Simulation Step 8 Monte Carlo Optimisation

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1 Prelude

[1]:
```
import matplotlib.pyplot as plt
import pulp
import math
import random
import pandas as pd
import numpy as np
import time
import simpy
```

2 Utilities (as before)

2.1 Points and Distances

[2]:
```
def dist(p1, p2):
    (x1, y1) = p1
    (x2, y2) = p2
    return int(math.sqrt((x1-x2)**2+(y1-y2)**2))
```

2.2 PlotMap

[3]:
```
def plotMap(G, T=[], P=[], W=None,
            style='r-o', lw=1, ms=3,
            styleT='go', msT=5,
            styleP='b-o', lwP=3, msP=1,
            stylePT='go', msPT=7,
            styleW='bo', msW=9,
            text=None, grid=False):
    fig = plt.gcf()
    fig.set_size_inches(6, 6)
    V, E = G

    if not grid:
        plt.axis('off')
    plt.plot( [ p[0] for p in V ], [ p[1] for p in V ], 'ro', lw=lw, ms=ms)
    for (p, q) in E:
        plt.plot( [ p[0], q[0] ], [ p[1], q[1] ], 'r-o', lw=lw, ms=ms)
    for t in T:
        plt.plot( [ t[0] ], [ t[1] ],
                  styleT, ms=msT)
    plt.plot( [ p[0] for p in P ],
              [ p[1] for p in P ],
             styleP, lw=lwP, ms=msP)
    for p in P:
        if p in T:
            plt.plot( [ p[0] ], [ p[1] ],
                      stylePT, ms=msPT)
    if W is not None:
        plt.plot( [ W[0] ], [ W[1] ],
                      styleW, ms=msW)
    if text is not None:
        maxX = max([p[0] for p in V])
        plt.text(0.8*maxX, 0, text)
    if grid:
        plt.grid()
    plt.show()
```

2.3 Add Targets

[4]:
```
def addTargets(M, T):
    V, E = M
    E = E.copy()
    V = V.copy()
    for t in T:
        minD = math.inf
        minE = None
        for e in E:
            P, Q = e
            distT = dist(P, t)+dist(t, Q)-dist(P, Q)
            if distT < minD:
                minD = distT
                minE = e
        P, Q = minE
        E.remove( (P, Q) )
        E.append( (P, t) )
        E.append( (t, Q) )
        V.append(t)
    return V, E
```

2.4 Generate Warehouse Location

[5]:
```
def generateWarehouseLocation(M):
    V, _ = M
    W = random.sample(V, k=1)[0]
    return W
```

[ ]:
```

```

2.5 Time Handling

Convention: In this project we measure time in seconds. The simulation will start at 0:00. Time related methods will be added as they are needed.

`timestamp(t)` generates a timestamp string in the form `[dd] hh:mm:ss.d`

[6]:
```
    def timestamp(t):
        day = int(t)//(24*3600)
        t = t - day*24*3600
        hour = int(t)//3600
        t = t - hour*3600
        mins = int(t)//60
        t = t - mins*60
        secs = int(math.floor(t))
        t = int(math.floor((t - secs)*10))
        return f"[{day:2d}] {hour:02d}:{mins:02d}:{secs:02d}.{t:1d}"
```

[7]:
```
timestamp(24*3600*3+17*3600+615.1)
```

[7]:
```
'[ 3] 17:10:15.0'
```

[8]:
```
def nextHour(env, hour):
    beginningOfDay = int(env.now//(24*3600))*24*3600
    timeOfDay = env.now-beginningOfDay
    if hour*3600 > timeOfDay:
        return hour*3600 - timeOfDay
    else:
        return hour*3600 + 24*3600 - timeOfDay
```

[9]:
```
def day(now):
    return int(now//(24*3600))
```

2.6 Plotting Routines

[10]:
```
import scipy.stats as stats

def histplot(data, title="", xlabel="",
             width=None, height=None):

    minx = min(data)
    maxx = max(data)
      = np.mean(data)
      = np.std(data)

    fig = plt.figure()
    fig.set_figwidth(width if width is not None else 4)
    fig.set_figheight(height if height is not None else 2.5)
    ax = fig.gca()

    bins=(maxx-minx)//5+1
    hist=plt.hist(data, density=True, bins=bins)
    plt.xlabel(xlabel)
    plt.ylabel('Density')
    plt.title(title)

    x = np.linspace(minx, maxx, 100)
    y = [ stats.norm(loc= , scale= ).pdf(p) for p in x]
    ax.plot(x, y, lw=1, color='red')
    ax.axvline(x= , color='red')
    maxy = max(max(y), max(hist[0]))
    ax.text(maxx, maxy,
            f' ={ :2.2f}\n ={ :2.2f}',
            ha='right', va='top',
            color='red', fontsize=12)
```

```
    ax.grid(True)
    plt.show()
```

[11]:
```
def dailyPlot(data,
              title="", ylabel="",
              width=None, height=None):

    days = len(data)

    fig = plt.figure()
    fig.set_figwidth(width if width is not None else 6)
    fig.set_figheight(height if height is not None else 2)

    ax = fig.gca()
    diff = (max(data)-min(data))*0.1
    ymin = int(math.floor(min(data)-diff))
    ymax = int(math.ceil(max(data)+diff))
    ax.set_xlim(-1, days)
    ax.set_ylim(ymin, ymax)
    ax.grid(True)

    ms = 2 if len(data)>100 else 5
    lw = 0.5 if len(data)>100 else 1

    x = np.arange(0, len(data))
    y = np.array([ y for y in data ])
    b, m = np.polynomial.polynomial.polyfit(x, y, 1)

    plt.plot(x, y, 'bo-', linewidth=lw, markersize=ms)
    plt.plot(x, m*x+b, 'r-')

    plt.xlabel('Day')
    plt.ylabel(ylabel)
    plt.title(title)
    plt.show()
```

3 Finding Shortest Path (as before)

[12]:
```
def dist(p1, p2):
    (x1, y1) = p1
    (x2, y2) = p2
    return int(math.sqrt((x1-x2)**2+(y1-y2)**2))
```

[13]:
```
def pathLength(P):
    return 0 if len(P)<=1 else \
            dist(P[0], P[1])+pathLength(P[1:])
```

[14]:
```
def shortestPath(M, A, B):

    def h(p):
        return pathLength(p)+dist(p[-1],B)

    # candidates C are pairs of the path so far and
    # the heuristic function of that path,
    # sorted by the heuristic function, as maintained by
    # insert function
    def insert(C, p):
        hp = h(p)
        c = (p, hp)
        for i in range(len(C)):
            if C[i][1]>hp:
                return C[:i]+[c]+C[i:]
        return C+[c]

    V, E = M
    assert(A in V and B in V)
    C = insert([], [A])

    while len(C)>0:
        # take the first candidate out of the list of candidates
        path, _ = C[0]
        C = C[1:]
        if path[-1]==B:
            return path
        else:
            for (x, y) in E:
                if path[-1]==x and y not in path:
                    C = insert(C, path+[y])
                elif path[-1]==y and x not in path:
                    C = insert(C, path+[x])
    return None
```

4 Finding Shortest Delivery Route (as before)

[15]:
```
def createTables(M, T):

    def reverse(P):
        return [ P[-i] for i in range(1,len(P)+1) ]

    def index(x, L):
        for i in range(len(L)):
            if x==L[i]:
                return i
```

```
        return None

    n = len(T)
    d = [ [ math.inf for t in T ] for t in T ]
    p = [ [ None for t in T ] for t in T ]
    for i in range(n):
        d[i][i] = 0
        p[i][i] = [ T[i] ]
    for i in range(n):
        for j in range(n):
            if p[i][j] is None:
                s = shortestPath(M, T[i], T[j])
                d[i][j] = d[j][i] = pathLength(s)
                p[i][j] = s
                p[j][i] = reverse(s)
                for m in range(len(s)-1):
                    smi = index(s[m], T)
                    if smi is None:
                        continue
                    for l in range(m+1, len(s)):
                        sli = index(s[l], T)
                        if sli is None:
                            continue
                        sub = s[m:l+1]
                        if p[smi][sli] is None:
                            p[smi][sli] = sub
                            p[sli][smi] = reverse(sub)
                            d[smi][sli] = d[sli][smi] = pathLength(sub)
    return d,p
```

[16]:
```
def roundtrips(x, n):

    def isElem(x, l):
        for i in range(len(l)):
            if l[i]==x:
                return True
        return False

    def startpoint(trips):
        for i in range(n):
            for t in trips:
                if isElem(i, t):
                    break
            else:
                return i

    def totalLength(trips):
```

```
        s=0
        for i in range(0, len(trips)):
            s += len(trips[i])-1
        return s

    trips = []
    while totalLength(trips)<n:
        start = startpoint(trips)
        trip = [ start ]
        i = start
        while len(trip) < n-totalLength(trips):
            for j in range(0, n):
                if pulp.value(x[i][j])==1:
                    trip.append(j)
                    i=j
                    break
            if pulp.value(x[trip[-1]][start])==1:
                trip.append(start)
                break
        trips.append(trip)
    return sorted(trips, key=lambda t: len(t), reverse=True)

def createLoop(M, T):
    D, P = createTables(M, T)
    n = len(T)
    # create variables
    x = pulp.LpVariable.dicts("x", ( range(n), range(n) ),
                              lowBound=0, upBound=1, cat=pulp.LpInteger)
    # create problem
    prob = pulp.LpProblem("Loop",pulp.LpMinimize)
    # add objective function
    prob += pulp.lpSum([ D[i][j]*x[i][j]
                         for i in range(n) for j in range(n) ])
    # add constraints
    constraints=0
    for j in range(n):
        prob += pulp.lpSum([ x[i][j] for i in range(n) if i!=j ]) ==1
    constraints += n
    for i in range(n):
        prob += pulp.lpSum([ x[i][j] for j in range(n) if i!=j ]) ==1
    constraints += n
    for i in range(n):
        for j in range(n):
            if i!=j:
                prob += x[i][j]+x[j][i] <= 1
                constraints += 1
    # initialise solver
```

```
    solvers = pulp.listSolvers(onlyAvailable=True)
    solver = pulp.getSolver(solvers[0], msg=0)
    # print(f"{constraints:d} Constraints")
    prob.solve(solver)
    trips = roundtrips(x, n)
    while len(trips)>1:
        for t in trips:
            prob += pulp.lpSum([ x[t[i]][t[i+1]] + x[t[i+1]][t[i]]
                           for i in range(0,len(t)-1) ]) <= len(t)-2
            constraints += 1
        # print(f"{constraints:d} Constraints")
        prob.solve(solver)
        trips = roundtrips(x, n)
    trip = trips[0]
    # print(trip)
    loop = []
    for k in range(len(trip)-1):
        sub = P[trip[k]][trip[k+1]]
        loop += sub if len(loop)==0 else sub[1:]
    return loop
```

5 Class Recorder

We willl use a class Recorder as a reference point for capturing data during the simulation. There will be only one recorder. It will be created at the beginning of every simulation run. Every entity will carry a reference to the Recorder.

