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EXPERIMENT - 6

Aim :- Simulate and analyze discrete probability distributions, focusing on Bernoulli, Binomial, and Poisson distributions using R.

Introduction :- The aim of this experiment is to simulate and analyze discrete probability distributions, with a specific focus on the Bernoulli, Binomial and Poisson distributions, using the R programming language. The primary goal is to understand the principles of these distributions, simulate random variables and analyze their properties.

Software Required :-

1. R Statistical Software
2. RStudio.

Relevance of the Experiment :- Understanding and simulating discrete probability distributions is fundamental in various fields such as statistics, data analysis and operations research. This experiment is relevant for individuals who need to model and analyze scenarios where outcomes are discrete and probabilistic. It provides essential skills for handling randomness & making informed decisions based on probability distribution.

Description :- The experiment begins with a brief overview of the Bernoulli, Binomial and Poisson distributions. Participants will then learn to simulate random variables from these distributions, calculate probabilities and analyze

Teacher's Signature:

Aim:- Simulate and analyze discrete probability distributions, focusing on Bernoulli, Binomial and Poisson distributions using R.

```r
# Load required libraries
library(ggplot2)

# Set seed for reproducibility
set.seed(123)

# Define parameters
n_trials <- 1000  # number of trials

# Simulate Bernoulli distribution
bernoulli_data <- rbinom(n = n_trials, size = 1, prob = 0.5)

# Simulate Binomial distribution
binomial_data <- rbinom(n = n_trials, size = 10, prob = 0.5)

# Simulate Poisson distribution
poisson_data <- rpois(n = n_trials, lambda = 5)

# Plot probability mass functions
ggplot() +
  geom_histogram(aes(x = bernoulli_data), binwidth = 1, fill = "cyan", color = "violet", alpha = 0.6) +
  labs(title = "Bernoulli Distribution", x = "Outcome", y = "Frequency") +
  theme_minimal()

ggplot() +
  geom_histogram(aes(x = binomial_data), binwidth = 1, fill = "green", color = "brown", alpha = 0.6) +
  labs(title = "Binomial Distribution", x = "Outcome", y = "Frequency") +
  theme_minimal()

ggplot() +
  geom_histogram(aes(x = poisson_data), binwidth = 1, fill = "red", color = "yellow", alpha = 0.6) +
  labs(title = "Poisson Distribution", x = "Outcome", y = "Frequency") +
  theme_minimal()
```

```
> rbinom(n = n_trials, size = 1, prob = 0.5)
  [1] 0 1 1 1 0 0 1 0 1 1 0 1 0 0 0 1 1 1 0 0 0 1 1 0 1 1 1 1 0 1 1 0 0 1 0 1 0 1 1 1 1 1 1 0 1 0 1 0 0 1 0
