DIGITAL IMAGE PROCESSING

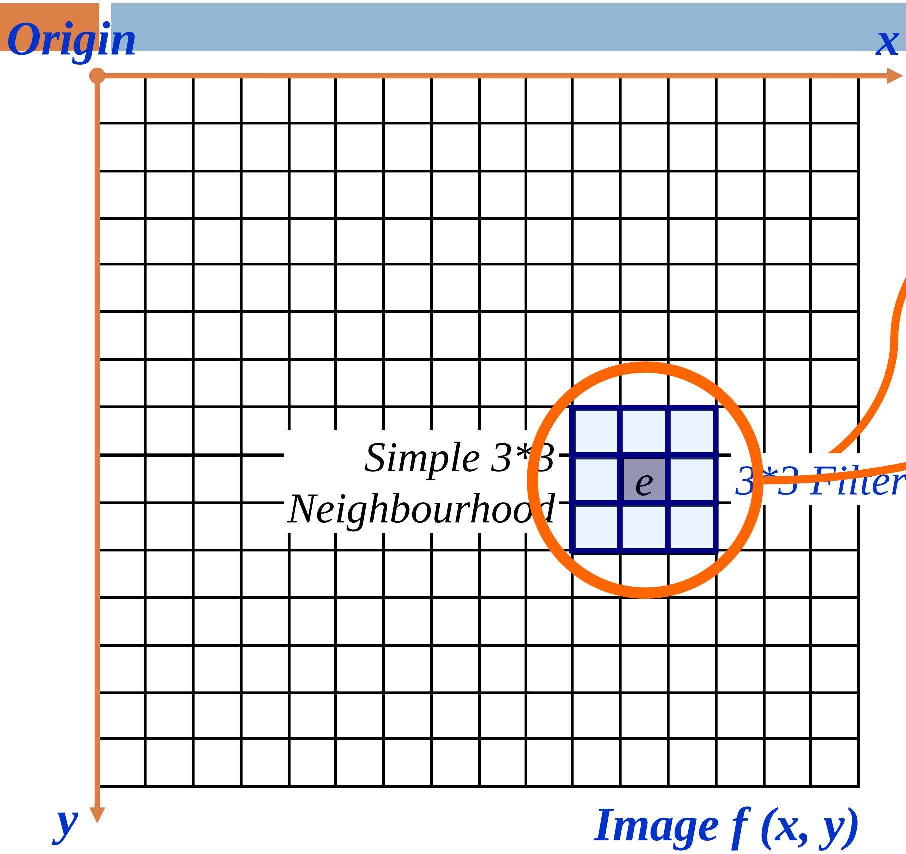
Image Enhancement
(Spatial Filtering 2)

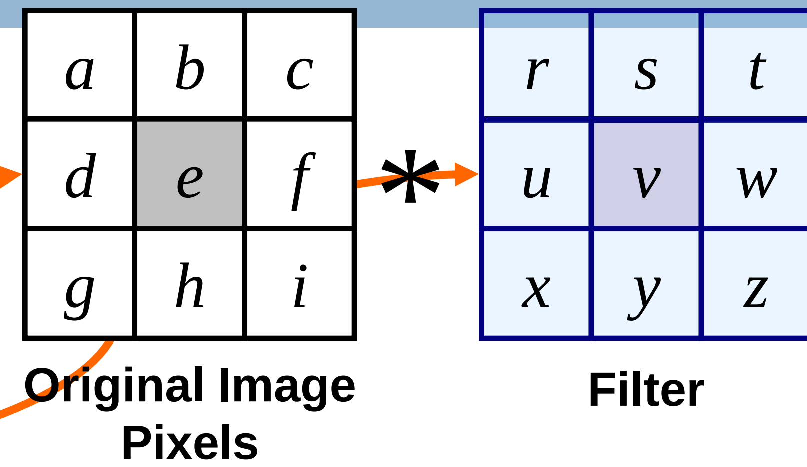
Contents

In this lecture we will look at more spatial filtering techniques

- Spatial filtering refresher
- Sharpening filters
  - 1^{st} derivative filters
  - 2^{nd} derivative filters
- Combining filtering techniques

Spatial Filtering Refresher





$$
\begin{aligned}
e_{processed} = {} & v*e + \\
& r*a + s*b + t*c + \\
& u*d + w*f + \\
& x*g + y*h + z*i
\end{aligned}
$$

The above is repeated for every pixel in the original image to generate the smoothed image

Sharpening Spatial Filters

Previously we have looked at smoothing filters which remove fine detail

Sharpening spatial filters seek to highlight fine detail

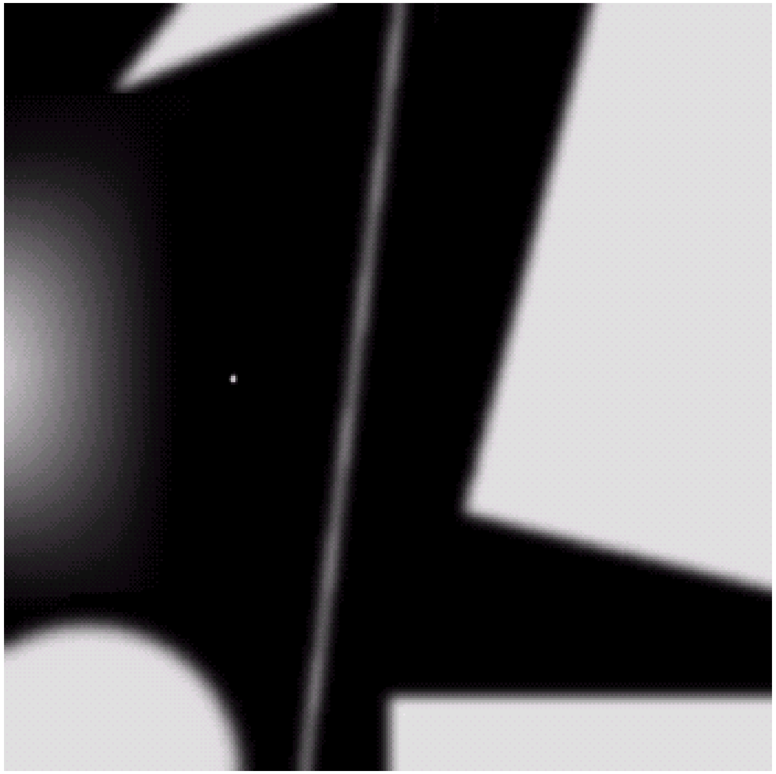
- Remove blurring from images
- Highlight edges