[17]:
```
class Recorder:

    def __init__(self, env, M, W, C, days,
                 log=False, plot=False, timing=False):
        self.env = env
        self.M = M
        self.W = W
        self.C = C
        self.days = days
        self.log = log
        self.plot = plot

        self.timing = timing
        self.start_time = time.time()
        self.last_time = self.start_time
        self.cum_timer = {}

        Customer.REGISTER = []
        Parcel.REGISTER = []
```

```
        # create a data frame for records per working day
        self.daily = pd.DataFrame()
        self.daily['begin work at'] = [None]*days
        self.daily['end work at'] = [None]*days
        self.daily['dist'] = [None]*days
        self.daily['left'] = [None]*days

    def timer(self, s):
        t = time.time()
        Δt = t-self.last_time
        if self.timing:
            print(f"==== t: {t-self.start_time:6.2f}s "
                  f"Δt: {Δt:6.2f}s [{s:s}]")
        if s in self.cum_timer:
            self.cum_timer[s] += Δt
        else:
            self.cum_timer[s] = Δt
        self.last_time = t

    def reportTimer(self):
        print(f"==== t: {self.total_time:6.2f}s Total")
        for k in sorted(self.cum_timer, key=lambda x: self.cum_timer[x],␣
↪reverse=True):
            print(f"==== ΣΔt: {self.cum_timer[k]:6.2f}s "+ k)

    def trace(self, event):
        if self.log:
            print(timestamp(self.env.now), event)

    def recordDriverBeginsWork(self):
        self.trace("Driver arrives for work")
        self.daily.at[day(self.env.now), 'begin work at'] = int(round(self.env.
↪now))

    def recordDriverEndsWork(self):
        self.trace("Driver goes home")
        self.daily.at[day(self.env.now), 'end work at'] = int(round(self.env.
↪now))

    def recordTourLength(self, length):
        self.daily.at[day(self.env.now), 'dist'] = int(length)

    def recordParcelsLeftOver(self, numberOfParcels):
        self.trace(f"{numberOfParcels:d} left over for next day")
        self.daily.at[day(self.env.now), 'left'] = numberOfParcels

    def finish(self):
```

```
        self.total_time = time.time()-self.start_time
        self.daily['working time'] = (self.daily['end work at']-self.
↪daily['begin work at'])//60
        # simulation is finished for good
        # by removing the simulation environment we can
        # pickle recorder
        self.env = None

    def __stats__(self, column):
        d = self.daily[column].copy()
        return d.mean(), d.median(), d.std()

    def statsWorkingTime(self):
        return self.__stats__('working time')

    def histWorkingTime(self):
        histplot(self.daily['working time'],
                 xlabel='Working Time [min]',
                 title='Daily Working Time')

    def plotWorkingTime(self):
        dailyPlot(self.daily['working time'],
                  ylabel='Working Time [min]',
                  title='Daily Working Time')

    def statsTourLength(self):
        return self.__stats__('dist')

    def histTourLength(self):
        histplot(self.daily['dist'],
                 xlabel='Tour Length [m]',
                 title='Daily Tour Length')

    def plotTourLength(self):
        dailyPlot(self.daily['dist'],
                  ylabel='Tour Length [m]',
                  title='Daily Tour Length')

    def statsLeftOver(self):
        return self.__stats__('left')

    def histLeftOver(self):
        histplot(self.daily['left'],
                 xlabel='Left-Over Parcels',
                 title='Daily Left-Over Parcels')

    def plotLeftOver(self):
```

```
        dailyPlot(self.daily['left'],
                  ylabel='Number of Parcels',
                  title='Daily Left-Over Parcels')
```

6 Class Parcel

Parcels follow through a sequence of states: - processing - in transit (from manufacture to distribution centre) - arrived in distribution centre - ready for delivery - out for delivery - customer not present - returned to distribution centre - delivered

[18]:
```
class Parcel:

    REGISTER = []

    def __init__(self, rec, i, cust, custIndex):
        self.rec = rec
        self.i = i  # row index in data frames of input data
        self.dest = cust.location
        self.custIndex = custIndex
        self.status = [ 'processing' ] # status record and
        self.timing = [ self.rec.env.now ]       # timing
        assert(len(Parcel.REGISTER)==i)
        Parcel.REGISTER += [ self ]

    # factory method ensures that there is only
    # one Parcel per location
    def getParcel(rec, i, location, custIndex):
        for p in Parcel.REGISTER:
            if p.i == i:
                return p
        return Parcel(rec, i, location, custIndex)


    def __str__(self):
        return f"Parcel: {self.i:3d} ({self.custIndex:3d})"

    def index(self):
        return self.i

    def destination(self):
        return self.dest

    def __reg(self, state):
        self.status += [ state ]
        self.timing += [ self.rec.env.now ]
        self.rec.trace(str(self)+" "+state)
```

```
    def arrivedAtDeliveryCentre(self):
        self.__reg('arr at delivery centre')

    def outForDelivery(self):
        self.__reg('out for delivery')

    def returnFromDelivery(self):
        self.__reg('return from delivery')
```

7 Class Customer

[19]:
```
class Customer:

    REGISTER = []

    def __init__(self, rec, location):
        self.rec = rec
        self.location = location
        self.i = len(Customer.REGISTER)
        Customer.REGISTER += [ self ]
        self.atHome = True
        self.answersDoor = False
        self.parcelsReceived = []
        rec.env.process(self.process())

    def __str__(self):
        return f"Customer: {self.i:2d} {str(self.location):s}"

    # factory method ensures that there is only
    # one customer per location
    def getCustomer(rec, location):
        for c in Customer.REGISTER:
            if c.location == location:
                return c
        return Customer(rec, location)

    def leaveHouse(self):
        assert(self.atHome and not self.answersDoor)
        # self.rec.trace(str(self)+" leaves house")
        self.atHome = False

    def returnHome(self):
        assert(not self.atHome)
        # self.rec.trace(str(self)+" returns home")
        self.atHome = True
```

```
    def answerDoor(self):
        if self.atHome:
            yield self.rec.env.timeout(random.expovariate(1/
↪AVERAGE_TIME_ANSWER_DOOR))
            self.rec.trace(str(self)+" answers door")
            self.answersDoor = True
        else:
            yield self.rec.env.timeout(WAIT_TIME_IF_CUSTOMER_DOESNT_ANSWER_DOOR)
            self.rec(str(self)+" not at home")

    def acceptParcel(self, parcel):
        assert(self.answersDoor)
        self.parcelsReceived += [parcel]
        self.rec.trace(str(self)+" accepts "+str(parcel))

    def signOff(self):
        assert(self.answersDoor)
        self.rec.trace(str(self)+" signs off")
        self.answersDoor = False

    def process(self):
        yield self.rec.env.timeout(nextHour(self.rec.env, 8))
        while day(self.rec.env.now)<self.rec.days:
            # in a refinement we may use random times
            self.leaveHouse()
            yield self.rec.env.timeout(nextHour(self.rec.env, 18))
            self.returnHome()
            yield self.rec.env.timeout(nextHour(self.rec.env, 8))
```

8 Class Driver

[20]:
```
class Driver:

    def __init__(self, rec, DC):
        self.rec = rec
        self.DC = DC
        self.location = None
        self.parcels = None
        self.tour = None
        self.rec.env.process(self.process())

    # activity
    def __drive(self, target):
        assert(self.tour[0] == self.location)
        while self.location!=target:
```

```
            d = dist(self.location, self.tour[1])
            yield self.rec.env.timeout(d / AVERAGE_SPEED)
            self.location = self.tour[1]
            self.tour = self.tour[1:]
        assert(self.tour[0] == self.location == target)

    def arriveForWork(self):
        self.location = self.DC.W
        self.parcels = []
        self.returns = []
        self.tour = [ self.DC.W ]
        self.rec.recordDriverBeginsWork()

    def leaveForDelivery(self, tour, parcels):
        self.tour, self.parcels = tour, parcels
        self.rec.trace(f"Driver leaves for delivery " \
                       f"of {len(parcels):d} parcels")

    def process(self):
        yield self.rec.env.timeout(nextHour(self.rec.env, 18))
        while day(self.rec.env.now)<self.rec.days:
            self.arriveForWork()
            tour, parcels = self.DC.sendForDelivery()
            yield self.rec.env.timeout(PREP_TIME_PER_PARCEL*len(parcels))
            self.rec.recordTourLength(pathLength(tour))
            self.leaveForDelivery(tour, parcels)
            while len(self.parcels)>0:
                # drive to customer
                custLocation = self.parcels[0].dest
                cust = Customer.getCustomer(self.rec, custLocation)
                self.rec.trace("Driver drives to "+str(cust))
                yield from self.__drive(custLocation)
                self.rec.trace("Driver arrived at "+str(cust))
                # call at customer
                yield from cust.answerDoor()

                if cust.answersDoor:
                    while len(self.parcels)>0 and \
                            custLocation == self.parcels[0].dest:
                        cust.acceptParcel(self.parcels[0])
                        yield self.rec.env.timeout(random.expovariate(1/10))
                        self.parcels = self.parcels[1:]
                    cust.signOff()
                    yield self.rec.env.timeout(random.expovariate(1/10))
                else:
                    while len(self.parcels)>0 and  \
                            custLocation == self.parcels[0].dest:
```

```
                        self.returns += self.parcels[0]
                        self.parcels = self.parcels[1:]

            # return to delivery centre
            self.rec.trace("Driver returns to delivery centre")
            yield from self.__drive(self.DC.W)
            self.rec.trace("Driver arrived at delivery centre")

            for parcel in self.returns:
                self.DC.returnFromDelivery(parcel)
                yield self.rec.env.timeout(RETURN_TIME_PER_PARCEL)
            yield self.rec.env.timeout(600)

            self.rec.recordParcelsLeftOver(len(self.DC.parcels)+
                                           len(self.DC.leftOver))

            self.rec.recordDriverEndsWork()

            self.rec.trace(self.DC.inventory())

            yield self.rec.env.timeout(nextHour(self.rec.env, 18))
```

9 Class Delivery Centre

[21]:
```
class DeliveryCentre:

    def __init__(self, rec, M, W):
        self.rec = rec
        self.M = M
        self.W = W
        self.limit = 35000


        self.leftOver = []     # list of parcels
        self.parcels = []      # list of parcels scheduled for delivery
        self.dest = []         # list of unique customer destinations
        self.tour = [W]        # tour planned for delivery

    def __accept(self, parcel):
        custLoc = parcel.dest
        if custLoc not in self.dest:
            start_time = time.time()
            MT = addTargets(self.M, self.dest + [custLoc])
            self.rec.timer("addTarget")
            S = createLoop(MT, [self.W] + self.dest + [custLoc])
            self.rec.timer("createLoop")
```

```
            if pathLength(S)<self.limit:
                self.parcels.append(parcel)
                self.dest += [custLoc]
                self.tour = S
            else:
                self.leftOver.append(parcel)
        else:
            self.parcels.append(parcel)

    def acceptParcel(self, parcel):
        parcel.arrivedAtDeliveryCentre()
        self.__accept(parcel)

    def sendForDelivery(self):
        parcels = []
        tour = self.tour
        addresses = self.dest

        # pick parcels in sequence to be delivered
        for i in range(1, len(tour)-1):
            dest = tour[i]
            for p in self.parcels:
                if p.dest == dest and p not in parcels:
                    parcels += [p]
                    p.outForDelivery()

        # arrange the left overs
        L = self.leftOver
        self.tour = [self.W]
        self.parcels = []
        self.leftOver = []
        self.dest = []
        for p in L:
            self.__accept(p)

        if self.rec.plot:
            plotMap(self.rec.M, T=addresses, P=tour, W=tour[0],
                    text=f"Day {day(self.rec.env.now):2d}, {pathLength(tour):
↪,d}m")

        return tour, parcels

    def returnFromDelivery(self, parcel):
        parcel.returnFromDelivery()
        self.__accept(parcel)

    def getInventory(self):
```

```
        acc = ""
        for p in self.parcels:
            if len(acc)>0:
                acc += ", "
            acc += f"{p.custIndex:d}"
        acc += " / "
        for p in self.leftOver:
            if len(acc)>0:
                acc += ", "
            acc += f"{p.custIndex:d}"
        return acc

    def inventory(self):
        return f"Delivery Centre Inventory: " \
               f"{len(self.parcels)+len(self.leftOver):d}"
```

10 Simulation

10.1 Parameters from Specification

The time required for driving is based on the distance between way points at an average speed of 15km/h.