 [52] 1 0 0 0 1 1 0 0 1 0 0 0 0 0 1 1 1 0 0 0 1 1 0 0 1 1 1 1 1 0 1 1 0 1 1 0 1 0 1 1 1 0 0 0 0 0 1 0 1 0 1 0 0
[103] 0 1 1 0 1 1 0 1 1 0 0 1 1 0 1 1 1 1 1 1 1 0 0 1 1 1 1 1 0 0 0 0 1 1 0 0 0 0 0 1 1 1 0 0 0 1 0 0 1 0 0 0 1
[154] 1 0 1 0 1 0 1 0 0 0 0 0 1 0 0 0 0 1 0 1 0 0 0 1 0 0 0 0 1 1 1 1 0 0 0 0 0 1 0 1 0 1 0 0 1 1 1 1 0 0 1 1
[205] 1 1 0 1 1 0 0 1 0 1 0 1 1 0 1 0 0 0 0 0 0 0 0 1 0 1 0 0 1 0 1 1 0 0 0 1 0 0 1 0 0 1 0 1 0 1 1 1 0 0 1 0
[256] 1 1 1 1 0 0 0 0 1 0 1 1 0 0 1 0 1 1 0 0 0 1 0 0 0 1 1 0 1 0 1 0 1 1 1 0 0 0 0 0 1 0 0 1 0 1 1 1 1 1 1 1
[307] 1 0 1 0 0 1 0 0 0 1 0 1 1 0 0 0 1 1 0 0 0 1 1 0 0 1 1 1 0 0 1 0 0 0 0 1 0 0 0 1 0 1 0 0 0 0 0 1 1 0 1
[358] 0 1 1 1 0 0 1 0 1 1 1 1 1 0 0 0 1 1 1 0 0 1 1 1 0 0 0 0 1 1 0 1 1 1 1 0 1 0 0 1 1 1 1 1 1 1 0 0 0 1 1 1
[409] 1 1 0 0 1 0 0 1 0 0 1 1 1 1 1 1 0 1 1 0 1 0 0 0 1 0 0 1 0 0 1 1 0 0 1 1 0 0 0 0 0 1 1 0 1 1 0 1 1 0 0
[460] 0 0 1 0 0 1 1 1 1 1 1 1 1 0 0 1 0 1 0 0 0 0 0 1 1 1 0 0 1 0 1 0 0 1 0 0 0 1 1 0 0 0 0 0 1 0 0 0 0 1 1
[511] 1 0 0 1 0 0 0 0 1 1 0 1 0 1 0 0 0 1 1 0 1 1 1 1 0 0 0 0 1 1 0 0 0 0 1 1 0 1 1 0 0 1 0 0 0 0 1 1 0 0 0
[562] 0 0 0 0 1 0 1 1 1 0 1 1 0 1 1 1 1 1 0 0 0 1 1 1 1 0 1 1 1 1 0 1 1 0 0 0 0 1 1 1 1 0 0 1 0 1 1 0 0 1 0
[613] 1 1 1 1 0 0 0 1 1 0 1 1 1 1 0 1 0 1 1 1 1 0 1 1 1 0 0 1 0 0 0 1 1 0 1 1 1 1 0 0 1 1 0 0 1 0 1 1 1 1 1
[664] 0 0 0 0 1 0 1 0 1 1 0 0 0 0 1 1 0 0 0 0 1 0 1 1 1 1 0 0 0 0 0 0 1 0 0 0 1 0 1 1 0 1 1 0 1 1 1 1 0 1 0
[715] 0 0 0 0 0 1 1 0 1 0 1 1 0 0 0 1 1 1 1 1 0 1 1 1 0 1 0 0 0 1 0 1 1 0 1 1 1 1 1 1 1 1 1 1 0 0 0 1 0 1 0
[766] 0 1 1 0 1 1 1 1 1 0 0 0 0 0 1 1 1 0 0 0 1 0 1 1 0 0 0 0 0 1 1 1 0 1 1 1 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 1
[817] 0 1 1 0 0 0 0 1 1 1 1 0 0 1 0 0 0 0 0 1 0 1 1 0 1 0 1 0 0 0 1 0 1 1 1 0 1 1 0 0 0 0 1 1 0 0 1 1 0 1 1 1
[868] 0 0 0 1 1 1 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 0 0 1 1 0 0 0 1 1 1 1 0 0 0 1 1 0 0 1 1 0 0 0 1 1
[919] 0 0 1 0 0 0 0 0 1 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1 1 0 1 0 1 1 0 1 1 0 1 1 0 1 0 1 1 0 0 1 1 0 0 0
[970] 0 1 0 1 1 0 1 0 1 0 0 1 0 1 1 0 0 1 0 0 0 1 0 1 1 0 0 1 0 1 0
> rbinom(n = n_trials, size = 10, prob = 0.5)
  [1]  4  9  6  6  4  3  4  5  6  4  4  8  5  8  7  8  3  4  7  6  8  8  6  5  6  4  4  7  6  6  4  6  5  4
 [35]  4  7  6  2  6  7  7  3  8  7  4  7  7  5  3  7  4  4  6  0  4  3  6  7  4  5  4  6  5  4  4  6  2  5
 [69]  5  5  6  5  5  7  4  2  6  6  7  3  1  5  7  5  2  5  4  5  3  5  4  5  7  5  6  3  6  6  2  2  5  6
[103]  6  4  6  5  3  5  6  6  3  3  8  3  4  5  6  7  6  5  3  7  7  4  5  5  6  7  5  6  6  4  5  4  7  6
