languag

Q2. let A, B, C be arbitrary sets :

(a) Prove that $A \times (B - C) = (A \times B) - (A \times C)$.

Ans. let $(x, y) \in A \times (B - C)$ be arbitrary

$\Rightarrow x \in A, \quad y \in (B - C)$

$\Rightarrow x \in A, \quad \langle y \in B$ but $y \notin C \rangle$

$\Rightarrow \langle x \in A, y \in B \rangle$ but $\langle x \in A, y \notin C \rangle$

$\Rightarrow x \in (A \times B)$

let, $(p, q) \in (A \times B) - (A \times C)$ be arbitrary.

$\Rightarrow (p, q) \in (A \times B)$ but $(p, q) \notin (A \times C)$

$\Rightarrow \langle p \in A, q \in B \rangle$ but $(p, q) \notin (A \times C)$

$\Rightarrow \langle p \in A, q \in B \rangle$ but $\langle p \in A, q \notin C \rangle$

$\Rightarrow p \in A, \quad \langle q \in B$ ~~by~~ but $q \in C \rangle$

$\Rightarrow p \in A, \quad q \in B - C$

$\Rightarrow (p, q) \in A \times (B - C)$

$\Rightarrow (p, q) \in (A \times |B - C|)$

$\cancel{\Rightarrow (p,q)} \in (A \times B) - (A \times C) \subseteq A \times (B - C)$ — (1)

$(A \times (B - C)) \subseteq (A \times B) - (A - C)$ — (2)

from eq. (1) and (2)

$A \times (B - C) = (A \times B) - (A \times C)$

Hence Proved

(b) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

After considering two cases : $x \in A$ & $x \in A$ we consider :

$x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow A \cup (B \cap C) \in (A \cup B) \cap (A \cup C)$

$\therefore A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ — (1)

Now let's consider y is arbitrary element.

$y \in (A \cup B)$ & $y \in (A \cup C)$

$\Rightarrow (A \cup B) \cap (A \cup C) \in A \cup (B \cap C)$ — (2)

from eq ① & ②

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Hence Proved.

Q3. Each of the following defines a relation on the set N of positive integers.

$R : x > y$

$S : x + y = 10$

$T : x + 4y = 10$ for all $(x, y) \in N$

Determine which of the relations are

(a) Reflexive (b) Symmetric (c) Transitive (d) antisymmetric

Ans. Consider I,

$R = \{(x, y) : x > y\}$

(a) Reflexive :- If $(x, x) \in R$ then $x > x$, which is not true for any $x \in N$.

$\therefore$ R is not reflexive.

(b) Symmetric :- Let $(x, y) \in R$

$x > y$

$\Rightarrow y > x$ which is not true for any $x \in N$

$\therefore$ R is not symmetric.

(c) Transitive :- Let $(x, y) \in R$ and $(y, z) \in R$

$x > y$ and $y > z$

$x > z$

$\therefore$ R is transitive.

Consider II,

$S = \{(x, y) : x + y = 10\}$

Let $S = \{(1,9), (2,8), (3,7), (4,6), (5,5), (6,4), (7,3), (8,2), (9,1)\}$
Clearly $(1,1) \notin S$
$\therefore$ S is not reflexive.
Now $(1,9) \in S$ and $(9,1) \in S$, but $(1,1) \notin S$
$\therefore$ S is not transitive.
Now $x+y = 10$. also $y+x = 10$ $y, x \in S$
S is symmetric

Consider III :-
$T = \{(x,y) :- x + 4y = 10\}$
$\therefore T = \{(2,2), (6,1)\}$

(a) Reflexive
Clearly $(1,1) \in T$
$\therefore$ T is not reflexive.

(b) Here $(6,1) \in T$ but $(1,6) \notin T$
$\therefore$ T is not transitive.

(c) $x, y \in T$
$x + 4y = 10$
$(y,z) \in T$
$y + 4z = 10$
$x - 16z = 30$
$(x,z) \notin T$ $\therefore$ T is not transitive.

Q4. Consider an algebraic system $(G, *)$, where G is the set of all non zero real numbers and $*$ is a binary operation defined by $a*b = ab/$
Show that $(G, *)$ is an abelian group.

Ans. To show that $(G, *)$ is an abelian group, we need to verify that it satisfies the 4 group :-

1. <u>Closure</u> :- for any a, b in G $a*b = ab/4$ is also a non-zero real number.

$\therefore (G,*)$ is closed under $*$.

(2) <u>Associativity</u> :- for any a, b, c in G, $(a*b)*c$.
$= (ab/4) * c = abc/16$ and $a*b*c = a*bc/4$
$= abc/16$

Hence $*$ is associativity.

(3) <u>Identity</u> :- there exist an element e in G such that a in G,
$a*e = e*a = a$

let a be any element of G, Then $a*e = ae/4$.
(by the definition of $*$) we want $ae/4$ to equal a for all a in G.

This is only possible if $e=4$.

Thus $e=4$ is identity element in $(G,*)$

(4) <u>Inverse</u> :- for any a in G there exist an element b in G such that $a*b = b*a = e$.

where e is identity.

$a*b = b*a = 4$

$a*b = ab/4 = 4$

This implies $b = 16/a$

(5) Commutative :- For any a, b in G, $a*b$.

$ab/4 = ba/4 = b*a$.

Hence, the operation $*$ is commutative

Since, $(4,*)$ satisfies all the group.

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Q5. (a) If H and K are two subgroups of G then prove $H \cap K$ is also a subgroup of G.

Ans. Let H & K are subgroup of G. Let e be identity element of G.

$$e \in H \cap K$$

$$\Rightarrow a * b \in H \cap K$$

Thus $a \in H \cap K$, then $a \in H$ and $a \in K$.

$$a^{-1} \in H \ , \ a^{-1} \in K$$

$$a^{-1} \in H \cap K$$

Thus, all conditions of subgroup are satisfied.

Hence, $H \cap K$ is also subgroup of G.

$\therefore$ If H and K are subgroup of G then, $H \cap K$ is subgroup of G.

(b) Suppose $f : R \to R$, $g : R \to R$, where R is the set of real numbers given by $f(x) = x^2 - 2$ and $g(x) = x + 4$. find fog and gof, state whether these functions are bijective or not.

Ans.

$$f(x) = x^2 - 2$$

$$g(x) = x + 4$$

$$fog(x)$$

$$f(g(x)) = f(x+4) = (x+4)^2 - 2$$

$$\therefore fog(x) = x^2 + 8x + 14$$

Again, $gof(x)$

$$= g(f(x))$$

$$= g(x^2 - 2) - x^2 - 2 + 4 = x^2 + 2$$

$$gof(x) = x^2 + 2.$$

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Dated.........................

- Injective :

$$fog(x) = x^2+8x+14.$$

Discriminant $= 8 > 0$

So, both roots are real

Suppose roots are a & b

$fog(a) = fog(b) = 0$

but $a \neq b$

So, function is not ~~bt~~ bijective

→ Surjective :

Cleary -20 is the codomain set R has no pre image in domain set R

So, function is not surjective.

Thus, fog is not bijective

Similarly when checking for gof it is also not bijective.