[22]:
```
AVERAGE_SPEED = 15/3.6
```

The **cumulative preparation time** (route planning and sorting of the parcels in the delivery order and packing the cargo-bike) is assumed to be 50 sec per parcel to be delivered.

[23]:
```
PREP_TIME_PER_PARCEL = 50
```

Additional assumption: The time to **process returned parcels** in the delivery centre is 30 sec per parce.

[24]:
```
RETURN_TIME_PER_PARCEL = 30
```

The average time to answer the door.

[25]:
```
AVERAGE_TIME_ANSWER_DOOR = 40
```

[26]:
```
WAIT_TIME_IF_CUSTOMER_DOESNT_ANSWER_DOOR = 60
```

10.2 Generate Input Data

[27]:
```
def generateDeliveryData(p, C, days, seed=0):
    ## p is the average number of parcels per day per customer
    ## C is the number of customers to be served
    ## days is the number of days for which data are to be generated.
    np.random.seed(seed)
    R = np.random.poisson(lam=len(C)*p, size=days)
```

```
    D = [ sorted(list(np.random.choice(range(len(C)), size=i))) for i in R ]
    return D
```

[28]:
```
def generateInputData(D, log=False):

    R = [ len(d) for d in D ]
    N = sum(R)

    DAY_LENGTH = 24*3600    # measured in minutes
    DAY_START = 8*3600      # first delivery in the morning
    DAY_END = 17*3600     # last delivery during day time

    x = pd.DataFrame()

    x['iarr'] = [None]*N
    x['time'] = [None]*N
    x['day'] = [None]*N
    x['dest'] = [None]*N

    current_day = 0
    last_time = 0
    i = 0

    for d in D: # for each day

        if log:
            print("generating for day: ",current_day, D[current_day])

        time = current_day*DAY_LENGTH + DAY_START

        for c in d: # for each customer that should get a

            IARR = (DAY_END-DAY_START-2*3600) / len(d)  # estimated average IAT␣
↪for the current day

            iat = random.expovariate(1.0/IARR)
            new_time = time + iat

            x.at[i, 'iarr'] = round(new_time - last_time,1)
            x.at[i, 'time'] = round(new_time - current_day*DAY_LENGTH , 1)
            x.at[i, 'day'] = current_day
            x.at[i, 'dest'] = c

            i += 1
            last_time = time = new_time

        current_day += 1
```

```
    return x
```

10.3 Simulation Routine

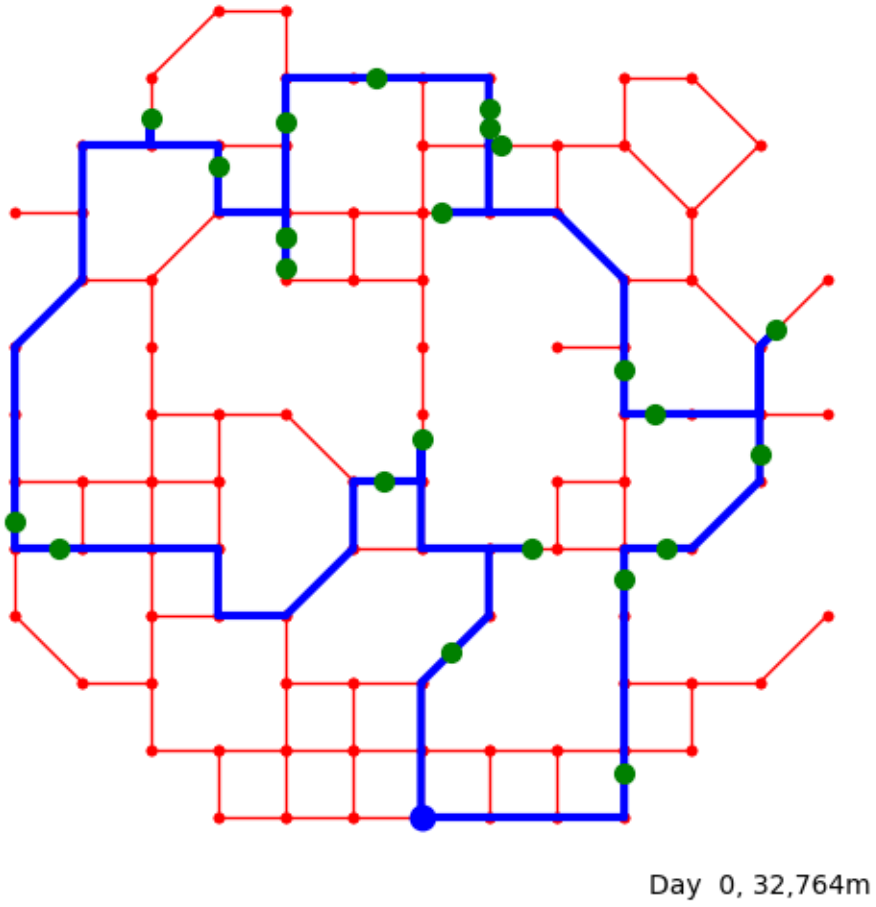
[45]:
```
def simulation(M, W, C, p=0.2, days=10, seed=0,
               title=None, log=False, plot=False, timing=False):

    if timing:
        start_time = time.time()

    random.seed(seed)
    D = generateDeliveryData(p, C, days, seed)
    X = generateInputData(D, log=log)

    env = simpy.Environment()
    rec = Recorder(env, M, W, C, days, log=log, plot=plot, timing=timing)

    if title is not None:
        print(title)
    else:
        print(f"Simulating delivery of {len(X):d} parcels "
              f"over {len(D):d} days to {len(C):d} customers")

    for c in C:
        Customer.getCustomer(rec, c)
    DC = DeliveryCentre(rec, M, W)
    D = Driver(rec, DC)

    def generatorProcess(env):
        # generate the parcels based on input data x
        for i in range(len(X)):
            yield env.timeout(X.at[i, 'iarr'])
            custIndex = X.at[i, 'dest']
            custLoc = C[custIndex]
            cust = Customer.getCustomer(rec, custLoc)
            p = Parcel.getParcel(rec, i, cust, custIndex)
            DC.acceptParcel(p)

    env.process(generatorProcess(env))
    env.run()

    if log:
        print(DC.inventory())

    rec.finish()
    return rec
```

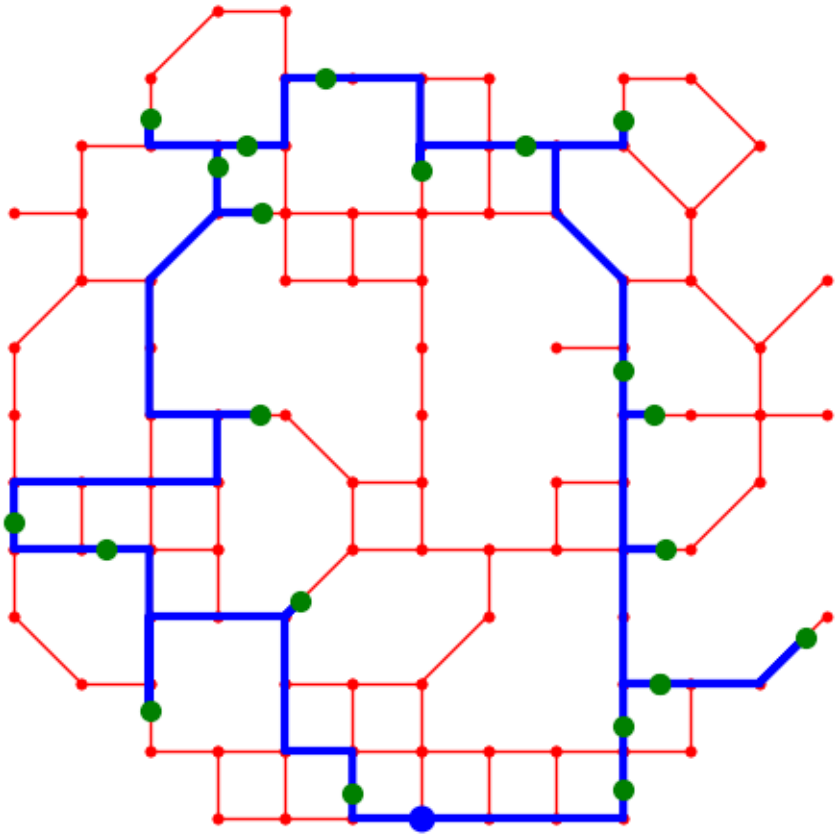
10.4 Small Simulation Run

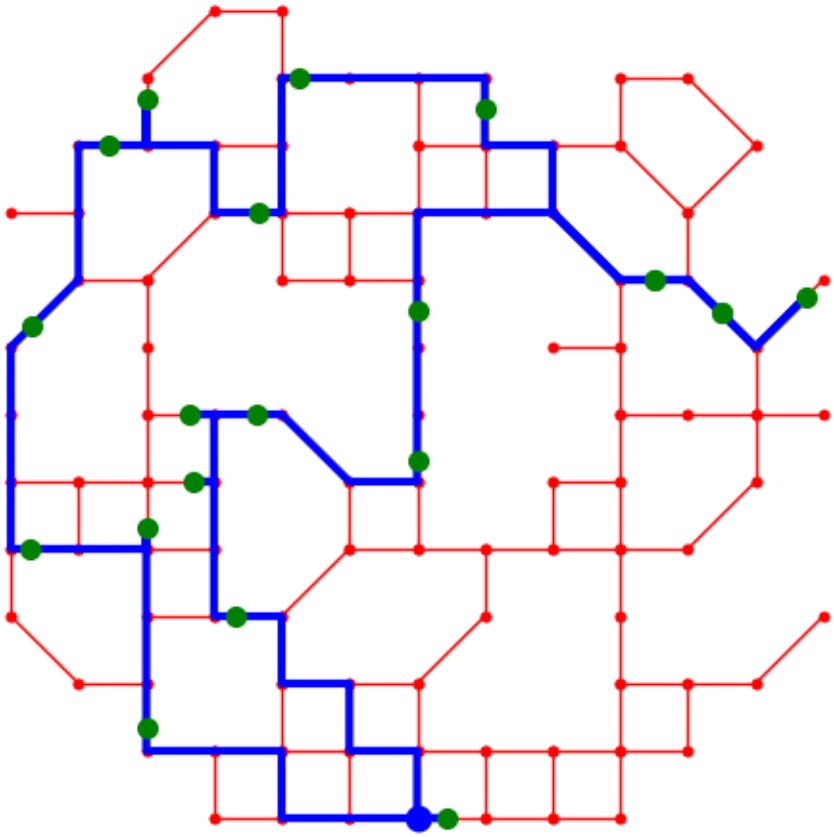
[30]:
```
import pickle
with open('data.pickled', 'rb') as f:
    M, C = pickle.load(f)
```

