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ASSIGNMENT - 1

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Subject :- Discrete Mathematics and Graph Theory

Q. Show that these conditional statements are tautology, contradiction or Contingency by using truth tables.

(a) $[p \vee (p \vee q)] \rightarrow q$

p	q	$(p \vee q)$	$(p \vee (p \vee q))$	$[p \vee (p \vee q)] \rightarrow q$
T	T	T	T	T
T	F	T	T	F
F	T	T	T	T
F	F	F	F	T

Contingency

(b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \Leftrightarrow (p \rightarrow r)$

p	q	r	$(p \rightarrow q)$	$(p \rightarrow r)$	$(q \rightarrow r)$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$(p \rightarrow q) \wedge (q \rightarrow r) \Leftrightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	F
T	F	F	F	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Contingency

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(c) $[p \wedge (p \to q)] \to q$

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Tautology

(d) $[(p \vee q) \wedge (p \Leftrightarrow r) \wedge (q \to r)] \to r$

p	q	r	$(p \vee q)$	$(p \Leftrightarrow r)$	$(q \to r)$	$(p \vee q) \wedge (p \Leftrightarrow r)$	$(p \vee q) \wedge (p \Leftrightarrow r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \Leftrightarrow r) \wedge (q \to r)] \to r$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	T	F	T	F	F	T
F	T	F	T	T	F	T	F	T
F	F	T	F	F	T	F	F	T
F	F	F	F	T	T	F	F	T

Tautology

Q2. Use truth tables to verify these logical equivalences.

(a) $(p \wedge q) \Leftrightarrow p \equiv p \to q$

p	q	$(p \wedge q)$	$(p \wedge q) \Leftrightarrow p$	$p \to q$
T	T	T	T	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

From column 4 and 5 we get $(p \wedge q) \Leftrightarrow p \equiv p \to q$

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b) $(p \wedge q) \to r \equiv p \to (\sim q \vee r)$

p	q	r	$(p \wedge q)$	$(p \wedge q) \to r$	$\sim q$	$(\sim q \vee r)$	$p \to (\sim q \vee r)$
T	T	T	T	T	F	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	F	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

From column 5 and 8 we get $(p \wedge q) \to r \equiv p \to (\sim q \vee r)$

c) $(p \to \sim q) \wedge (p \to \sim r) \equiv \sim [p \wedge (q \vee r)]$

p	q	r	$\sim q$	$(p \to \sim q)$	$\sim r$	$(p \to \sim r)$	$(p \to \sim q) \wedge (p \to \sim r)$	$(q \vee r)$	$p \wedge (q \vee r)$	$\sim [p \wedge (q \vee r)]$
T	T	T	F	F	F	F	F	T	T	F
T	T	F	F	F	T	T	F	T	T	F
T	F	T	T	T	F	F	F	T	T	F
T	F	F	T	T	T	T	T	F	F	T
F	T	T	F	T	F	T	T	T	F	T
F	T	F	F	T	T	T	T	T	F	T
F	F	T	T	T	F	T	T	T	F	T
F	F	F	T	T	T	T	T	F	F	T

By column 8 or 11 we get $(p \to \sim q) \wedge (p \to \sim r) \equiv \sim [p (q \vee r)]$

Q3. $p \leftrightarrow q$

$q \to r$

$r \vee \sim s$

$\sim s \to q$

$\therefore s$

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Translate the given premises into statement and identify that this argument is valid or not.

Ans. The given premises can be expressed in symbols as:

$p \leftrightarrow q$

$q \rightarrow r$

$r \vee \neg s$

$\neg s \rightarrow q$

To determine whether the argument is valid or not, we need to check whether the conclusion follows logically from the premises.

Using the rules of logical inference, we can simplify the premises as follows:

From $p \leftrightarrow q$, we can infer $p \rightarrow q$ and $q \rightarrow p$.
From $q \rightarrow r$ and $q \rightarrow p$, we can infer $p \rightarrow r$.
From $p \rightarrow r$ and $r \vee \neg s$, we can infer $p \vee \neg s$.
From $\neg s \rightarrow q$, we can infer $\neg q \rightarrow s$.

Now, we can use a proof by contradiction to show that the argument is valid:
Assume $\neg s$.
Then from $p \vee \neg s$, we have p.
From $p \rightarrow q$ and $\neg q \rightarrow s$, we can infer s.
This contradicts our assumption that $\neg s$, so we conclude that s must be true.

Therefore, the argument is valid.

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Q4. A box contains 6 red, 8 green, 10 blue, 12 yellow and 15 white pens. What is the minimum no. of pen we have to choose randomly from the box to ensure that we get 9 pens of same colour?

Ans.

$n = 5$

$K+1 = 9$

$K = 8$

$Kn+1$ (When we put in the formula)

$5 \times 9 + 1$

$= 46.$

∴ 46 is the minimum number of pen we have choose.

Q5. If $(Kn+1)$ pigeons are kept in n pigeon holes where K is a positive integer, what is the average no. of pigeons per pigeons hole?

Ans. average number of pigeons per hole =

$(Kn+1)/n = K+1/n.$

[redacted]

Therefore there will be at least one pigeonhole which will contain at least $(K+1)$ pigeons i.e.,

ceil $[K+1/n]$ and remaining will contain at most K i.e.,

floor $[K+1/n]$ pigeons.

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the minimum number of pigeons required to ensure that at least one pigeon hole contains (k+1) pigeons is (kn+1).

Q6.

(a) How can this English sentence be translated into a logical expression?

"You can access the Internet from campus only if you are a computer Science major or you are not a freshman."

Ans.

Let p be the proposition

"You can access the Internet from campus" and

let q be the proposition

"You are a computer Science major", and

let r be the proposition

"You are a freshman".

Using these propositions, we can translate the given English sentence into a logical expression:

$$p \rightarrow (q \vee \neg r)$$

This logical expression represents the conditional statement "if you can access the internet from campus (p), then you are either a computer Science major (q) or you are not a freshman ($\neg r$)."

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(b) How can this English sentence be translated into a logical expression?

"You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.

Ans. Let p = "You can ride the roller coaster",
q = "You are under 4 feet tall",
r = "You are older than 16 years old".

Using these propositions, we can translate the given English sentence into a logical expression:

$$\neg p \leftrightarrow (q \wedge \neg r)$$

This logical expression represents the biconditional statement "You cannot ride the roller coaster ($\neg p$) if and only if you are under 4 feet tall (q) and not older than 16 years old ($\neg r$)".

Alternatively, we can express the same logical expression as a conditional statement as:

$$p \rightarrow (\neg q \vee r)$$

This logical expression represents the conditional statement "If you can ride the roller coster (p), then you are either not under 4 feet tall ($\neg q$) or you are older than 16 years old. (r)".