[137]  4  9  6  4  6  4  6  4  4  3  5  1  4  4  6  3  4  5  8  4  6  3  5  5  7  4  5  5  6  2  6  6  6  5
[171]  4  4  3  4  2  5  4  4  6  4  5  6  3  3  8  8  6  2  8  6  7  4  4  5  4  6  3  7  6  7  8  4  4  3
[205]  3  3  6  7  3  6  8  4  6  5  2  5  6  6  5  7  1  5  5  6  4  5  4  6  5  4  6  5  8  4  1  4  6  5
[239]  3  3  7  3  6  5  7  5  3  5  6  5  5  5  7  4  5  6  7  5  4  5  4  3  5  2  3  4  4  4  7  4  8  5
[273]  7  4  4  3  6  4  5  4  3  4  5  6  4  5  3  4  6  5  4  6  5  3  3  8  5  5  6  6  6  7  3  4  7  4
[307]  7  4  6  5  5  5  5  5  5  4  6  6  5  4  6  7  4  5  7  5  5  5  2  5  5  5  6  6  7  6  5  4  2  3
[341]  5  5  3  4  3  4  3  5  4  3  7  4  4  4  4  6  5  3  6  7  4  6  4  7  4  6  3  7  6  7  2  2  4  4
[375]  3  6  2  6  5  6  7  5  8  4  4  8  4  6  4  6  6  4  1  5  5  5  4  5  4  4  2  5  5  8  2  6  5  1
> rpois(n = n_trials, lambda = 5)
  [1]  4  4  4  5  2  6  3  7 12  2  5  6  7  7  4  9  6  4  5  9  8  3  4  4  2  9  7  6  4  4  8  4  5  5
 [35]  8  5  6  3  4  8  7  7  4  2  9  5  5  3  1  3  4  5  2  3  1  3  5  4  6  9  3 10  2  6  4  4  2  3
 [69]  6  4  6  3  2  6  4  4  7  0  4  2  5 10  6  8  6  5  8  5  7  5  2  5  4  5  6  3  5  2  1  2  3  3
[103]  7  4  5  5  2  7  5  7  4  6  6  6 10  4  2  4  4  4  5  6  1  3  5  8  4  4 11  5  0  2  8  5  7  7
[137]  7  5  5  5  3  6  7  7  3  4  6  2  6  6  7  3  4  7  4  7  6  3  6  2  8  8  6  9  5  7  7  4  4  3
[171]  7  6  3  7  6  5  2  4  5  9  2  5  4  5  1  7  3  5  3  9  6  4  4  6  3  4  8  5  9  7  7  5  2  5
[205]  8  1  2  6  5 11  7  7  4 12  4  9  5  6  6  7  6  0  2  8  4  6  5  9  6  3  4  6  3  5  2  9  3  4
[239]  5  5  3  3  6  4  6  6  2  5  6  4  6  8  4  4  5  2  4  3  4  4  3  2  6  4  7  5  4  2  3  5  3  5
[273]  4  7  4  5  3  4  4  6  2  6  5  4  6  4  2  4  2  3 11  2  5  6  5  4  4  5  6 10  5  5  4  3  4  4
[307]  4  5  4  4  3  2  7  5  5  6  3  5  6  1  6  6  5  7  6  5  3  6  4  2  3  7  2  5  4  5  5  9  3  4
[341]  4  7  1  1  4  4  8  5  3  2  2  7  4 10  8  4  8  6 12  2  7 10  6  7 10  4  4  3  4  4  5  4  2  6
[375]  1  3  9  5  4  0  4  8  3  4  5  5  6  7  4  3  5  4  7  3  5  2  4  7  5  5  3  3  5  6  3  3  8  3
[409]  0  4  6  5  4  3 10  6  4  2  6  5  6  6  5  2  6  1 10  9  7  3  5  4  9  7  6  7  6  4  8  2  5  4
[443]  7  6  4  7  5  7  5  6  3  4 10 11  4  5  4 11  8  5  5  4  8  8  4  7  5  6  4  2  3  1  4  6  3  4
[477]  7  7  4  6  5  3  3  2  5  9  2  6  0 11 10  6  7  5  6  6  1  5  6  1  6  7  5  3  7  4  8  6  5  5
[511] 10  6  7  4  8  1  4  6  5  3  3  6  6  4  5  4  5  7  3  6  4  5 10  5  3  8  4  7  2  2  2  7  4  8
```