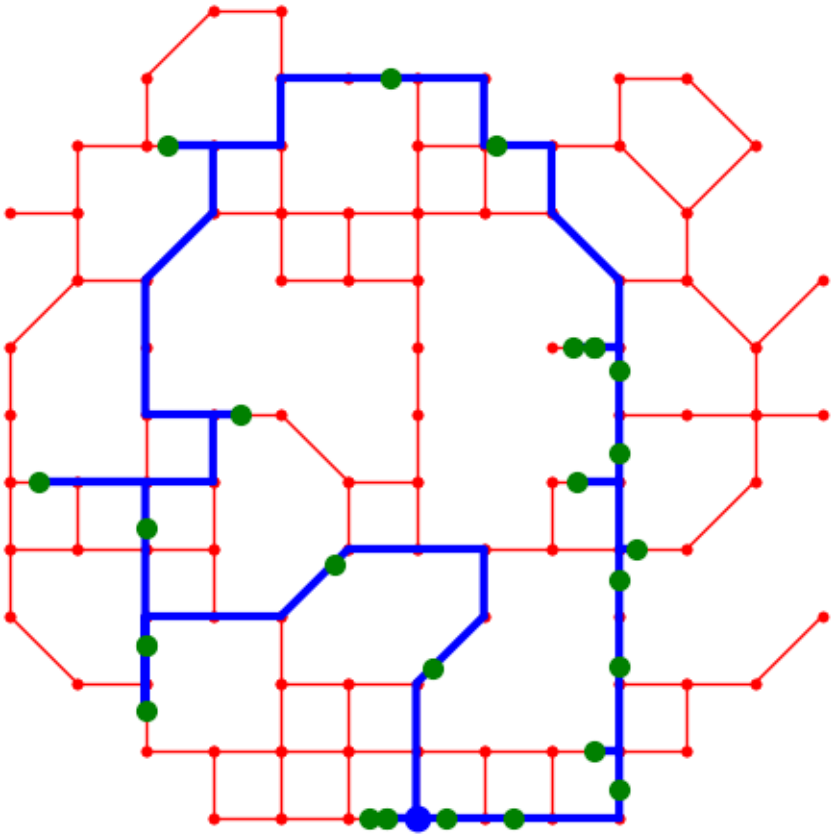
[31]:
```
random.seed(0)
W = generateWarehouseLocation(M)
rec = simulation(M, W, C, p=0.16, days=20, plot=True)
with open('rec.pickled', 'wb') as f:
            pickle.dump(rec, f)
```

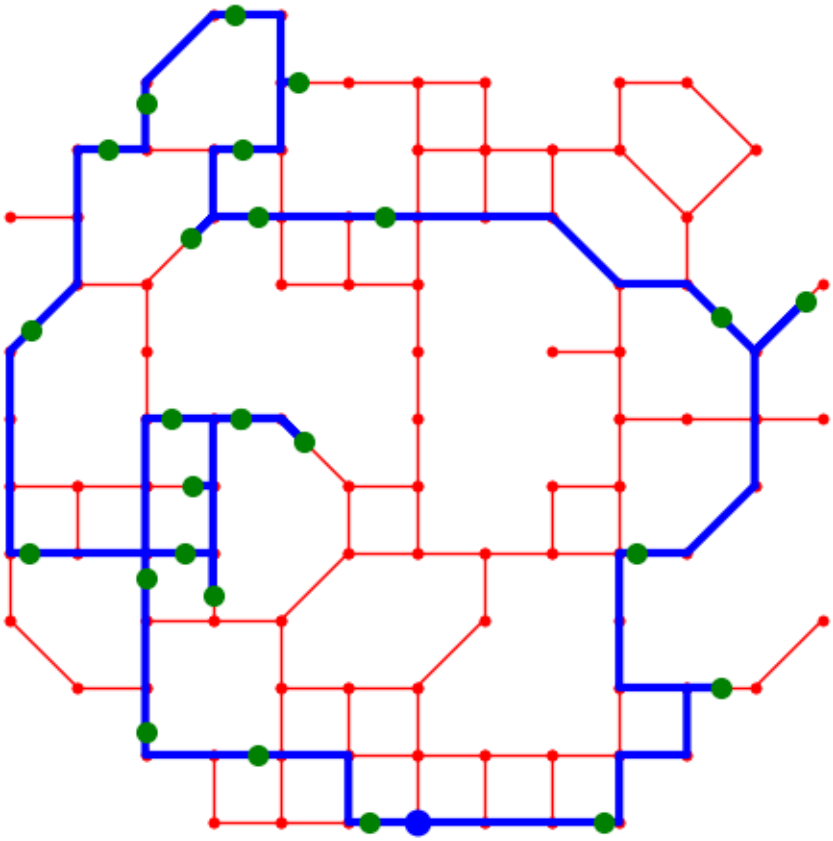
```
Simulating delivery of 484 parcels over 20 days to 150 customers
```