Sharpening filters are based on *spatial differentiation*

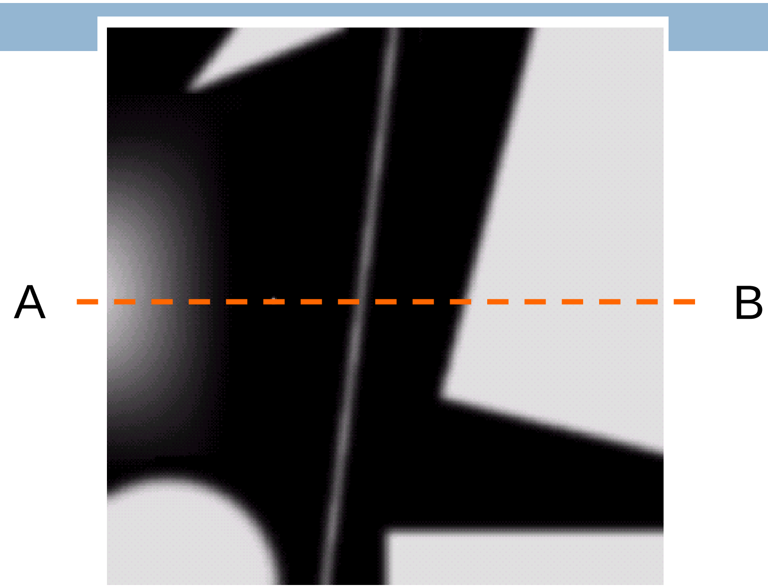
Spatial Differentiation

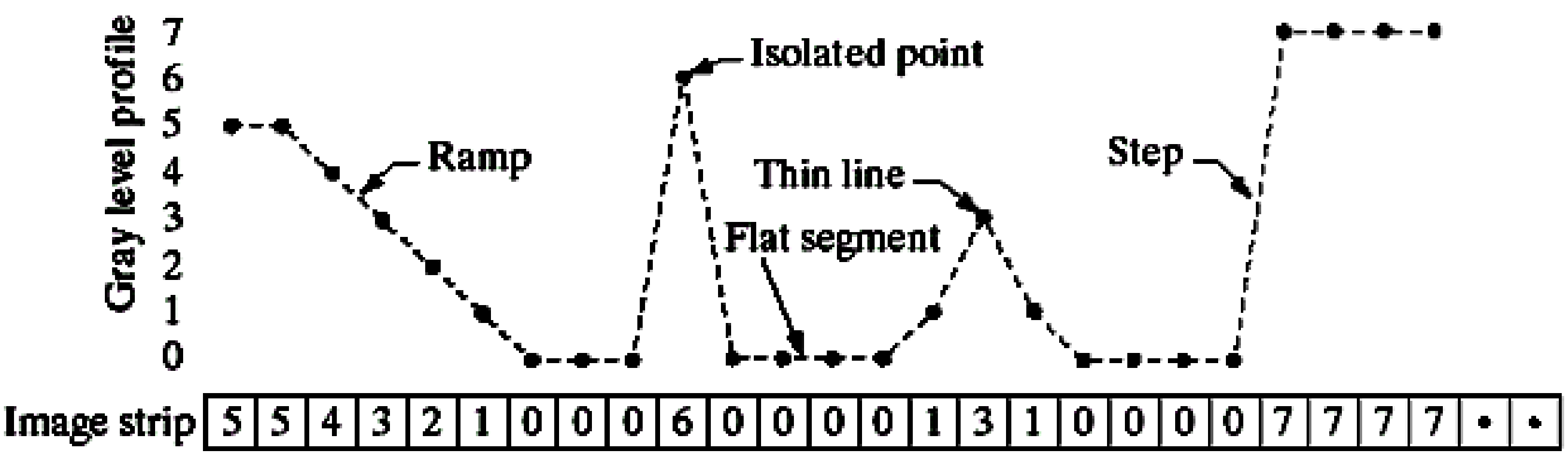
Differentiation measures the *rate of change* of a function

Let's consider a simple 1 dimensional example



Spatial Differentiation





Images taken from Gonzalez & Woods, Digital Image Processing (2002)

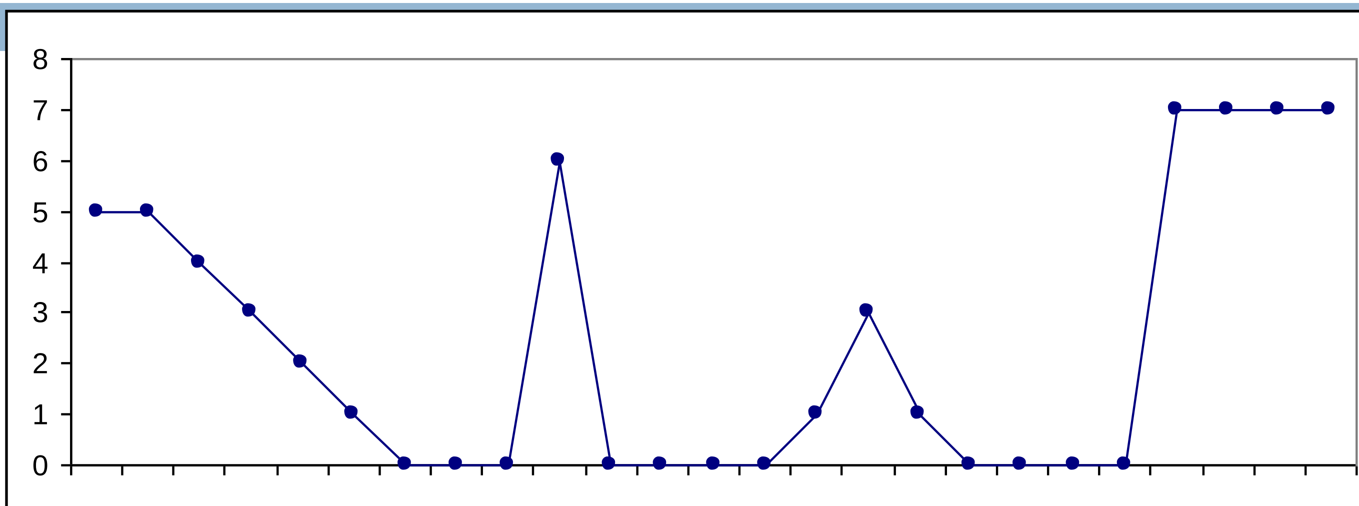
1st Derivative

The formula for the 1st derivative of a function is as follows:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