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the properties of each distribution. Emphasis will be placed on understanding the parameters that govern each distribution and how they influence the shape and characteristics of the distribution.

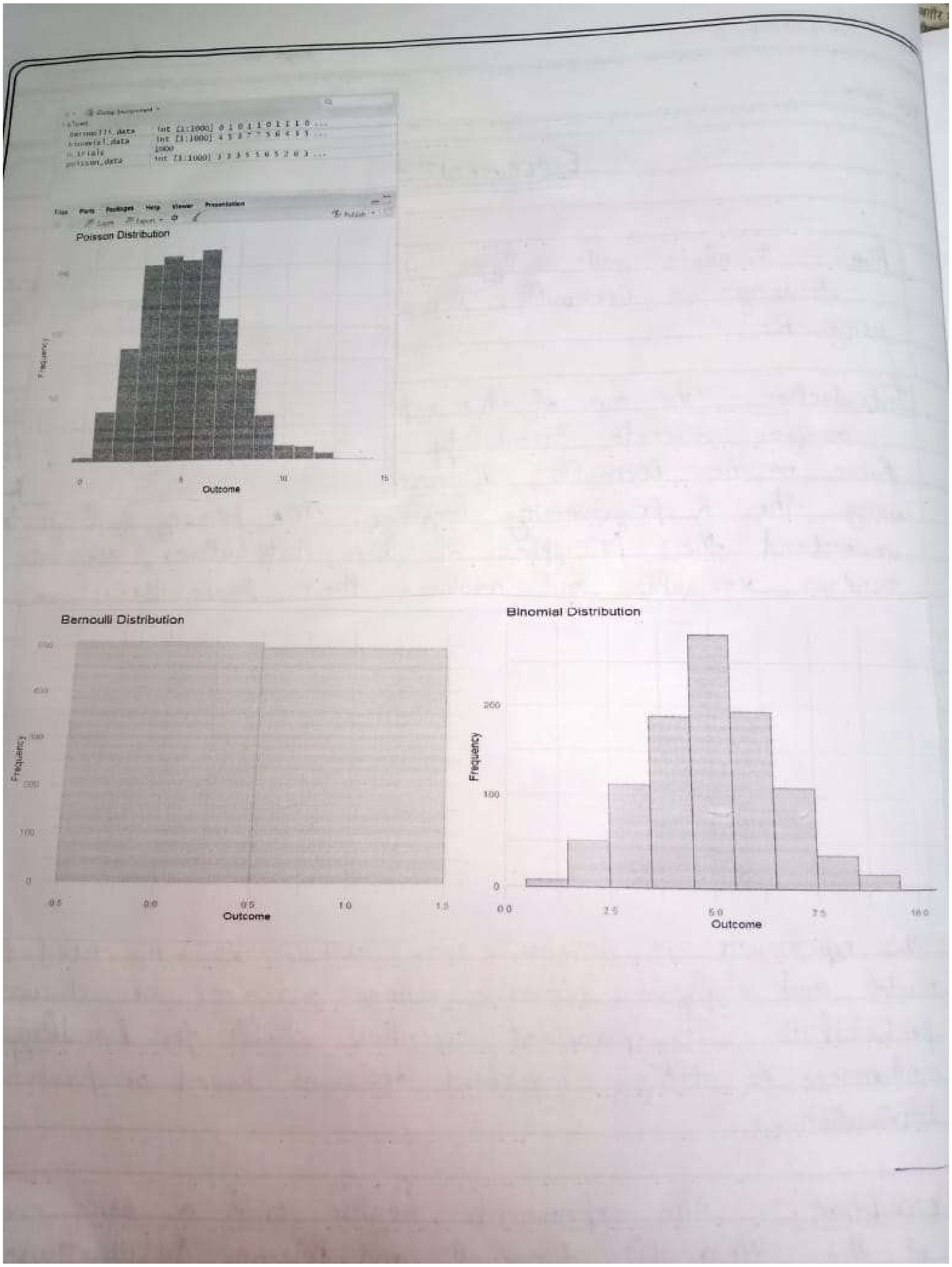
Pseudocode / Steps :-

1. Select the probability distribution to simulate (Bernoulli, Binomial or Poisson).
2. Specify the parameters for the chosen distribution (eg. probability of success for Bernoulli, number of trials for Binomial)
3. Generate random variables based on the chosen distribution using appropriate R functions.
4. Calculate and visualize probability mass functions (PMFs) or cumulative distribution functions (CDFs) for the simulated data.