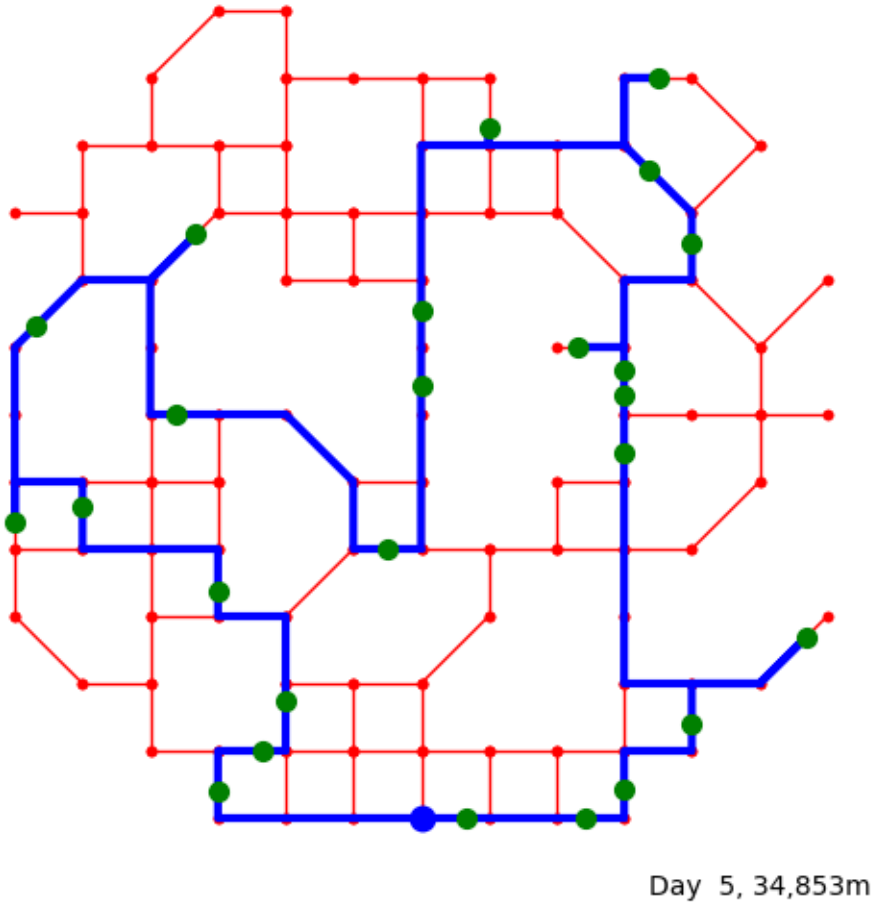


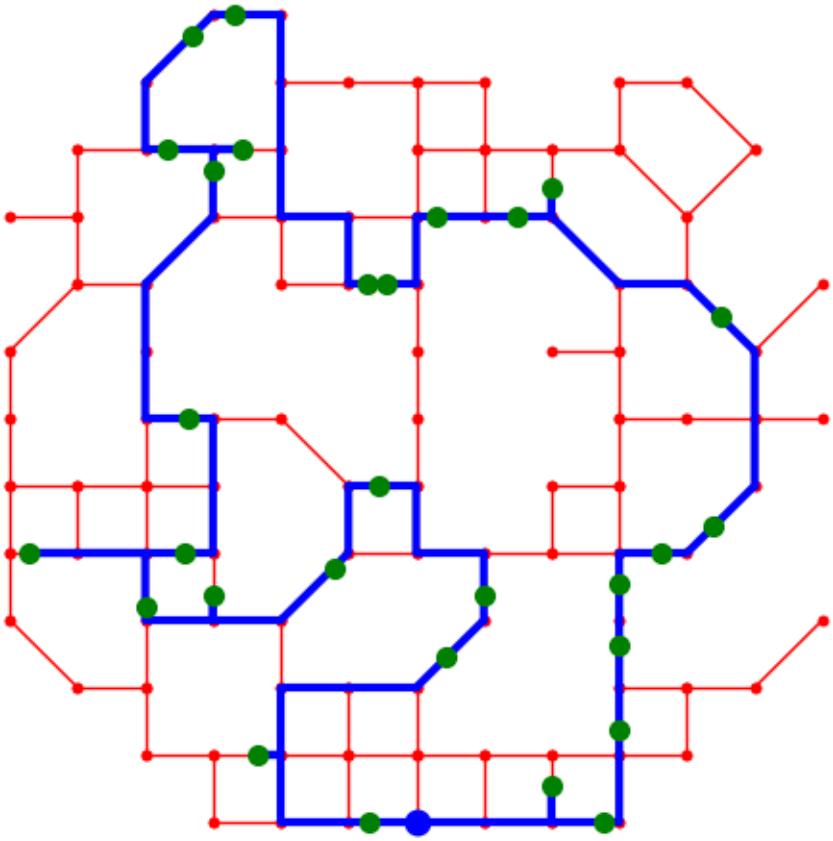

Day 1, 34,286m


Day 2, 34,060m

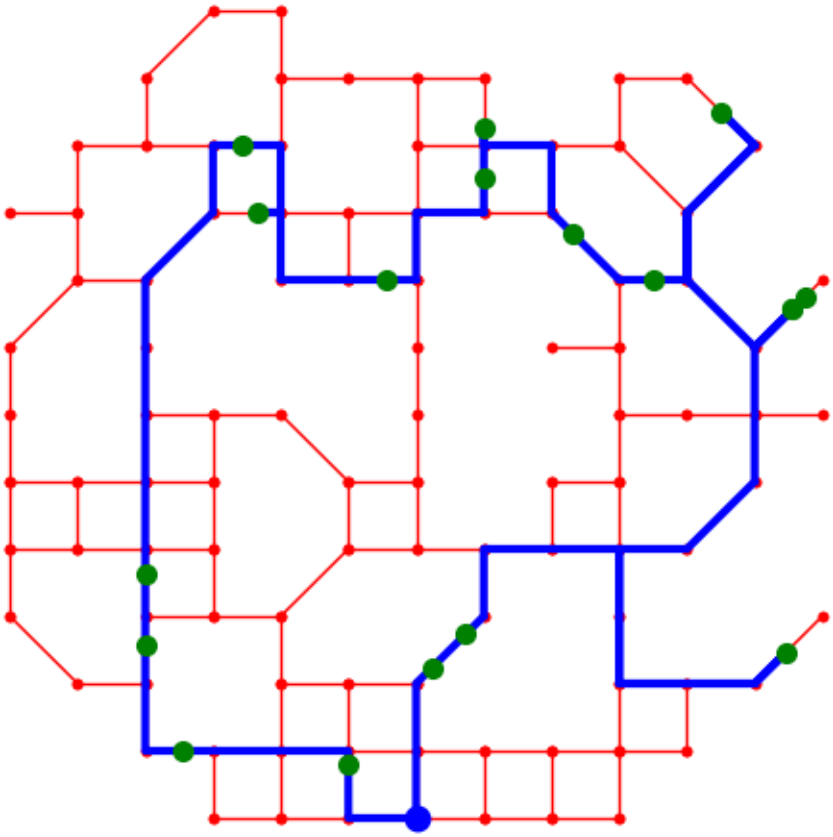

Day 3, 29,662m

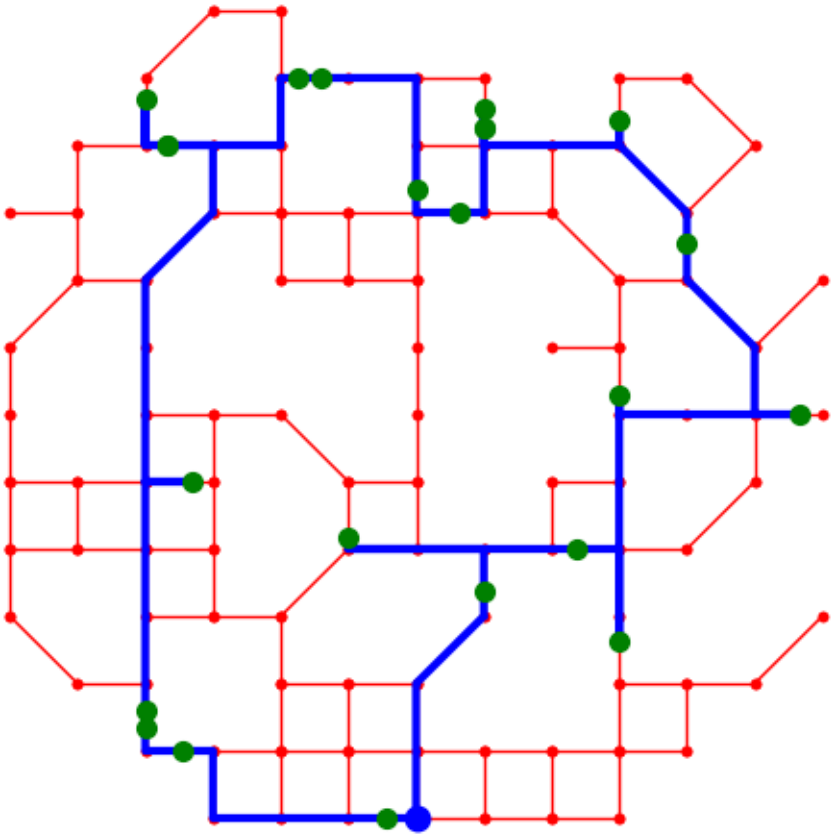

Day 4, 34,963m

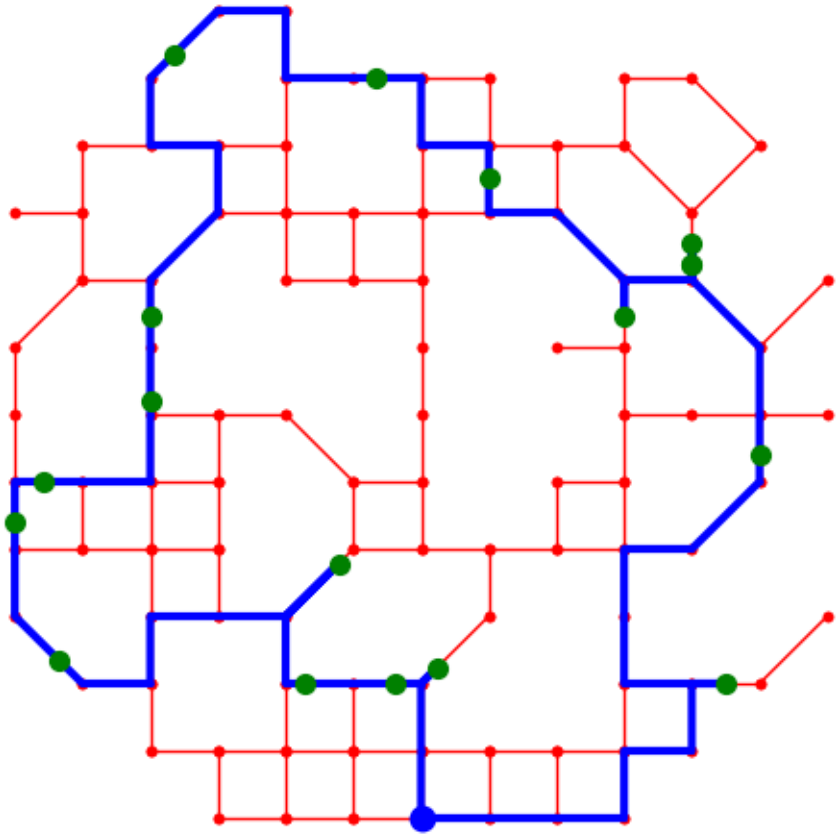

Day 5, 34,853m

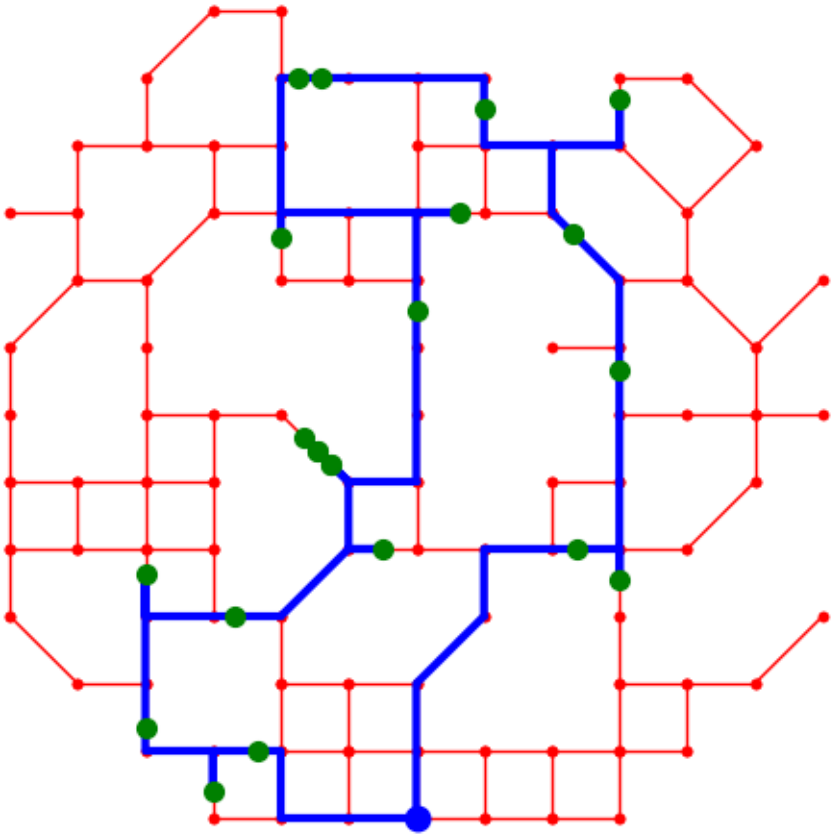


Day 6, 34,460m

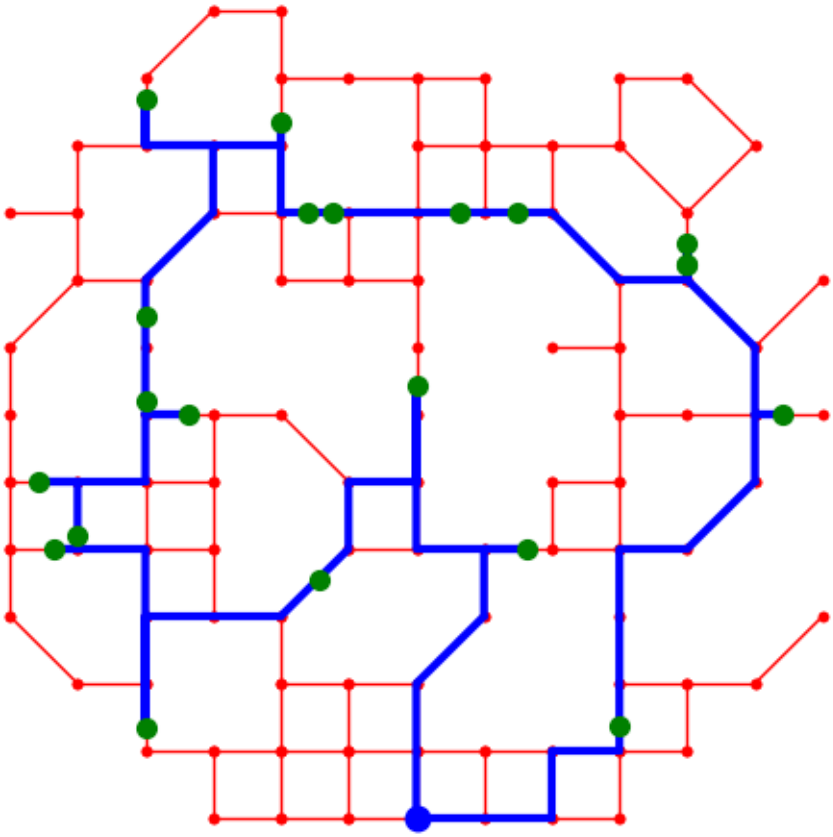

Day 7, 32,396m

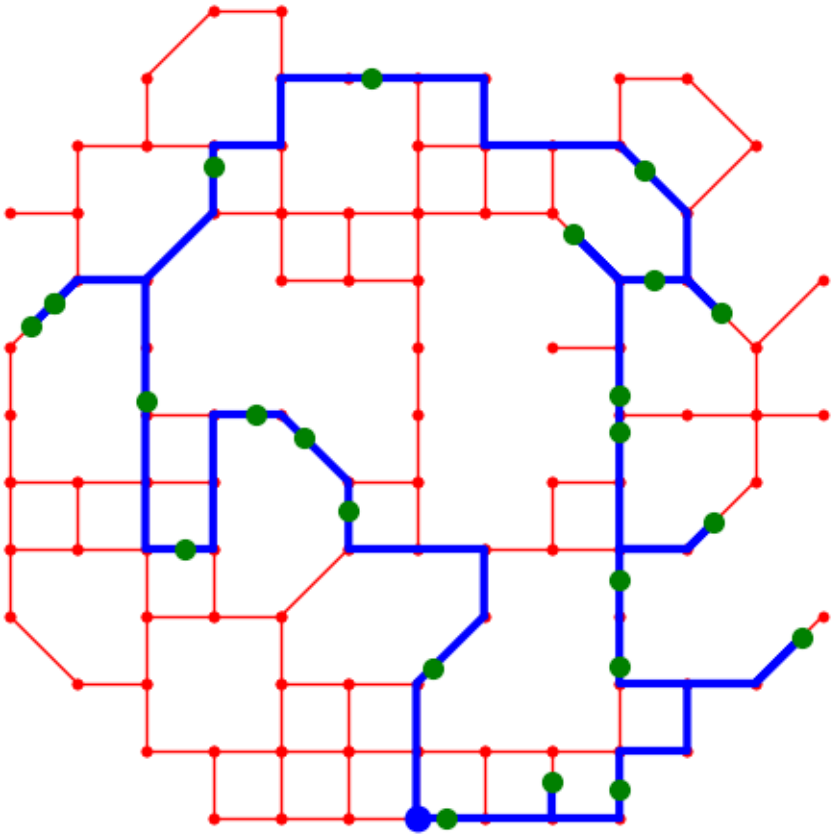