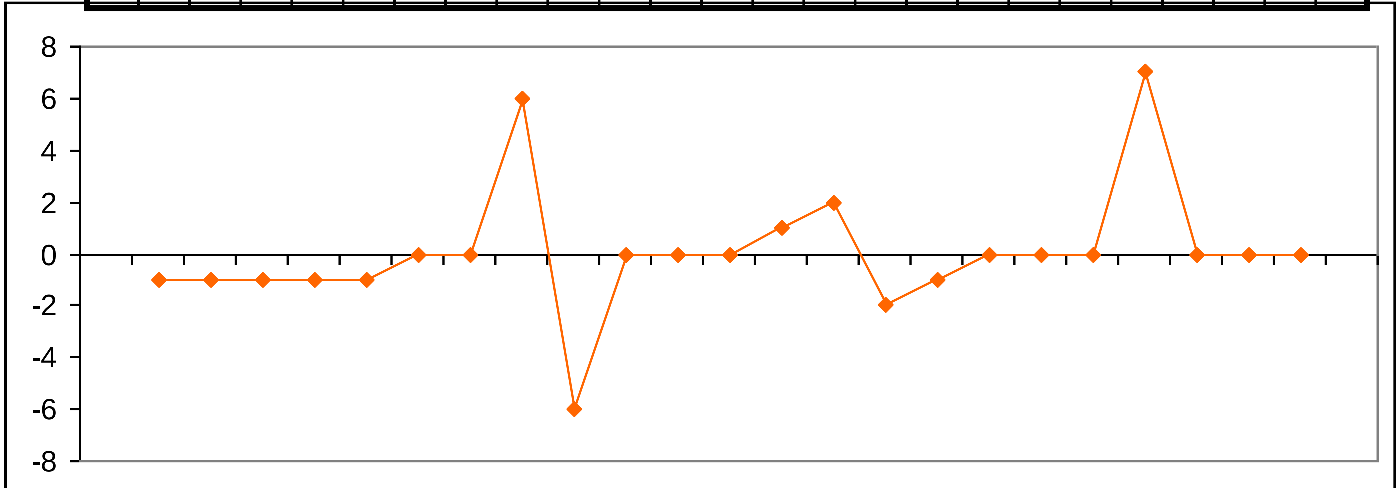
It's just the difference between subsequent values and measures the rate of change of the function

1 st Derivative (cont…)



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7

	0	-1	-1	-1	-1	0	0	6	-6	0	0	0	1	2	-2	-1	0	0	0	7	0	0	0	



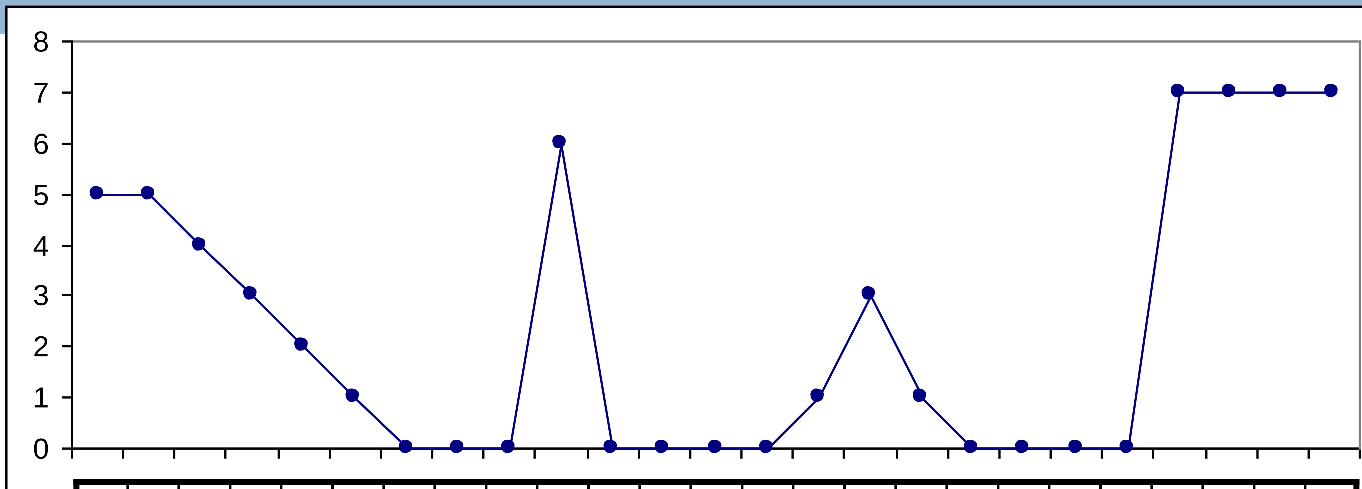
2^{nd} Derivative

The formula for the 2^{nd} derivative of a function is as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

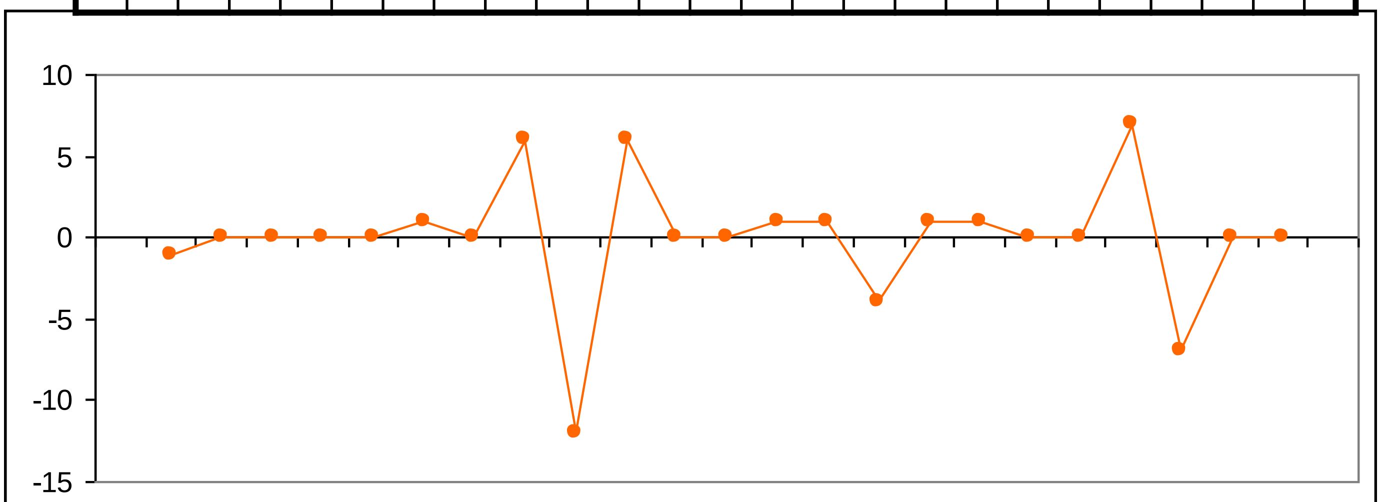
Simply takes into account the values both before and after the current value