5. Analyze and interpret the results, comparing them with theoretical expectations.

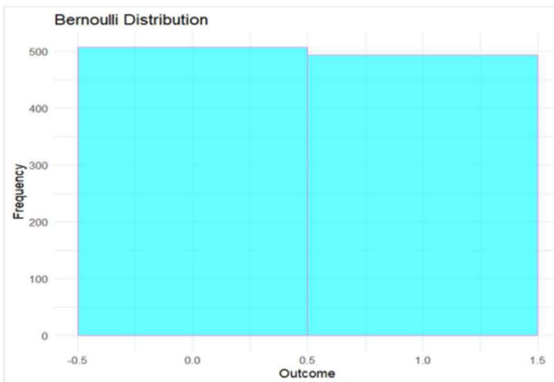
Learning Outcomes :-

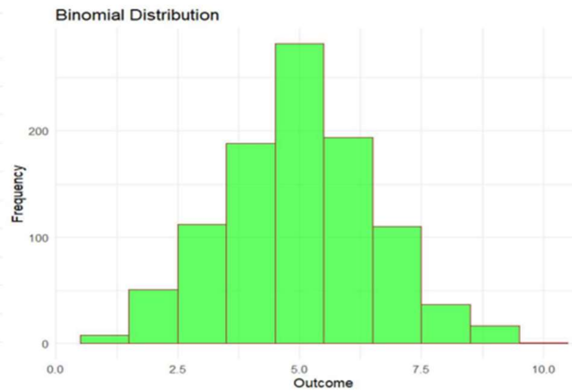
1. Understanding the principles of discrete probability distributions: Bernoulli, Binomial, and Poisson.
2. Proficiency in simulating random variables from these distributions using R.
3. Analyzing and interpreting the properties of simulated data
4. Comparing simulated results with theoretical expectation.
5. Gaining insights into practical applications of discrete probability distribution.

Teacher's Signature:



SCREENSHOTS:





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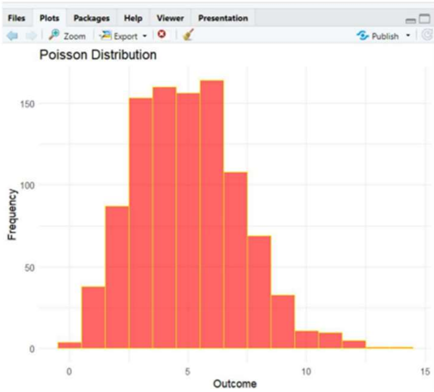
Values