Day 8, 30,924m


Day 9, 30,534m

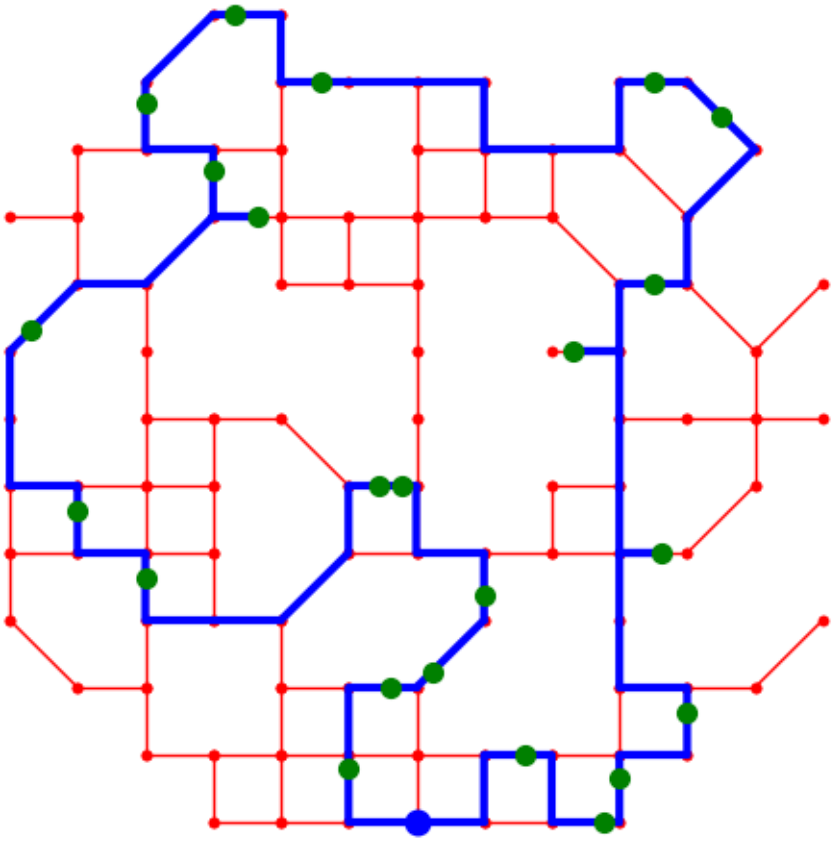


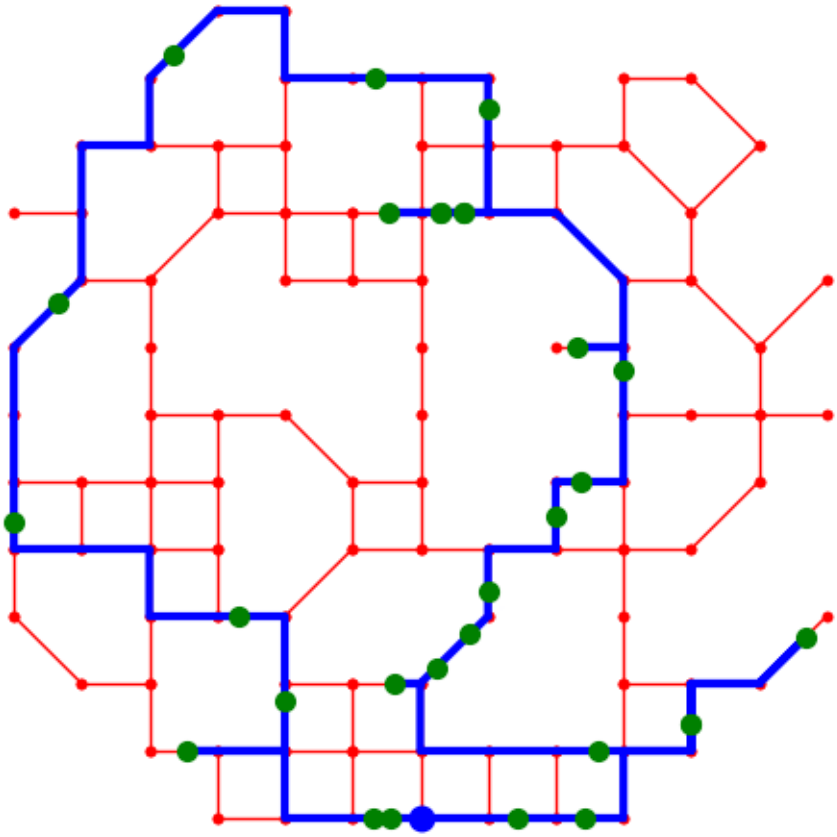
Day 10, 27,891m


Day 11, 33,062m

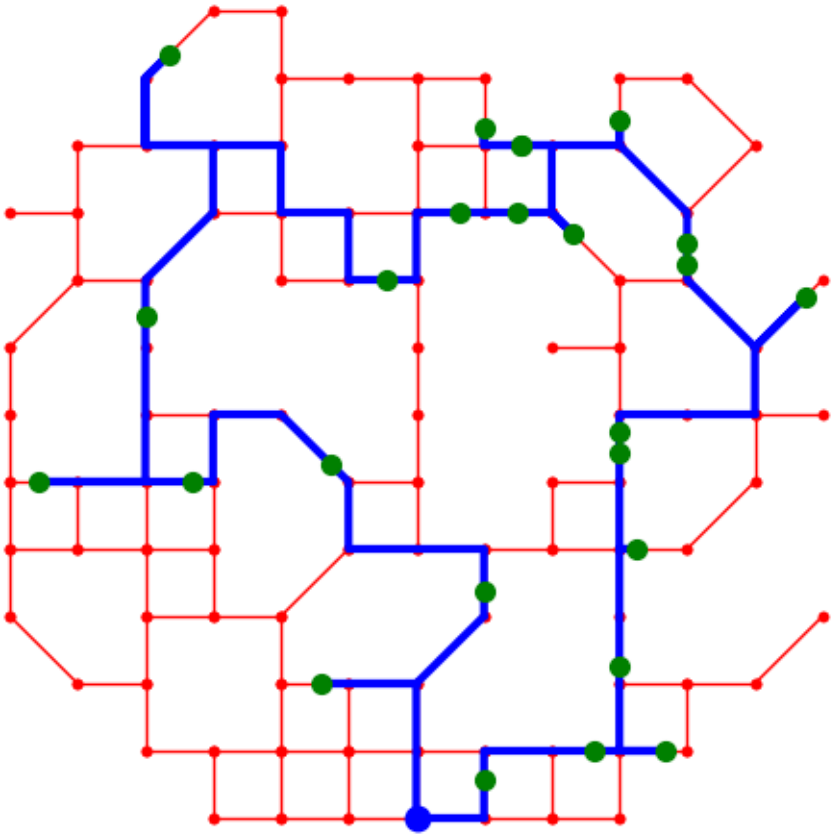


Day 12, 33,106m

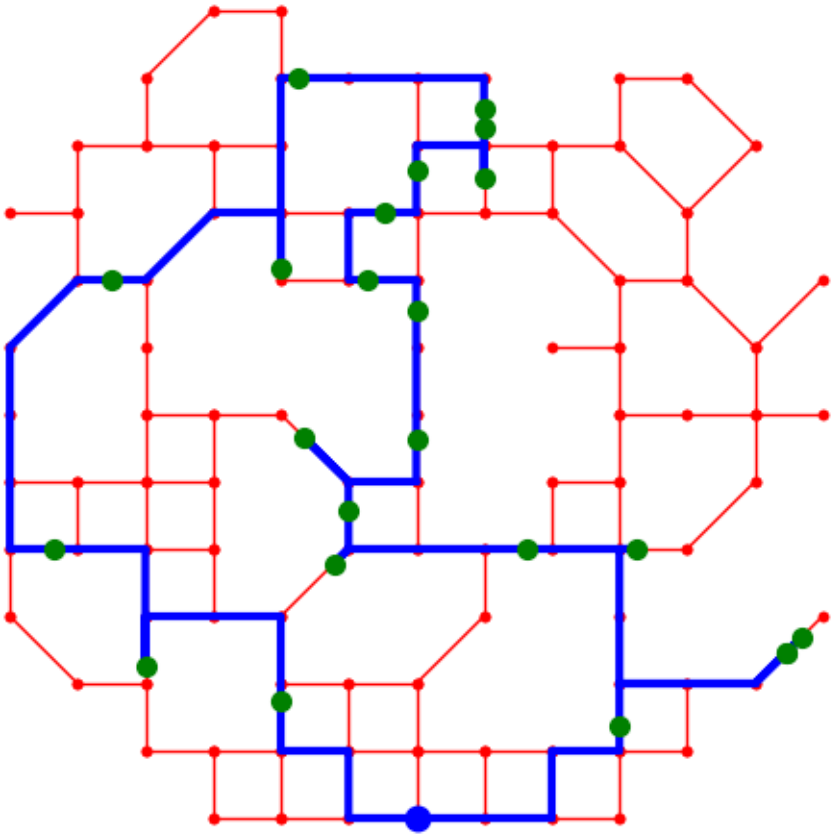

Day 13, 34,629m



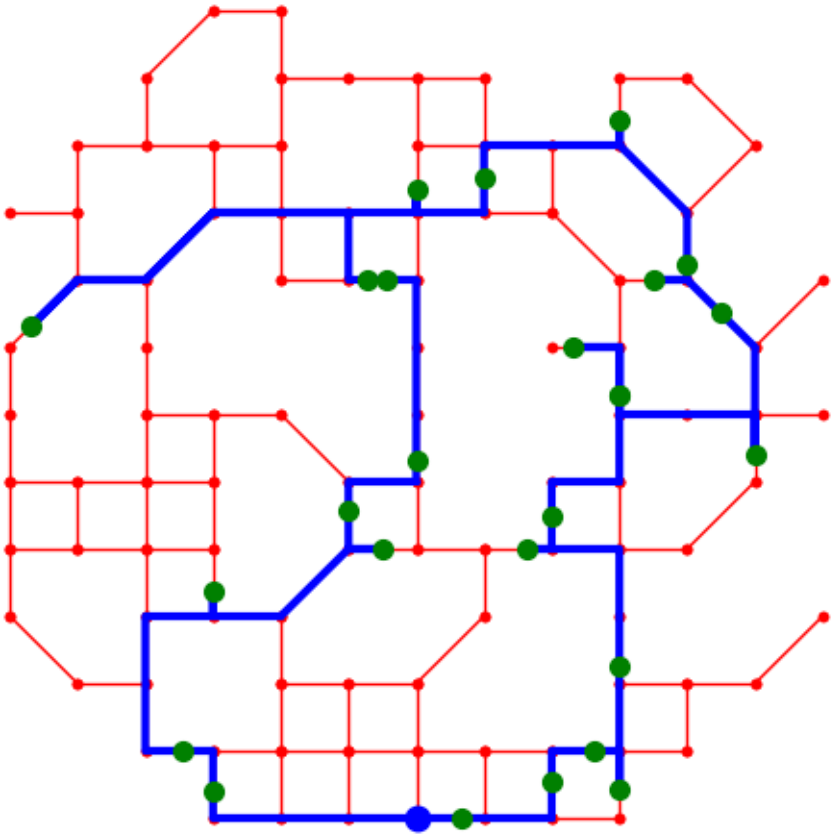
Day 14, 34,475m



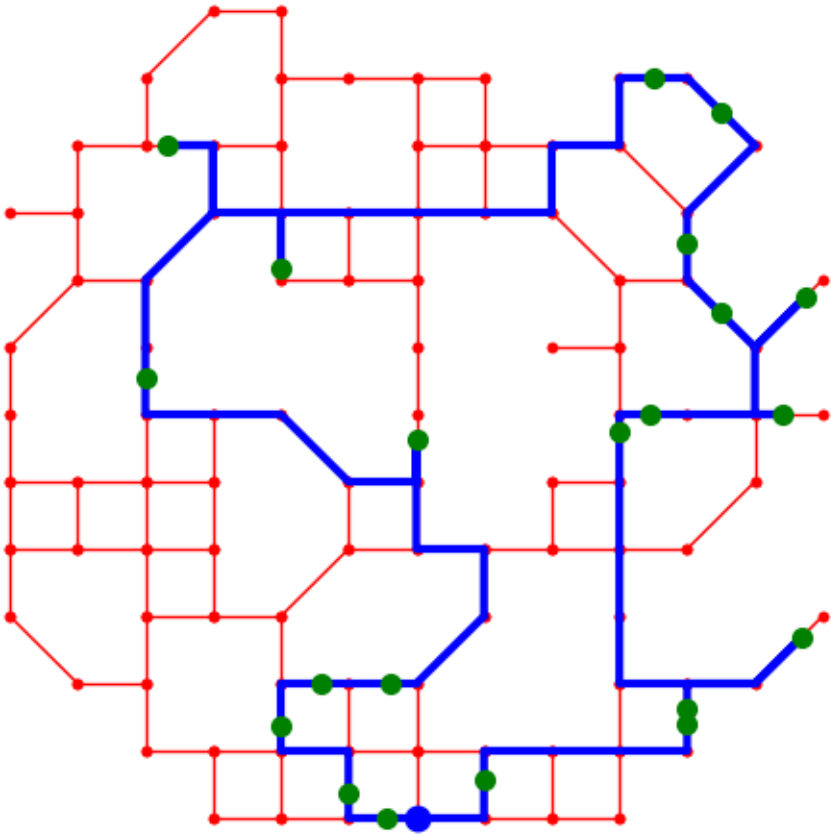
Day 15, 34,773m



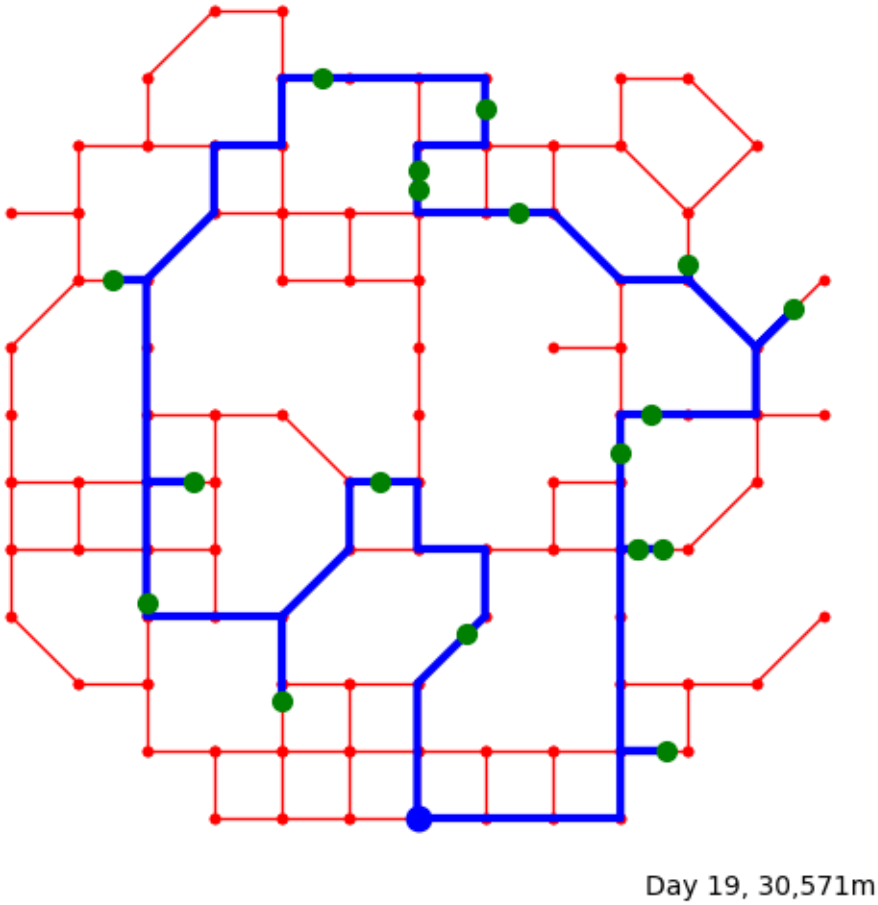