2^{nd} Derivative (cont…)



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7

	-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0	



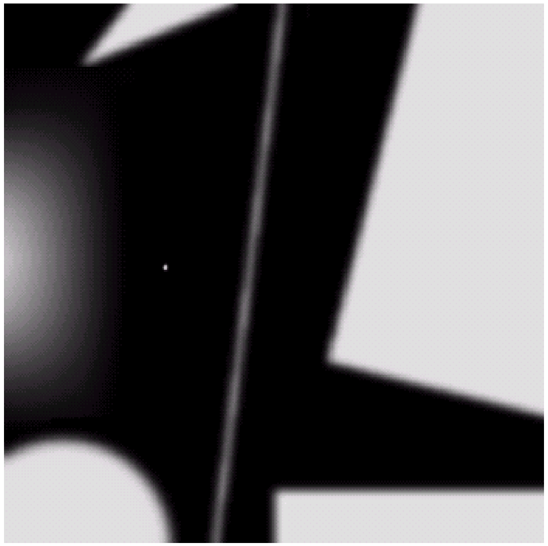
Sharpening Spatial filters

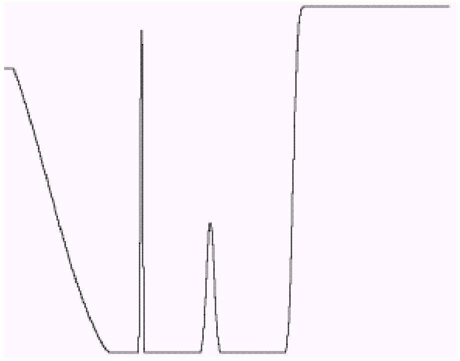
$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

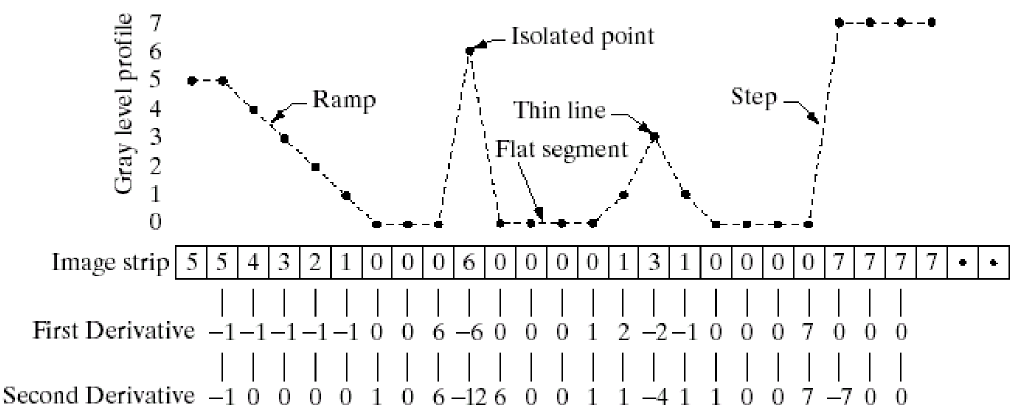
$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

First order is non-zero along the entire ramp
Second order is non-zero only in the beginning and end

First order – thick edges
Second order – fine edges
Second order – stronger response to fine detail







Sharpening Spatial filters

Sharpening filters using first- second order derivatives
First derivative:

1. Zero in flat segments (constant gray level)
2. Non-zero at start of a gray-level step/ ramp
3. Non-zero along ramps

Second derivative:

1. Zero in flat areas
2. Non-Zero at start and end of step/ramps
3. Zero along ramps of constant slope

The aim is to develop a technique which can identify changes (of different nature) in the gray levels

1st & 2nd Derivatives

Comparing the 1st and 2nd derivatives we can conclude the following:

- 1st order derivatives generally produce thicker edges
- 2nd order derivatives have a stronger response to fine detail e.g. thin lines
- 1st order derivatives have stronger response to grey level step
- 2nd order derivatives produce a double response at step changes in grey level

Using Second Derivatives For Image Enhancement

The 2nd derivative is more useful for image enhancement than the 1st derivative

- Stronger response to fine detail
- Simpler implementation
- We will come back to the 1st order derivative later on

The first sharpening filter we will look at is the *Laplacian*

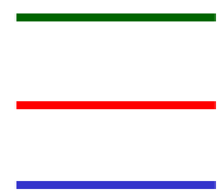
- Isotropic
- One of the simplest sharpening filters
- We will look at a digital implementation

Various situations encountered for derivatives

$$f' = \frac{\partial f}{\partial x} \qquad f'' = \frac{\partial^2 f}{\partial x^2}$$

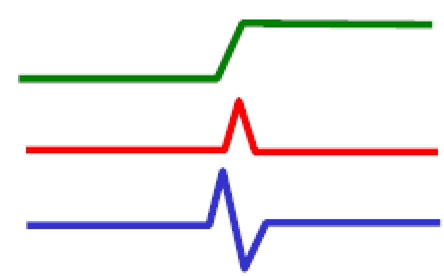
- Flat segment→ $(f')=0$; $(f'')=0$

f	0	0	0	0	0	
f'		0	0	0	0	
f''		0	0	0		



- Step→ (f'):{0,+,0}; (f''):{0,+,–,0}

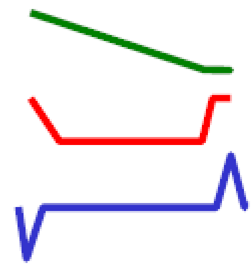
f	0	0	0	7	7	7	7	
f		0	0	7	0	0	0	0
f''		0	7	–7	0	0	0	



Various situations encountered for derivatives

•Ramp→ (f')≈constant; (f'')=0

f	5	4	3	2	1	0	0	
f'	0	–1	–1	–1	–1	–1	0	0
f''	–1	0	0	0	0	1	0	

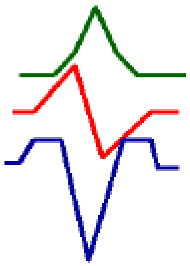


- *Ramps or steps* in the 1D profile normally characterize the edges in an image
- f'' is nonzero at the onset and end of the ramp: produce thin (double) edges
- f' is nonzero along the entire ramp produce thick edges

Various situations encountered for derivatives

- Thin lines

f	0	0	1	3	1	0	0
f'	0	0	1	2	–2	–1	0	0
f''	0	1	1	–4	1	1	0



- Isolated point

f	0	0	0	6	0	0	0
f'		0	0	6	–6	0	0	0
f''		0	6	–12	6	0	0

f'' responses much stronger than f' around the point

f'' enhances fine detail (including noise) much more than f'

The Laplacian

The Laplacian is defined as follows:

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

where the partial 2nd order derivative in the x direction is defined as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

and in the y direction as follows:

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

The Laplacian (cont…)

So, the Laplacian can be given as follows:

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

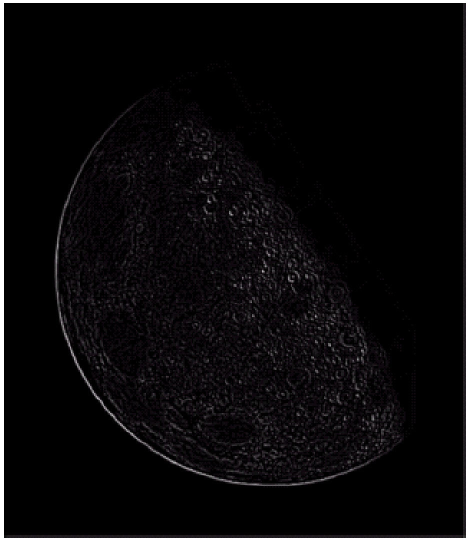
We can easily build a filter based on this

0	1	0
1	-4	1
0	1	0

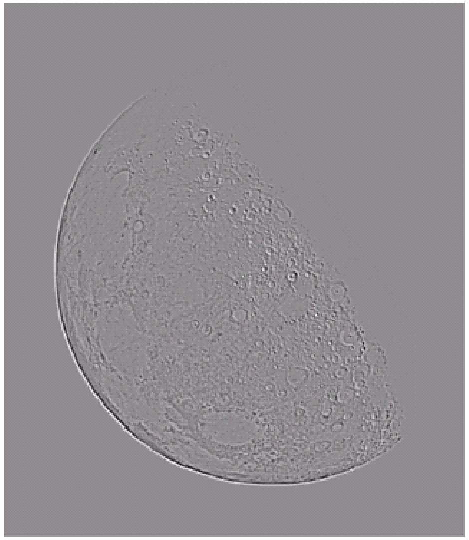
The Laplacian (cont…)

Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities

Original
Image



Laplacian
Filtered Image

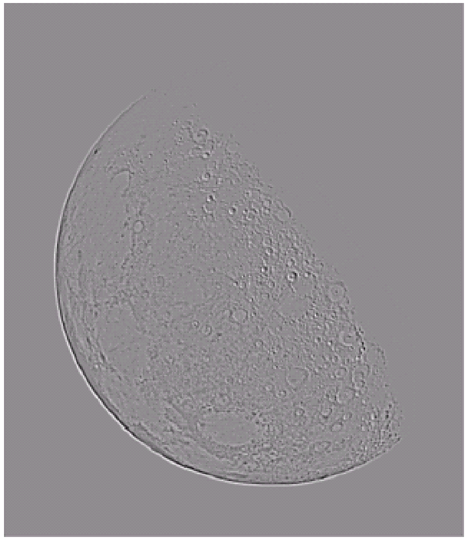


Laplacian
Filtered Image
Scaled for Display

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

But That Is Not Very Enhanced!

- The result of a Laplacian filtering is not an enhanced image
- We have to do more work in order to get our final image
- Subtract the Laplacian result from the original image to generate our final sharpened enhanced image



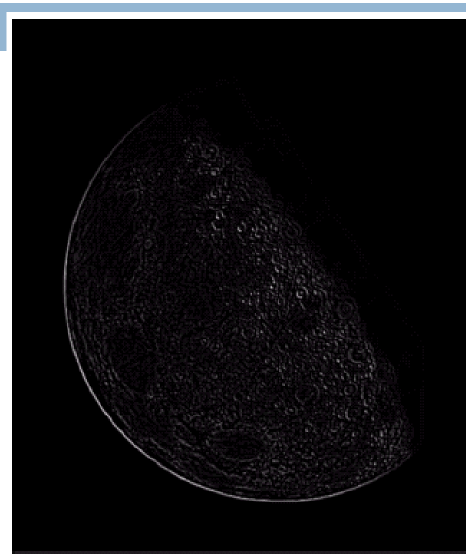
Laplacian Filtered Image Scaled for Display

$$g(x,y) = f(x,y) - \nabla^2 f$$

Laplacian Image Enhancement

Original Image

-



Laplacian Filtered Image

=

Sharpened Image

In the final sharpened image edges and fine detail are much more obvious

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Laplacian Image Enhancement

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Variant of Laplacian

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Isotropic for rotations in increments of 90°

$$+$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -4 & 0 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Isotropic for rotations in increments of 45°

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

a	b
c	d

FIGURE 3.39
(a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

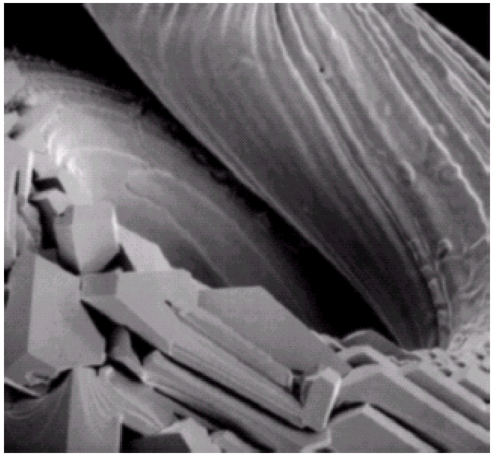
Simplified Image Enhancement

The entire enhancement can be combined into a single filtering operation

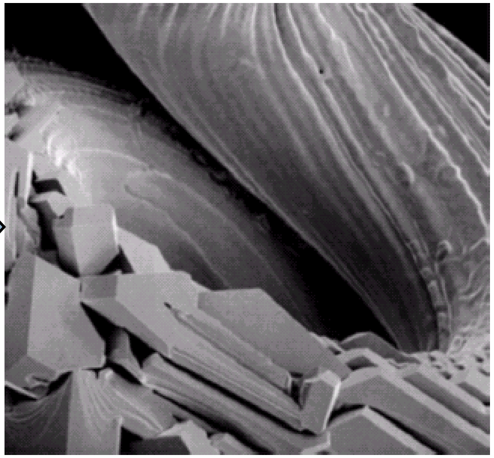
$$
\begin{aligned}
g(x,y) &= f(x,y) - \nabla^2 f \\
&= f(x,y) - [f(x+1,y) + f(x-1,y) \\
&\qquad + f(x,y+1) + f(x,y-1) \\
&\qquad - 4f(x,y)] \\
&= 5f(x,y) - f(x+1,y) - f(x-1,y) \\
&\qquad - f(x,y+1) - f(x,y-1)
\end{aligned}
$$

Simplified Image Enhancement (cont…)

This gives us a new filter which does the whole job for us in one step

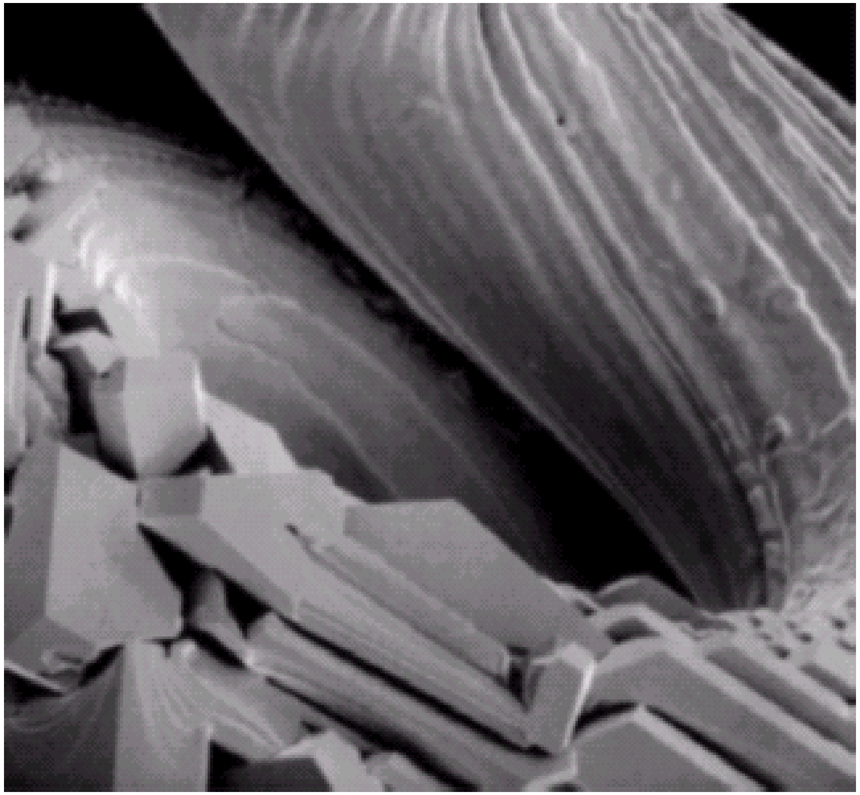


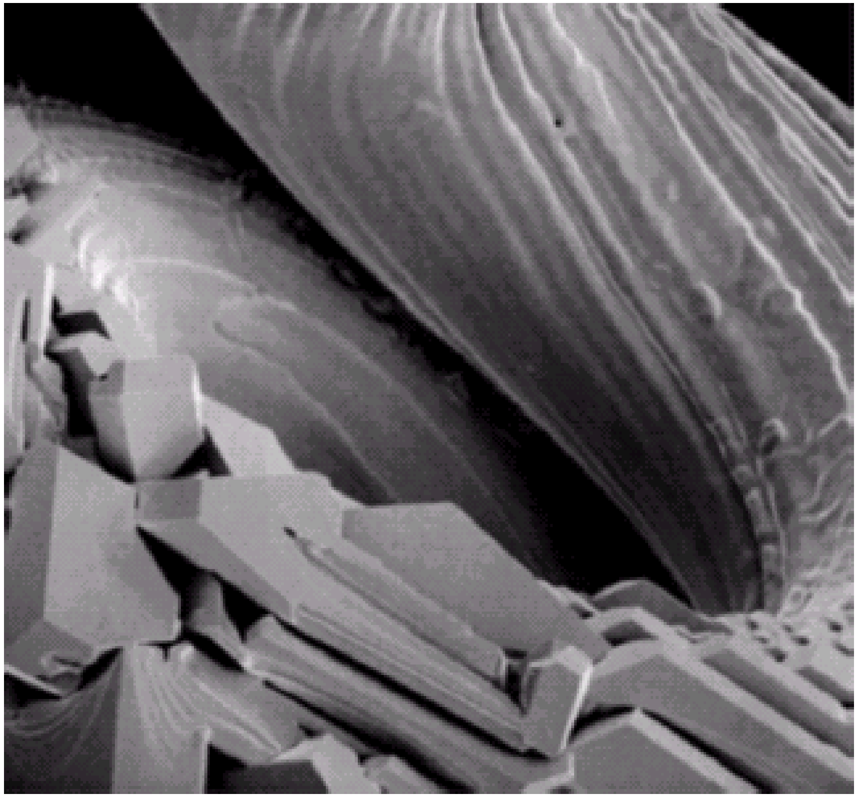
0	-1	0
-1	5	-1
0	-1	0



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Simplified Image Enhancement (cont…)





Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Variants On The Simple Laplacian

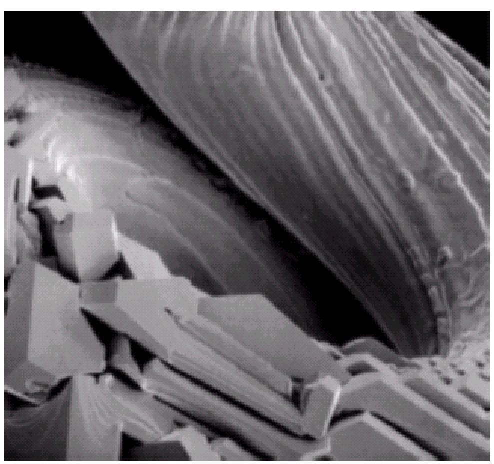
There are lots of slightly different versions of the Laplacian that can be used:

0	1	0
1	-4	1
0	1	0

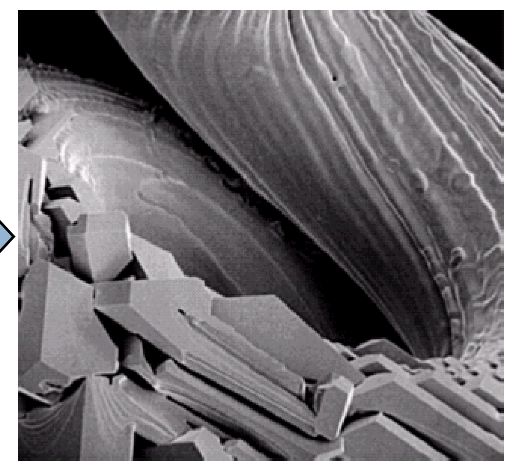
Simple Laplacian

1	1	1
1	-8	1
1	1	1

Variant of Laplacian



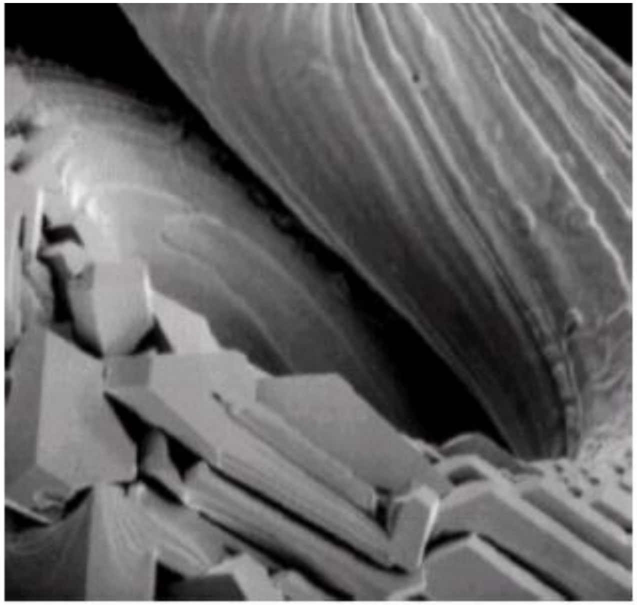
-1	-1	-1
-1	9	-1
-1	-1	-1

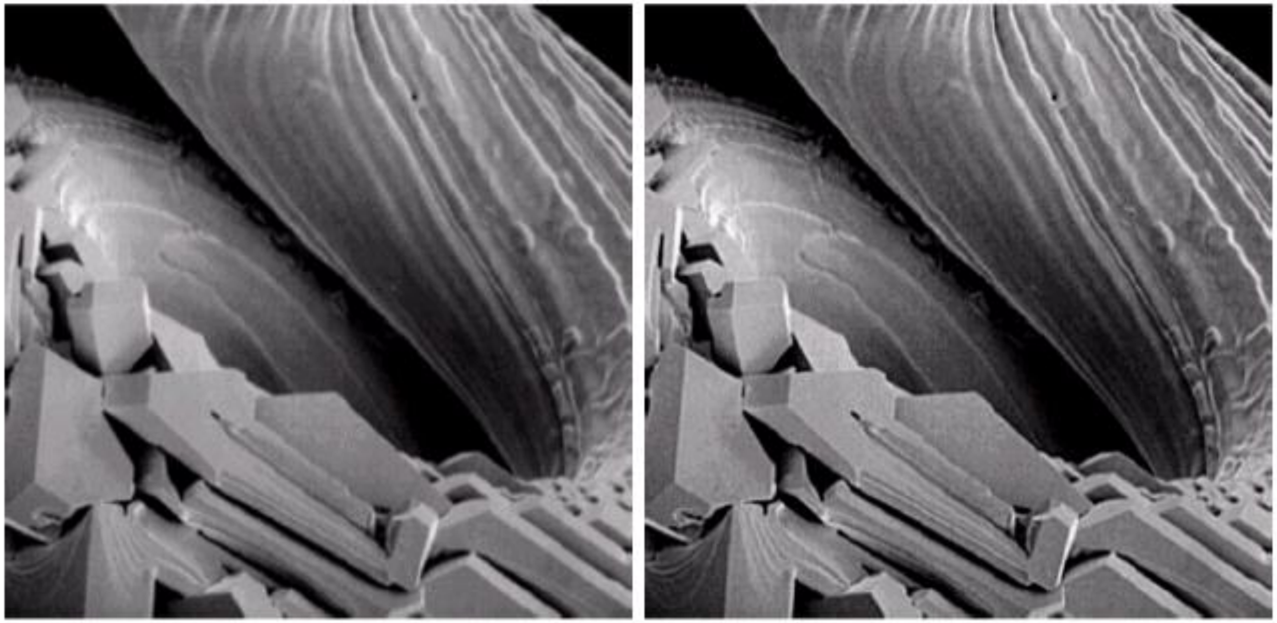


Images taken from Gonzalez & Woods, Digital Image Processing (2002)

0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1





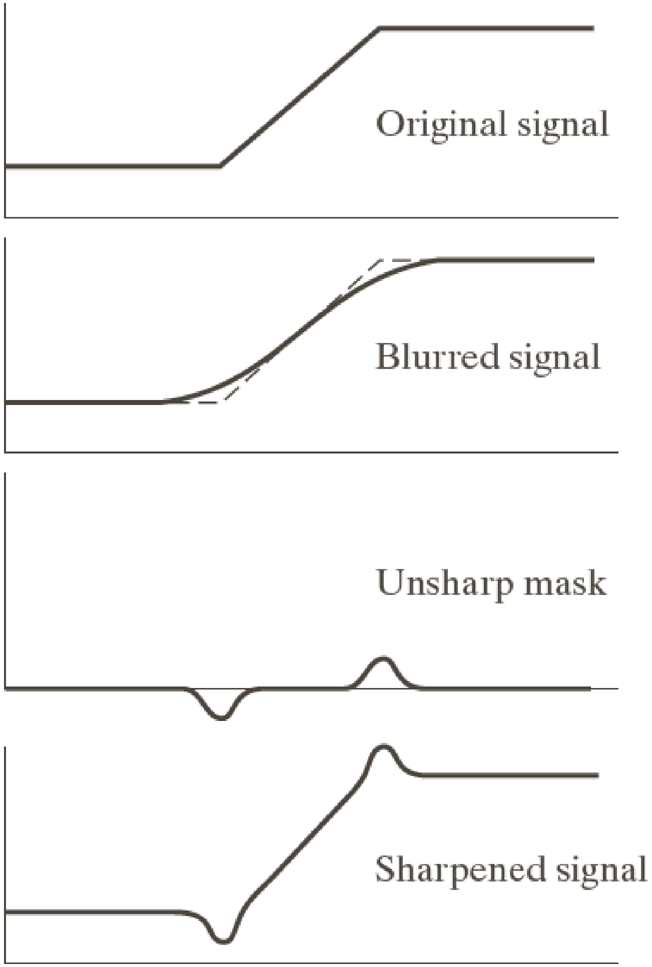
a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Unsharp Masking and Highboost filtering

- The process of Unsharp making consists of the following steps:
  - **Blur** the original image
  - **Subtract** the blurred image from the original (the result is called the **mask**)
  - **Add the mask** to the original image
- Let $\bar{f}(x,y)$ denote the blurred image
- The unsharp masking is: $g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$
- To obtain the output: $g(x,y) = f(x,y) + k * g_{mask}(x,y)$
- When 'k>1' the process is called Highboost filtering

Unsharp Masking and Highboost filtering



1st Derivative Filtering

Implementing 1st derivative filters is difficult in practice

For a function $f(x, y)$ the gradient of f at coordinates (x, y) is given as the column vector:

$$\nabla \mathrm{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

1st Derivative Filtering (cont…)

The magnitude of this vector is given by:

$$\begin{aligned}\nabla f &= mag(\nabla \mathrm{f}) \\ &= \left[G_x^2 + G_y^2\right]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}\end{aligned}$$

