Week 2 Practice Problem Solutions

1. $\boxed{f'(x) = 6x^2 - 12x^3}$ $f'(1) = 6(1)^2 - 12(1)^3 = \boxed{-6}$

   $\boxed{g'(x) = -x^{-2} + 3}$ $g'(1) = -(1)^{-2} + 3 = \boxed{2}$

2.
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import tensorflow as tf

x = tf.Variable(1.0)

with tf.GradientTape() as tape:
    f = 2 * tf.pow(x, 3) - 3 * tf.pow(x, 4) + 3
print(tape.gradient(f, x))   # ← outputs -6

with tf.GradientTape() as tape:
    g = tf.pow(x, -1) + 3 * x
print(tape.gradient(g, x))   # ← outputs 2
```

3. $f'(x) = 0$

   $6x^2 - 12x^3 = 0$

   $x^2 - 2x^3 = 0$

   $x^2(1-2x) = 0$

   $\boxed{x = \{0 \text{ or } \tfrac{1}{2}\}}$

   $g'(x) = 0$

   $-x^{-2} + 3 = 0$

   $-x^{-2} = -3$

   $x^{-2} = 3$

   $\frac{1}{x^2} = 3$

   $\frac{1}{3} = x^2$

   $\boxed{x = \pm\sqrt{1/3}}$

   If we plug in $\{0, \frac{1}{2}\}$ to $f(x)$ and $\pm\frac{1}{3}$ to $g(x)$, we notice that $f(0)$, $f(\frac{1}{2})$, $g(-\frac{1}{3})$, and $g(\frac{1}{3})$ are <u>all relative/local maximum/minimum</u> values. This is because the rate of change (i.e., the derivative) is 0 at the extrema. Therefore, the zeros of the derivative function correspond to min/max values.