Day 16, 32,374m



Day 17, 34,493m

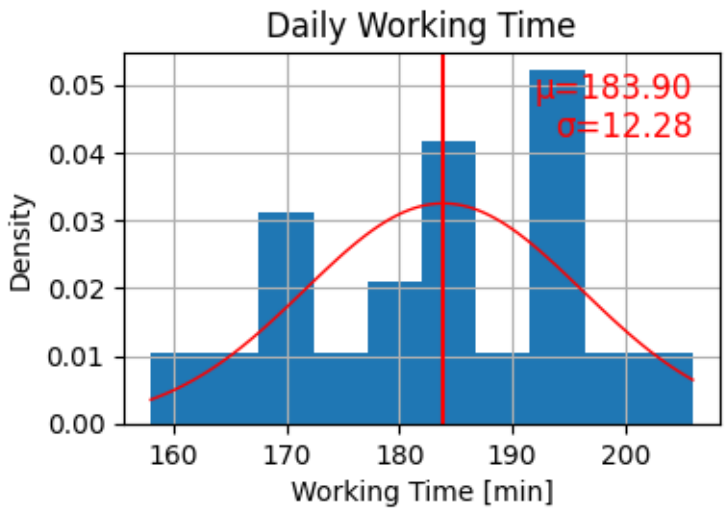


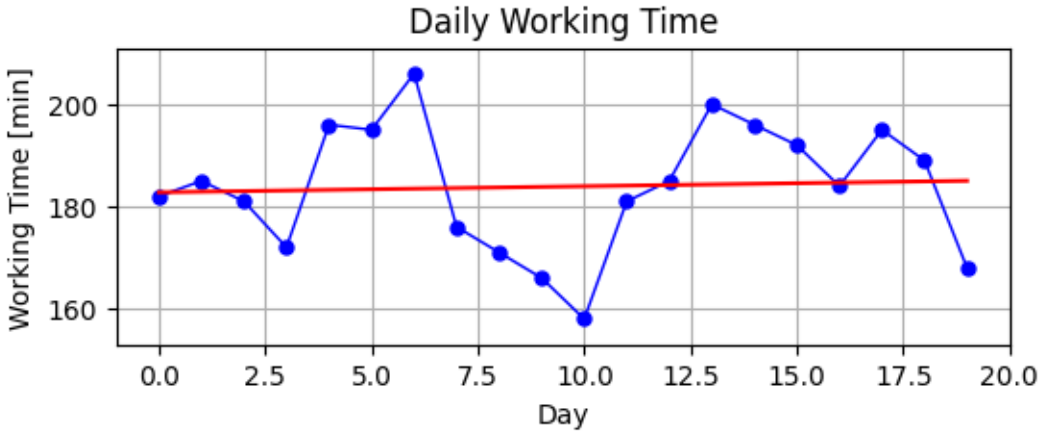
Day 18, 33,372m



[34]:
```
with open('rec.pickled', 'rb') as f:
    rec1 = pickle.load(f)
```

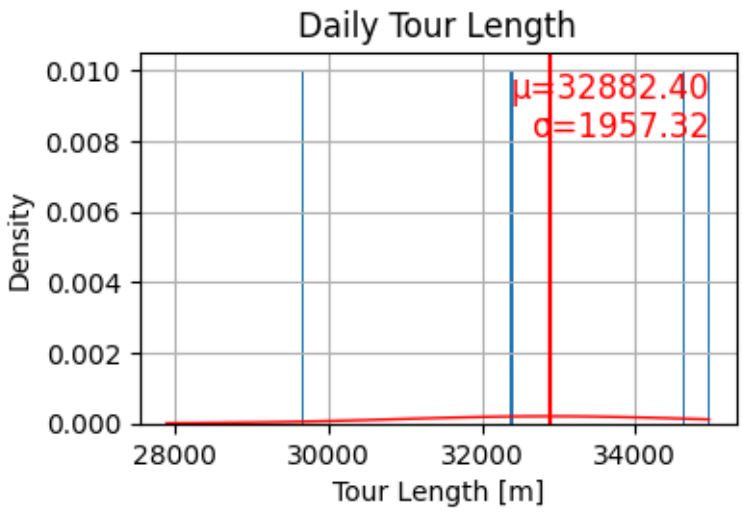
[35]:
```
rec1.histWorkingTime()
rec1.plotWorkingTime()
rec1.statsWorkingTime()
```

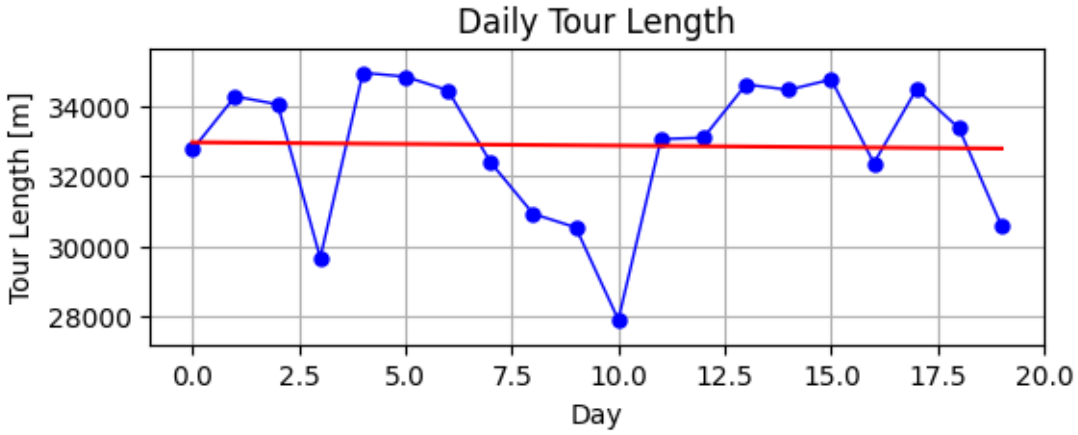




[35]: (183.9, 184.5, 12.598663254070795)