For practical reasons this can be simplified as:

$$\nabla f \approx \left|G_x\right| + \left|G_y\right|$$

1st Derivative Filtering (cont…)

There is some debate as to how best to calculate these gradients but we will use:

$$\nabla f \approx \left|(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)\right| + \left|(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)\right|$$

which is based on these coordinates

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Sobel Operators

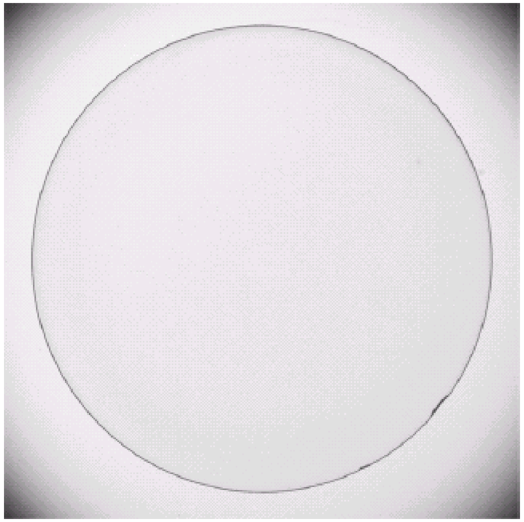
Based on the previous equations we can derive the *Sobel Operators*

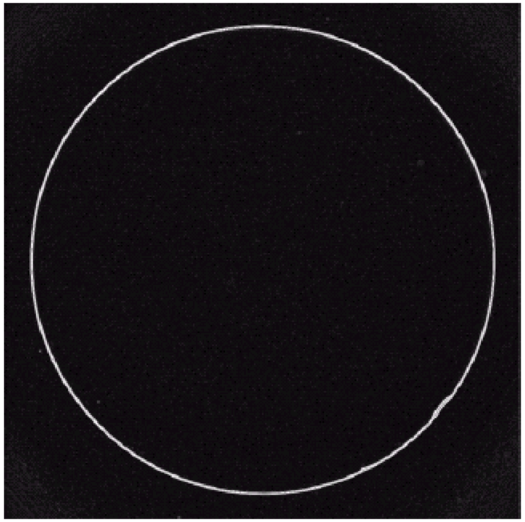
-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

To filter an image it is filtered using both operators the results of which are added together

Sobel Example





An image of a contact lens which is enhanced in order to make defects (at four and five o'clock in the image) more obvious

Sobel filters are typically used for edge detection

Directional Derivative

- Let f(x, y) be a function mapping two *real* numbers to a real number (intensity values).

- For the directional derivative of f along the x axis, we use notation df/dx.
  - Vertical edges correspond to points in g with high df/dx.
- For the directional derivative of f along the y axis, we use notation df/dy.
  - Horizontal edges correspond to points in g with high dg/dy.

Approximating df/dx via Filtering

- In the discrete domain df/dx is approximated by filtering with the right kernel:

```
dx = [-1 0 1;
      -2 0 2;
      -1 0 1];
dxgray = abs(filter2(gray, dx));
```

- Interpreting `filter2(gray, dx)`:
  - Results far from zero (positive and negative) correspond to strong vertical edges.
    - These are mapped to high positive values by **`abs`**.
  - Results close to zero correspond to weak vertical edges, or no edges whatsoever.

Result: Vertical/Horizontal Edges

```
gray = read_gray('data/myhand.bmp');
dx = [-1 0 1;
      -2 0 2;
      -1 0 1];
dy = dx'; % dy is the transpose of dx
dxgray = abs(imfilter(gray, dx, 'symmetric', 'same'));
dygray = abs(imfilter(gray, dy, 'symmetric', 'same'));
```

gray

dxgray
(vertical edges)

dygray
(horizontal edges)

Summary

In this lecture we looked at:

- Sharpening filters
  - 1st derivative filters
  - 2nd derivative filters
- Combining filtering techniques

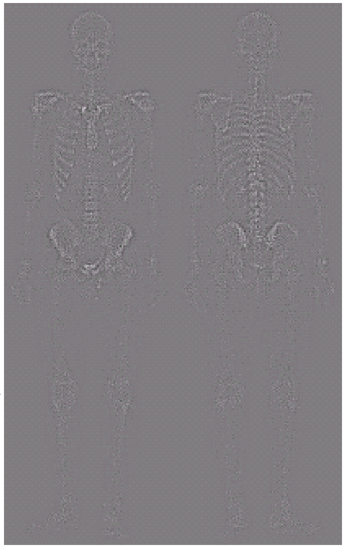
Combining Spatial Enhancement Methods

- Successful image enhancement is typically not achieved using a single operation
- Rather we combine a range of techniques in order to achieve a final result
- This example will focus on enhancing the bone scan to the right

Combining Spatial Enhancement Methods (cont…)

(a)

Laplacian filter of bone scan (a)



(b)

Sharpened version of bone scan achieved by subtracting (a) and (b)

(c)

Sobel filter of bone scan (a)

(d)

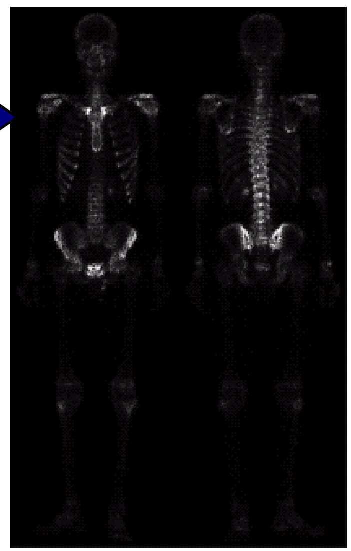
Combining Spatial Enhancement Methods (cont…)

Image (d) smoothed with a 5*5 averaging filter

(e)

The product of (c) and (e) which will be used as a mask

(f)

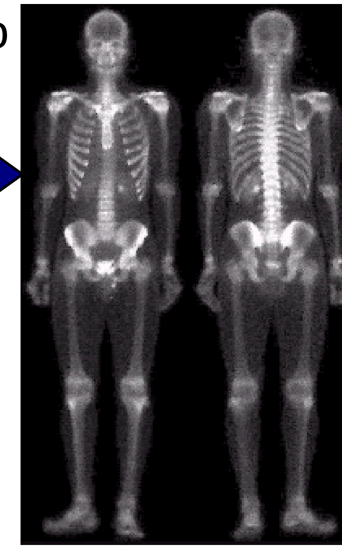


Sharpened image which is sum of (a) and (f)

(g)

Result of applying a power-law trans. to (g)

(h)



Combining Spatial Enhancement Methods (cont…)

Compare the original and final images