MEASURING THE VISCOSITY OF LIQUIDS USING STOKES LAW

Contents

Introduction

This project aims to measure fluid viscosity by analysing the drag force on particles in low Reynolds number conditions. In Part 1, we use Stokes' law to determine viscosity through the drag on a spherical particle, given by $F_d = 6\pi\mu aU$, where a is the radius, U is the velocity, and μ is the fluid viscosity. By measuring a sphere's terminal velocity in the fluid, we can calculate viscosity.

In Part 2, we repeat the experiment using a cylindrical particle with an aspect ratio of 10 to study the scaling of drag in this geometry, enhancing our understanding of drag behaviour across shapes and refining viscosity measurement techniques.

Aim

Part-1: The Stokes drag on a sphere of radius translating with a velocity U is given by $F_d = 6\pi\mu aU$. The drag is a good approximation for the sphere in the limit of Reynolds number going to zero. Design an experiment to measure the viscosity of a fluid using a spherical particle.

Part-2: Repeat the experiment and estimate the scaling for the drag in case of a long cylinder of aspect ratio 10.

Objectives

1. To construct a tank of capable of facilitating the controlled descent of the spherical and cylindrical shape.
2. To make a spherical ball of suitable radius so that we get favourable Reynolds number.
3. To make a cylinder of aspect ratio 10.
4. To measuring the viscosity of the Viscous fluid by accurately measuring the terminal velocity of the particles in the Viscous fluid using the Stokes Law.
5. To expand the study to measure the drag experienced by a long cylinder and understand how geometry influences drag force.

Theoretical Predictions and Calculations

- ### For Sphere

$$F_g = F_b + F_d$$

$$Fg = \left(\frac{4}{3}\right)\pi r^3 \rho s\ g$$

$$Fb = \left(\frac{4}{3}\right)\pi r^3 \rho f\ g$$

$$Fd = 6\pi\mu r U$$

Here U is the terminal velocity.

$$\mathrm{U} = \frac{2\mathrm{r}^2(\rho_s - \rho_f)g}{9\,\mu}$$

From here we can calculate the value of μ,

$$\mu = \frac{2\mathrm{r}^2(\rho_s - \rho_f)g}{9\,\mathrm{U}}$$

And drag force comes out to be, $\mathrm{Fd} = \left(\frac{4}{3}\right)\pi r^3(\rho_s - \rho_f)g$

Now when the terminal velocity hasn't been reached,

$$m\left(\frac{dv}{dt}\right) = F_g - F_b - F_d$$

And therefore, we get $\frac{dv}{dt} = g\left(\frac{\rho_s - \rho_f}{\rho_s}\right) - \frac{9\mu v}{2\, r^2 \rho_s}$

Solving this by taking the value $v = 0.99v_t$,

We get depth value before reaching, $h = 3.615 \frac{4r^4\rho_s(\rho_s - \rho_f)g}{81\, \mu^2}$

Density (in SI units) of: 1. Steel =7850, 2. Teflon = 2200, 3. Water = 1000, and 4. Glycerine = 1260

Viscosity (in SI units) of: 1. Water = 0.001, and 2. Glycerine = 1.41

Radius (in SI units) of: 1. Teflon ball = 0.0075, and 2. Steel ball = 0.003

Combinations of the things suggested but we got depth value before reaching, h

1. Teflon and water, h = 14628.55 m
2. Teflon and glycerine, h = 5.8 mm
3. Steel and water, h = 7627.77 m
4. Steel and glycerine, h = 3.7 mm

- For Cylinder

  For low Reynold's number, $C_D = \frac{Re}{24}$ and Re = $\frac{\rho UL}{\mu}$

  And the value of the drag force is $F_d = \frac{1}{2}C_D\rho U^2 A$. Here, A = 2rL

  Using the above two, we get

$$\mu = \frac{F_d}{12UL}$$

  Using $F_g = F_b + F_d$, we get

$$\mu = \frac{\pi r^2 g\,(\rho_s - \rho_f)}{12U}$$

Our Initial Apparatus Design and Setup

- **Requirements**
  - Transparent tank of sufficient height to drop the particle, allow it to reach terminal velocity, and then let it drop certain distance to measure time and thus take experimental reading of terminal velocity.
- **Design:** Cylindrical tank with radius 7.5 cm and height 1m.
- **Material**: Acrylic.
- **Drainage Port**: Machined a hole in the base to allow fluid and particle drainage.