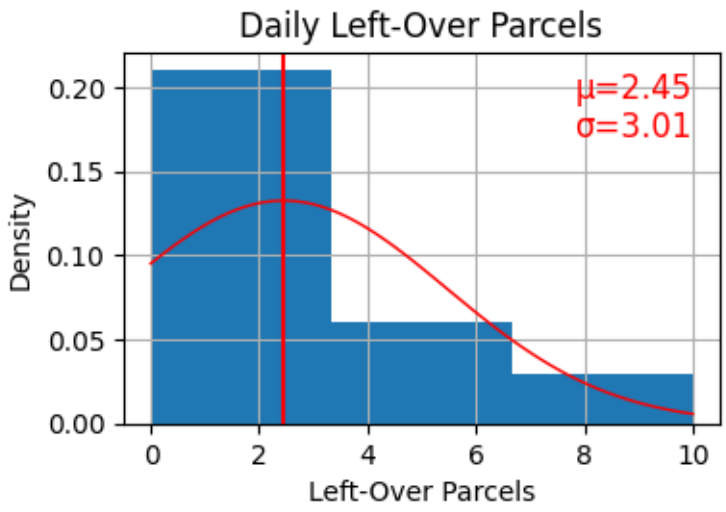
[36]:
```
rec1.histTourLength()
rec1.plotTourLength()
rec1.statsTourLength()
```

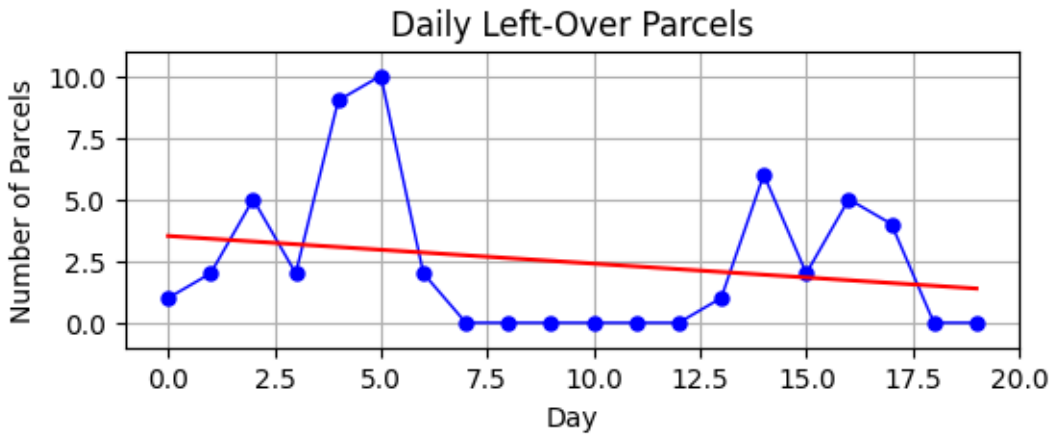




[36]: (32882.4, 33239.0, 2008.1697978114562)

[37]:
```
rec1.histLeftOver()
rec1.plotLeftOver()
rec1.statsLeftOver()
```





[37]: (2.45, 1.5, 3.086046696102688)

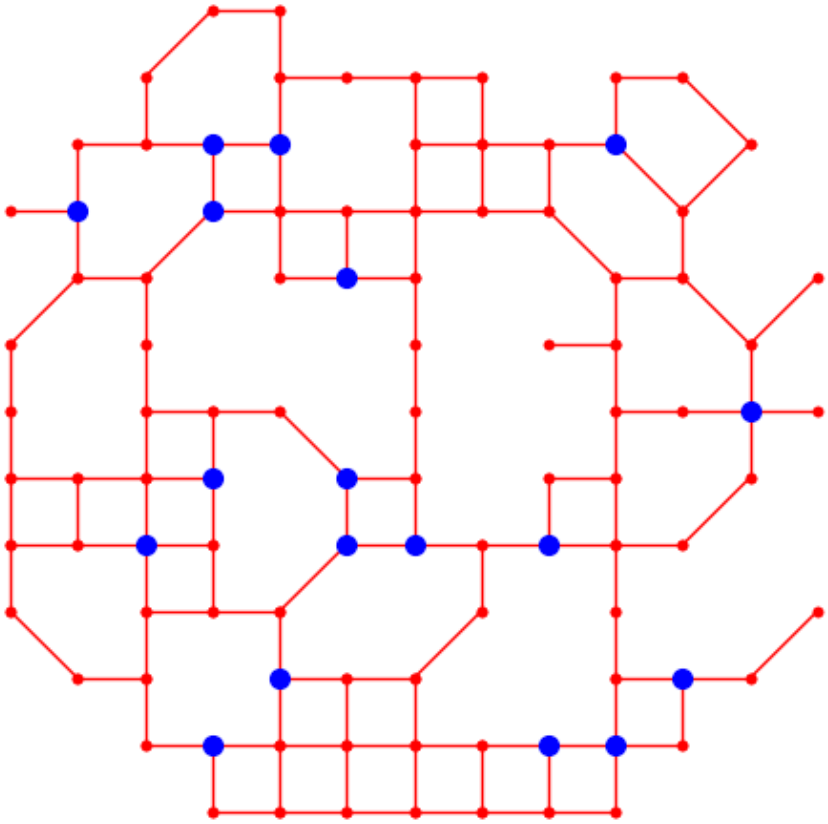
[38]:
```
rec1.reportTimer()
```

```
==== t: 304.70s Total
==== ΣΔt: 302.09s createLoop
==== ΣΔt:   2.52s addTarget
```

11 Optimisation

[46]:
```
import pickle

with open('data.pickled', 'rb') as f:
    M, C = pickle.load(f)

def generateWarehouseLocations(M, p):

    def numberOfEdges(v, E):
        count = 0
        for e in E:
            if e[0]==v or e[1]==v:
                count += 1
        return count

    V, E = M
    W = [ v for v in V if numberOfEdges(v, E)>2 ]
    return random.sample(W, k=int(round(len(W)*p)))

def formatLocation(w):
    return f"({w[0]:4d},{w[1]:4d})"

def objectiveFunction(w, seed=0):
    random.seed(seed)
    title = "Simulate Delivery from Warehouse "+formatLocation(w)
    rec = simulation(M, w, C, p=0.16, days=3, title=title)
    filename = "_/rec w="+formatLocation(w)+".pickled"
    with open(filename, 'wb') as f:
            pickle.dump(rec, f)
    return rec.statsTourLength()[0] # mean tour length

def monteCarlo(M, p):

    W = generateWarehouseLocations(M, p)
    plotMap(M, T=W, styleT='bo', msT=7,
            text="Warehouse Candidate Positions")

    minW, minP = None, math.inf
    for w in W:
        f = objectiveFunction(w)
        if f < minP:
            print(f"  improved position:   {formatLocation(w):s} f={f:7.2f}")
            minW, minP = [w], f
        elif f == minP:
            print(f"  equivalent position: {formatLocation(w):s} f={f:7.2f}")
            minW.append(w)
```

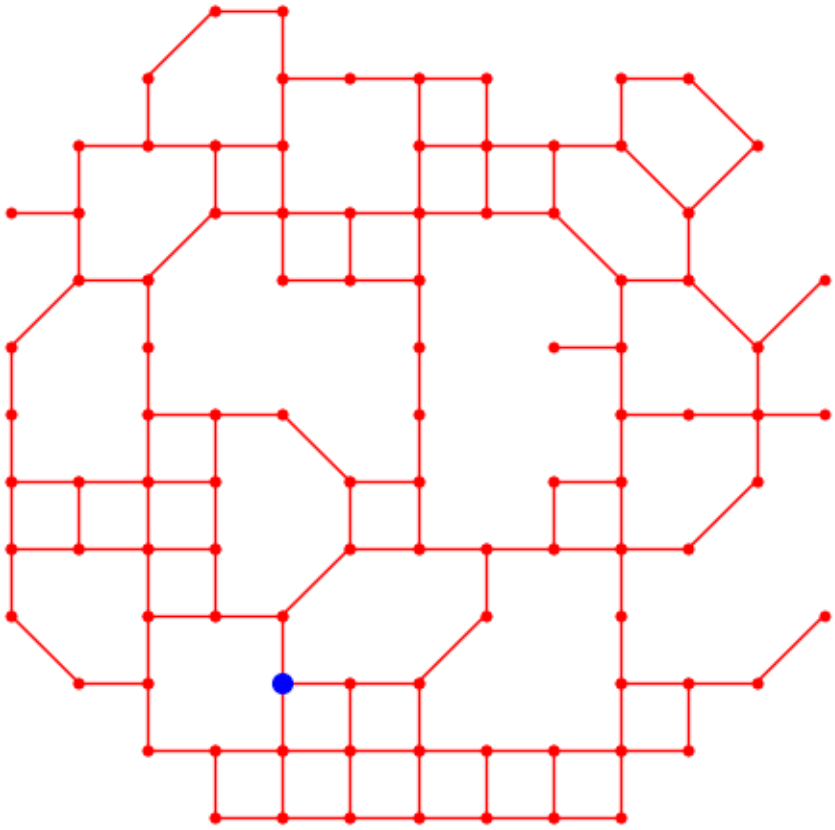
```
    return minW

random.seed(0)
W = monteCarlo(M, 0.28)
plotMap(M, T=W, styleT='bo', msT=7, text='Best Warehouse Positions')
```



Warehouse Candidate Positions

```
Simulate Delivery from Warehouse (5120,1200)
  improved position:   (5120,1200) f=34208.33
Simulate Delivery from Warehouse (5680,1200)
  equivalent position: (5680,1200) f=34208.33
Simulate Delivery from Warehouse (1200,5680)
Simulate Delivery from Warehouse (2320,3440)
Simulate Delivery from Warehouse (6240,1760)
Simulate Delivery from Warehouse (3440,5120)
Simulate Delivery from Warehouse (3440,3440)
Simulate Delivery from Warehouse (2880,6240)
Simulate Delivery from Warehouse (5120,2880)
```

```
Simulate Delivery from Warehouse (6800,4000)
Simulate Delivery from Warehouse (2320,6240)
  equivalent position: (2320,6240) f=34208.33
Simulate Delivery from Warehouse (3440,2880)
Simulate Delivery from Warehouse (2880,1760)
  improved position:   (2880,1760) f=33946.67
Simulate Delivery from Warehouse (4000,2880)
Simulate Delivery from Warehouse (2320,1200)
Simulate Delivery from Warehouse (5680,6240)
Simulate Delivery from Warehouse (1760,2880)
Simulate Delivery from Warehouse (2320,5680)
```



Best Warehouse Positions

[ ]: