3. (15 points) PCA and Hyperplane Fitting.

3.a)(5 points) How can principal component analysis (PCA) be used to best approximate a linear relationship between random variables X and Y . Describe the method clearly, using appropriate mathematical descriptions for clarity. Your description should be clear enough to lead to a programmable implementation

Solution:

⇒ Acc to given, we have 2 random variables X and Y and now we have sampling points (x,y)

⇒ Now that we have many points (x,y) we can find an approximate linear relation between X and Y and this is best accomplished by PCA.

⇒ In many Situations , we have large amounts of data and it is tough to handle such a huge amount of data , So we can do reduction of data such that the loss of information is minimized.

⇒ This can be done by following the below steps

⇒ step 1: First find the mean of the distribution (x0,y0) by just taking avg Separately on X and Y

⇒ Now just shift the points such that the mean is the origin.

⇒ Now that we have all points whose mean is (0,0). find a line that passes through the origin and exists in such a way that the sum of distances of projection of points from the origin is maximised. Or the distance of points from the line is minimised.

⇒ This is PCA1.

⇒ and line perpendicular to the PCA1 forms PCA2

⇒ Now the slope of PCA defines the linear relation between Y and X.

⇒ Now How can we easily execute the above process, PCA works here…

⇒ First standardise of the date (missing out will result in biased outcome)

⇒ Computing the covariance matrix. It is essential to identify heavily dependent variables because they contain biased and redundant info which reduces overall performance.

Covariance Variance and mean is computed in below script…

```
def find_ML_estimates(final_X):
    data_mean = np.matrix([[0.0], [0.0]])
    data_cov  = np.matrix([[0.0, 0.0], [0.0, 0.0]])
    n = len(final_X)
    for vec in final_X:
        data_mean += vec
```

```
        data_cov  += vec*vec.transpose()

    data_mean /= n
    data_cov /= n
    data_cov -= data_mean * data_mean.transpose()

    return (data_mean, data_cov)
```

⇒ Principal Components are basically a new set of variables that are obtained from the initial set . They compress and possess most of the useful information that was scattered among the initial values.

⇒ Now calculate eigenvalues and eigenvectors of the covariance matrix. These play a key role in determining PCA Components.

Computing eigenvalues and eigenvectors

```
[e_values, e_vectors] = np.linalg.eig(covar)
idx = np.argsort(e_values)[::-1]
e_values = e_values[idx]
e_vectors = e_vectors[:,idx]
```

Computing PCA1 and PCA2

```
PCA1 = e_values[0]*e_vectors[:,0]
PCA2 = e_values[1]*e_vectors[:,1]
```

⇒ For a note PCA1 is the most significant and stores the maximum possible info.similarly PCA2 is second most and so on..

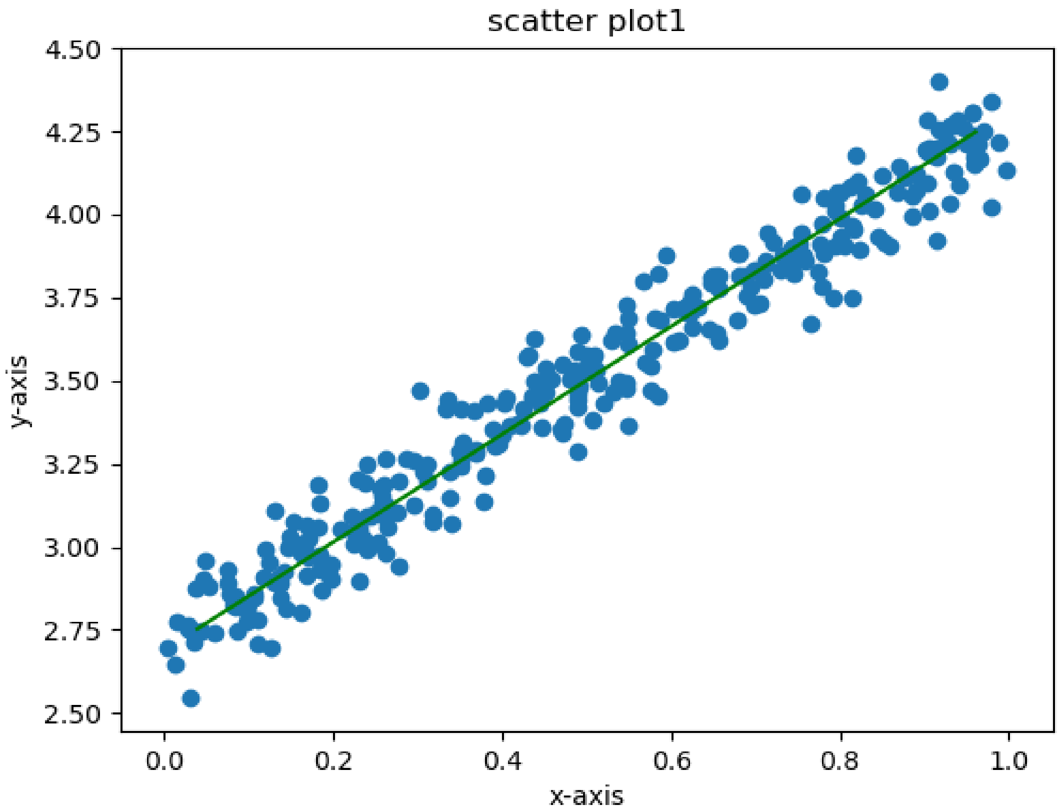
⇒ Now order the eigenvectors in descending order of eigenvalues such that the first one is PCA1 and so on

⇒ **So, linear relation between Y and X is the slope of PCA1**

3b)(5 points) Show a scatter plot of the points. Overlay on the scatter plot, the graph of a line showing the linear relationship between Y and X.

Plot:

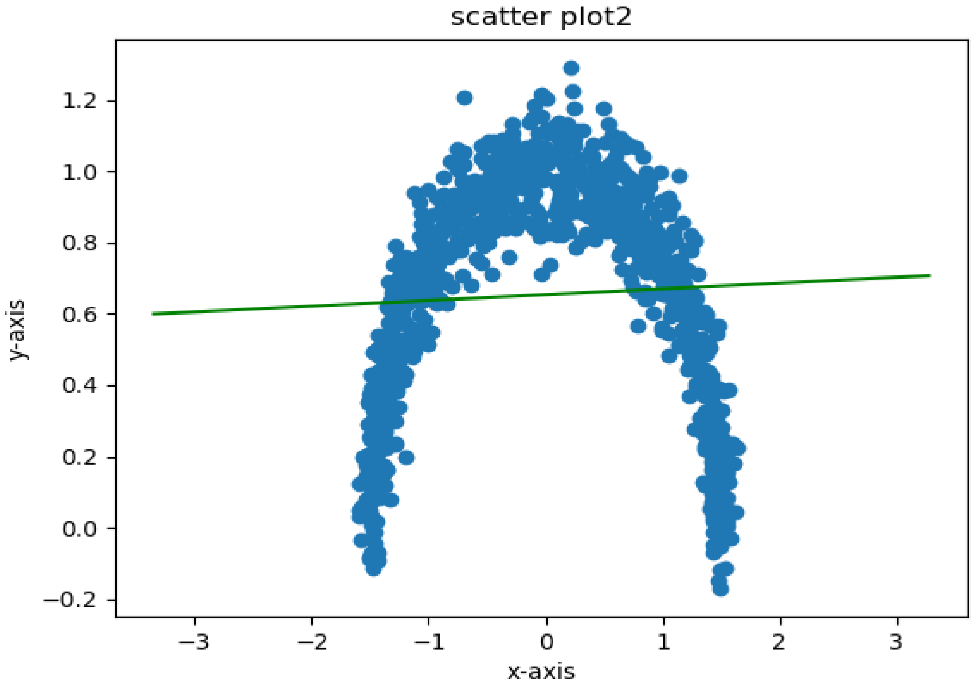
The green line is PCA1 and this is the scatter plot for `points2D_Set1.mat`



3c)(5 points) Repeat the same analysis for the set of points in "points2D_Set2.mat". Show a scatter plot of the points. Overlay on the scatter plot, the graph of a line showing the linear relationship between Y and X. Compared to the result on the other set of points, justify the quality of the approximation resulting in this question using logical arguments.

Solution:

The green line is PCA1 and this is the scatter plot for `points2D_Set2.mat`



⇒ **By Observing the above 2 plots we can say that in both cases Y is positively correlated with X Because the PCA1 has a positive slope.**

⇒ **But we can say that the quality of approximation is more for the first plot rather than the second plot.**

⇒ **This is because The line covers a lot of points in case 1 and also the graph is more linear and we can say that the distance of points from the line are minimized very well.**

⇒ **But in the second case we have the scatter plot in the shape of an arc, clearly the linear relation between X and Y is not very helpful to analyse the data.**