Reinforcement Learning
Tutorial for Lecture 4

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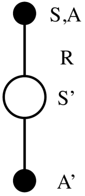
$$\pi(a \mid s) = \begin{cases} \frac{\epsilon}{m} + 1 - \epsilon & \text{if } a^* = \arg\max_{a \in \mathcal{A}} Q(s, a) \\ \frac{\epsilon}{m} & \text{otherwise} \end{cases}$$

SARSA

Simplest TD Q update formula is the **S A R S A** update. It is "on-policy".
Can you write down the update for Q?



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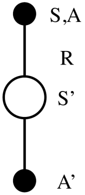

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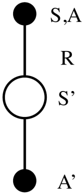

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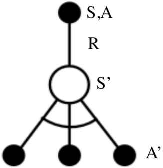

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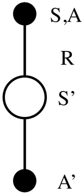

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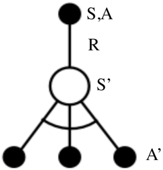

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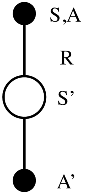

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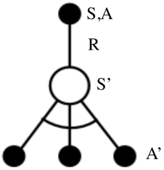

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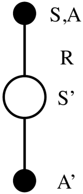

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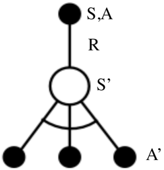
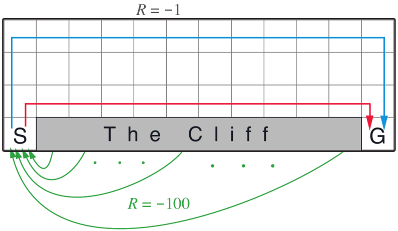


Why is one called on-policy and the other one off-policy? Solution: Sarsa uses the action A' produced by π (on-policy). Q-Learning uses the best local A' for value-backup (off-policy).

SARSA vs. Q: Cliff Walking Example



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1. Which path will be learned by Q-Learning?
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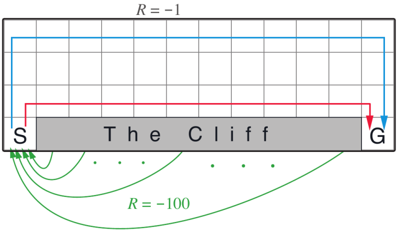

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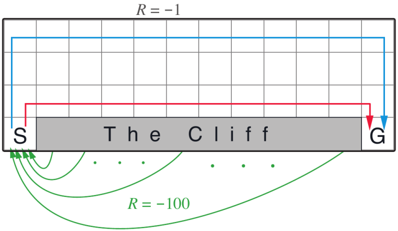

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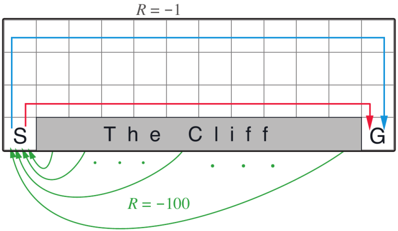

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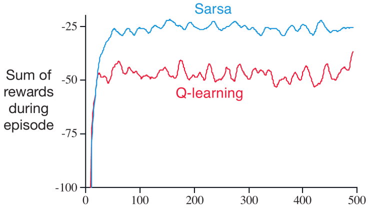
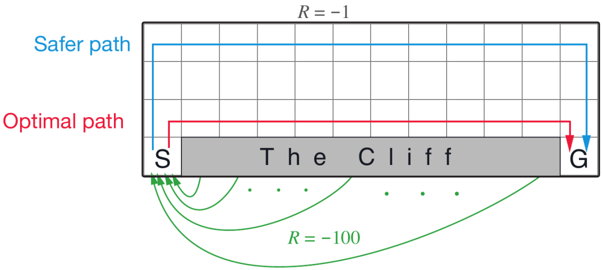
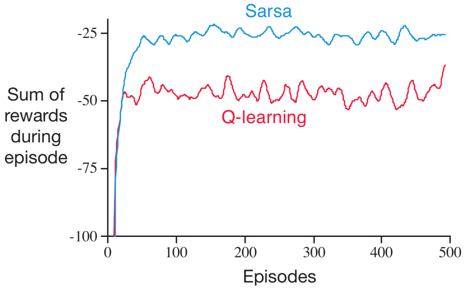
4. Are you sure after seeing these curves?



Cliff Walking Example (Slide from Lecture)

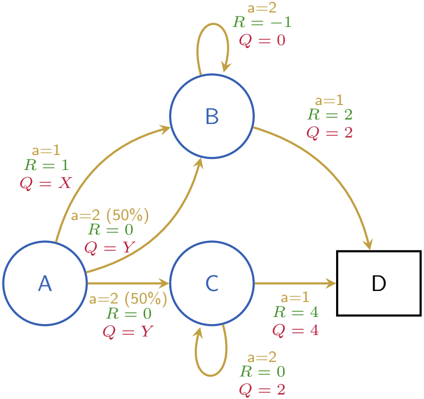




Q-learning learns the optimal policy, but shows lower online performance
(falls occationally off the cliff)

SARSA learns to cope with the ϵ-greedy policy: has to choose a saver but less optimal path.

Q-Update for SARSA and Q



The MDP on the left is unknown to us!

Let's say the Q value for state B,C,D are already estimated. Compute the Q values X,Y (initialized 0) from the following transitions from the ϵ-Greedy policy. $\alpha = 0.5, \gamma = 0.5$

	S	A	R	S'	A'	SARSA: $Q'(S, A)$	Q-learning: $Q'(S, A)$
τ_1	A	2	0	C	2		
τ_2	A	1	1	B	2		
τ_3	A	1	1	B	1		

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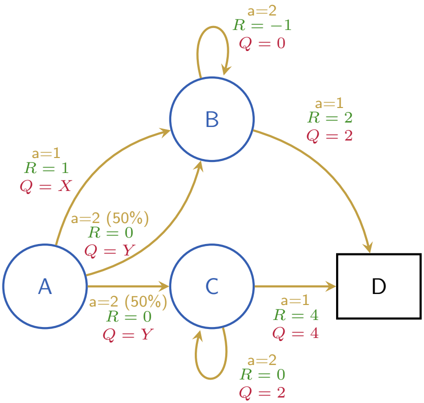

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τ_1	A	2	0	C	2	0.5 = 0.5(0+0.5*2)	1 = 0.5(0+0.5*4)
τ_2	A	1	1	B	2	0.25 = 0.5*(1 + 0.5*(-1))	1 = 0.5*(1 + 0.5*2)
τ_3	A	1	1	B	1	1.125 = 0.25 + 0.5*(1+0.5*2-0.25)	1.5 = 1 + 0.5*(1+0.5*2 - 1)