bernoulli_data	int [1:1000] 0 1 0 1 1 0 1 1 1 0 ...
binomial_data	int [1:1000] 4 5 3 7 7 5 6 4 3 5 ...
n_trials	1000
poisson_data	int [1:1000] 3 3 3 5 5 6 5 2 0 3 ...




```r
# Load required libraries
library(ggplot2)

# Set seed for reproducibility
set.seed(123)

# Define parameters
n_trials <- 1000  # Number of trials

# Simulate Bernoulli distribution
bernoulli_data <- rbinom(n = n_trials, size = 1, prob = 0.5)

# Simulate Binomial distribution
binomial_data <- rbinom(n = n_trials, size = 10, prob = 0.5)

# Simulate Poisson distribution
poisson_data <- rpois(n = n_trials, lambda = 5)

# Plot probability mass functions
ggplot() +
  geom_histogram(aes(x = bernoulli_data), binwidth = 1, fill = "cyan", color = "violet", alpha = 0.6) +
  labs(title = "Bernoulli Distribution", x = "Outcome", y = "Frequency") +
  theme_minimal()

ggplot() +
  geom_histogram(aes(x = binomial_data), binwidth = 1, fill = "green", color = "brown", alpha = 0.6) +
  labs(title = "Binomial Distribution", x = "Outcome", y = "Frequency") +
  theme_minimal()

ggplot() +
  geom_histogram(aes(x = poisson_data), binwidth = 1, fill = "red", color = "yellow", alpha = 0.6) +
  labs(title = "Poisson Distribution", x = "Outcome", y = "Frequency") +
  theme_minimal()
```

```
> rbinom(n = n_trials, size = 1, prob = 0.5)
   [1] 0 1 1 1 0 0 1 0 1 1 0 1 0 0 0 1 1 1 0 0 0 1 1 0 1 1 1 1 0 1 1 0 0 1 0 1 0 1 1 1 1 1 1 0 1 0 1 0 0 1 0
  [52] 1 0 0 0 1 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 1 1 1 1 0 1 1 0 1 1 0 1 0 1 1 1 0 0 0 0 0 1 0 1 0 1 0 0
 [103] 0 1 1 0 1 1 0 1 1 0 0 1 1 0 1 1 1 1 1 1 0 0 1 1 1 1 1 0 0 0 0 1 1 0 0 0 0 1 1 1 0 0 0 1 0 0 1 0 0 0 1
 [154] 1 0 1 0 1 0 1 0 0 0 0 1 0 0 0 0 1 0 1 0 0 0 1 0 0 0 0 1 1 1 1 0 0 0 0 0 1 0 1 0 1 0 0 1 1 1 1 0 0 1 1
 [205] 1 1 0 1 1 0 0 1 0 1 0 1 1 0 1 0 0 0 0 0 0 0 1 0 1 0 0 1 0 1 1 0 0 0 1 0 0 1 0 0 1 0 1 0 1 1 1 0 0 1 0
 [256] 1 1 1 1 0 0 0 0 1 0 1 1 0 0 1 0 1 1 0 0 0 1 0 0 0 1 1 0 1 0 1 0 1 1 1 0 0 0 0 1 0 0 1 0 1 1 1 1 1 1 1
 [307] 1 0 1 0 0 1 0 0 0 1 0 1 1 0 0 0 1 1 0 0 0 1 1 0 0 1 1 1 0 0 1 0 0 0 0 1 0 0 0 1 0 1 0 0 0 0 0 1 1 0 1
 [358] 0 1 1 1 0 0 1 0 1 1 1 1 1 0 0 0 1 1 1 0 0 1 1 1 0 0 0 0 1 1 0 1 1 1 1 0 1 0 0 1 1 1 1 1 1 0 0 0 1 1 1