- Intended to use a two-valve system for controlled drainage, but modified to a single-valve system due to budget constraints.
- **Ball and Cylinder material:** Steel
- **Fluid to be used:** Glycerine

Our Final Apparatus Design and Setup

- **Design:** Square base tank with side 15 cm and height 1.2 m.
- **Material**: Acrylic.
- **Joining:** With the help of screws at the edges.
- **Leak-proofing:** Silicone**,** araldite, duct tape**.**
- Single-valve system reduced precision in controlling fluid drainage, affecting particle retrieval.
- **Ball and Cylinder material:** Teflon and Steel respectively.
- **Fluid to be used:** Water

Planned Experimental Procedure

1. **Preparation of Fluid and Particle**:
    - Take a Viscous fluid with known density ρ.
    - Prepare a spherical particle, and a cylindrical particle with an aspect ratio of 10.
2. **Setting Up the Experiment**:
    - Place the Viscous fluid in container which is tall enough so that particle reach terminal velocity before hitting bottom.
    - Mark a reference starting depth near the top of the container, and mark incremental depths for later tracking particle descent.
3. **Measuring Terminal Velocity for the Sphere**:
    - Release the sphere from the starting depth and use a stopwatch to record the time taken to reach each marked depth.
    - Track the velocity profile to identify when the sphere reaches terminal velocity U.
    - Calculate terminal velocity by taking the average velocity over the depths where the sphere's speed stabilizes.
4. **Verifying Reynolds Number**:
    - Calculate the Reynolds number R_e (= $\frac{\rho U D}{\mu}$) using the terminal velocity U and ensure it remains in the low range (laminar flow conditions) to validate Stokes' law applicability.
5. **Calculating Viscosity**:
    - Use Stokes' law, $F_d = 6\pi\mu a U$, to calculate the fluid viscosity μ based on the measured terminal velocity U.
6. **Measuring Drag and Terminal Velocity for the Cylinder**:
    - Repeat the release and timing steps for the cylindrical particle, ensuring it descends axially.

- Record the time taken to reach terminal velocity and calculate the terminal velocity for the cylinder.
- Calculate the drag force F_d on the cylinder using the appropriate drag coefficient C_d for a long cylinder at low Reynolds number conditions.

7. **Estimation and Analysis**:
    - Compare drag forces for the sphere and cylinder to analyse the scaling behaviour of drag between spherical and cylindrical shapes.
8. **Data Analysis and Conclusion**:
    - Plot and analyse depth vs. velocity profiles for both shapes, discussing any deviations from expected theoretical predictions.
    - Use the findings to conclude on fluid viscosity and the influence of particle geometry on drag, supported by theoretical calculations and observed data.

Sources of Error

1. Minor leaks or uneven joins may disturb fluid flow and particle path.
2. Minor inaccuracies in measuring descent times, especially at higher speeds.
3. Small variations in fluid density or viscosity due to impurities.
4. Misalignment of the cylinder may cause rotational motion, affecting drag readings.
5. Changes in fluid temperature may impact viscosity and Reynolds number.

Learning and take aways from the Project

1. This project improved my understanding of key fluid mechanics concepts, particularly the applicability of Stokes' law for calculating drag in low Reynolds number conditions. I gained insights into terminal velocity, settling time, settling depth, and the unique behaviour of drag on different shapes, including the influence of a cylinder's aspect ratio on its drag coefficient across flow regimes.
2. By calculating Reynolds numbers, I learned to distinguish between flow regimes (laminar vs. transitional and turbulent) and the significance of these distinctions when predicting particle behaviour in a Viscous fluid.
3. Constructing the experimental setup allowed me to develop practical skills in assembling a custom fluid tank. I learned to bond acrylic sheets with silicone for waterproofing and used araldite as a strong adhesive for base joints, ensuring the setup was leak-proof and structurally stable.
4. Working with laser-cutting equipment to cut acrylic sheets taught me precision fabrication techniques, while drilling and joining with screws improved my skills in constructing modular setups that could be assembled or disassembled as needed.

Objectives Completed

1. All required theoretical calculations for drag force, terminal velocity, viscosity, and depth before terminal velocity have been derived and documented for both spherical and cylindrical particles.
2. A transparent, waterproof tank was successfully designed and built using laser-cut acrylic sheets, with silicone and Araldite used to ensure water-tight joints.
3. Got a spherical particle of appropriate size to achieve favourable Reynolds number conditions and a cylindrical particle with an aspect ratio of 10, allowing for accurate drag comparisons.
4. The experiment setup, including release mechanisms and marking depths, was finalized to allow controlled measurements of particle descent and terminal velocity, supporting viscosity calculations.