 [409] 1 1 0 0 1 0 0 1 0 0 1 1 1 1 1 1 0 1 1 0 1 0 0 0 1 0 0 1 0 0 1 1 0 0 1 1 0 0 0 0 0 1 1 0 1 1 0 1 1 0 0
 [460] 0 0 1 0 0 1 1 1 1 1 1 1 1 0 0 1 0 1 0 0 0 0 0 1 1 1 0 0 1 0 1 0 0 1 0 0 0 1 1 0 0 0 0 0 1 0 0 0 0 1 1
 [511] 1 0 0 1 0 0 0 0 1 1 0 1 0 1 0 0 0 1 1 0 1 1 1 1 0 0 0 0 1 1 0 0 0 0 1 1 0 1 1 0 0 1 0 0 0 0 1 1 0 0 0
 [562] 0 0 0 0 1 0 1 1 1 0 1 1 0 1 1 1 1 1 0 0 0 1 1 1 1 0 1 1 1 1 0 1 1 0 0 0 0 1 1 1 1 0 0 1 0 1 1 0 0 1 0
 [613] 1 1 1 1 0 0 0 1 1 0 1 1 1 1 0 1 0 1 1 1 1 0 1 1 1 0 0 1 0 0 0 1 1 0 1 1 1 1 0 0 1 1 0 0 1 0 1 1 1 1 1
 [664] 0 0 0 0 1 0 1 0 1 1 0 0 0 0 1 1 0 0 0 0 1 0 1 1 1 1 0 0 0 0 0 0 1 0 0 0 1 0 1 1 0 1 1 0 1 1 1 1 0 1 0
 [715] 0 0 0 0 0 1 1 0 1 0 1 1 0 0 0 1 1 1 1 1 0 1 1 1 0 1 0 0 0 1 0 1 1 0 1 1 1 1 1 1 1 1 1 1 0 0 0 1 0 1 0
 [766] 0 1 1 0 1 1 1 1 1 0 0 0 0 1 1 1 0 0 0 1 0 1 1 0 0 0 0 1 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 1
 [817] 0 1 1 0 0 0 0 1 1 1 1 0 0 1 0 0 0 0 1 0 1 1 0 1 0 1 0 0 0 1 0 1 1 1 0 1 1 0 0 0 0 1 1 0 0 1 1 0 1 1 1
 [868] 0 0 0 1 1 1 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 1 1 1 0 0 0 0 1 1 0 0 0 1 1 1 1 0 0 0 1 1 0 0 1 1 0 0 0 1 1
 [919] 0 0 1 0 0 0 0 1 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 1 1 0 1 0 1 1 0 1 1 0 1 1 0 1 0 1 1 0 0 1 1 0 0 0
 [970] 0 1 0 1 1 0 1 0 1 0 0 1 0 1 1 0 0 1 0 0 0 1 0 1 1 0 0 1 0 1 0
> rbinom(n = n_trials, size = 10, prob = 0.5)
   [1]  4  9  6  6  4  3  4  5  6  4  4  8  5  8  7  8  3  4  7  6  8  8  6  5  6  4  4  7  6  6  4  6  5  4
  [35]  4  7  6  2  6  7  7  3  8  7  4  7  7  5  3  7  4  4  6  9  4  3  6  7  4  5  4  6  5  4  4  6  2  5
  [69]  5  5  6  5  5  7  4  2  6  6  7  3  1  5  7  5  2  5  4  5  3  5  4  5  7  5  6  3  6  6  2  2  5  6
 [103]  6  4  6  5  3  5  6  6  3  3  8  3  4  5  6  7  6  5  3  7  7  4  5  5  6  7  5  6  6  4  5  4  7  6
 [137]  4  9  6  4  6  4  6  4  4  3  5  1  4  4  6  3  4  5  8  4  6  3  5  5  7  4  5  5  6  2  6  6  6  5
 [171]  4  4  3  4  2  5  4  4  6  4  5  6  3  3  8  8  6  2  8  6  7  4  4  5  4  6  3  7  6  7  8  4  4  3
 [205]  3  5  6  7  3  6  8  4  6  5  2  5  6  6  5  7  1  5  5  6  4  5  4  6  5  4  6  5  8  4  1  4  6  5
 [239]  3  3  7  3  6  5  7  5  3  5  6  5  5  5  7  4  5  6  7  5  4  5  4  3  5  2  3  4  4  4  7  4  8  5
 [273]  7  4  4  3  6  4  5  4  3  4  5  6  4  5  3  4  6  5  4  6  5  3  3  8  5  5  6  6  6  7  3  4  7  4
 [307]  7  4  6  5  5  5  5  5  5  4  6  6  5  4  6  7  4  5  7  5  5  5  2  5  5  5  6  6  7  6  5  4  2  3
 [341]  5  5  3  4  3  4  3  5  4  3  7  4  4  4  4  6  5  3  6  7  4  6  4  7  4  6  3  7  6  7  2  2  4  4
 [375]  3  6  2  6  5  6  7  5  8  4  4  8  4  6  4  6  6  4  1  5  5  5  4  5  4  4  2  5  5  8  2  6  5  1
```

```
> rpois(n = n_trials, lambda = 5)
   [1]  4  4  4  5  2  6  3  7 12  2  5  6  7  7  4  9  6  4  5  9  8  3  4  4  2  9  7  6  4  4  8  4  5  5
  [35]  8  5  6  3  4  8  7  7  4  2  9  5  5  3  1  3  4  5  2  3  1  3  5  4  6  9  3 10  2  6  4  4  2  3
  [69]  6  4  6  3  2  6  4  4  7  0  4  2  5 10  6  8  6  5  8  5  7  5  2  5  4  5  6  3  5  2  1  2  3  3
 [103]  7  4  5  5  2  7  5  7  4  6  6  6 10  4  2  4  4  4  5  6  1  3  5  8  4  4 11  5  0  2  8  5  7  7
 [137]  7  5  5  5  3  6  7  7  3  4  6  2  6  6  7  3  4  7  4  7  6  3  6  2  8  8  6  9  5  7  7  4  4  3
 [171]  7  6  3  7  6  5  2  4  5  9  2  5  4  5  1  7  3  5  3  9  6  4  4  6  3  4  8  5  9  7  7  5  2  5
 [205]  8  1  2  6  5 11  7  7  4 12  4  9  5  6  6  7  6  0  2  8  4  6  5  9  6  3  4  6  3  5  2  9  3  4
 [239]  5  5  3  3  6  4  6  6  2  5  6  4  6  8  4  4  5  2  4  3  4  4  3  2  6  4  2  5  4  2  3  5  3  5
 [273]  4  7  4  5  3  4  4  6  2  6  5  4  6  4  2  4  2  3 11  2  5  6  5  4  4  5  6 10  5  5  4  3  4  4
 [307]  4  5  4  4  3  2  7  5  5  6  3  5  6  1  6  6  5  7  6  5  3  6  4  2  3  7  2  5  4  5  5  9  3  4
 [341]  4  7  1  1  4  4  8  5  3  2  2  7  4 10  6  4  8  6 12  2  7 10  6  7 10  4  4  3  4  4  5  4  2  6
 [375]  1  3  9  5  4  0  4  8  3  4  5  5  6  7  4  3  5  4  7  3  5  2  4  7  5  5  3  3  5  6  3  3  8  3
 [409]  0  4  6  5  4  3 10  6  4  2  6  5  6  6  5  2  6  1 10  9  7  3  5  4  9  7  6  7  6  4  8  2  5  4
 [443]  7  6  4  7  5  7  5  6  3  4 10 11  4  5  4 11  8  5  5  4  8  8  4  7  5  6  4  2  3  1  4  6  3  4
 [477]  7  7  4  6  5  3  3  2  5  9  2  6  0 11 10  6  7  5  6  6  1  5  6  1  6  7  5  3  7  4  8  6  5  5
 [511] 10  6  7  4  8  1  4  6  5  3  3  6  6  4  5  4  5  7  3  6  4  5 10  5  3  8  4  7  2  2  2  7  4  8
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