Poisson

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$x_1, \dots x_n$ are n data points such that are independent drawn from the above distribution

$$L(x,\lambda) = \prod_{i=1}^{n} P(X=x_i)$$

$$\log L(x,\lambda) = \sum_{i=1}^{n} (x_i \log\lambda - \lambda - \log(x_i!))$$

$$\frac{\partial}{\partial\lambda} \log(L(x,\lambda)) = 0$$

$$n = \frac{\sum_{i=1}^{n} x_i}{\lambda}$$

2) $$\boxed{\lambda = \frac{\sum_{i=1}^{n} x_i}{n}}$$

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Binomial

x_i = no. of success in n trials.

Let us record,

N observations , each observation has 'n' trials.

$$L(x_i, p) = \prod_{i=1}^{N} {}^{n}C_{x_i} p^{x_i} (1-p)^{n-x_i}$$

$$\log(L(x_i,p)) = \sum_{i=1}^{N} x_i \log p + \sum_{i=1}^{N} (n-x_i)\log(1-p) + \log {}^{n}C_{x_i}$$

$$\frac{\partial(\log(L(x_i,p)))}{\partial p} = \sum_{i=1}^{N} \frac{x_i}{p} - \sum_{i=1}^{N} \frac{n-x_i}{1-p} = 0$$

$$\frac{\sum x_i}{p(1-p)} = \frac{Nn}{(1-p)}$$

$$\Rightarrow p = \left(\frac{\sum x_i}{N}\right) \Big/ n$$

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Exponential

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{else} \end{cases}$$

Say $(x_i) > 0$ are some N observations

$$L(x;\lambda) = \prod_{i=1}^{N} \lambda e^{-\lambda x_i}$$

$$\log(L(x_i;\lambda)) = \sum_{i=1}^{N} (\log\lambda - \lambda x_i)$$

$$\frac{\partial \log(L(x_i;\lambda))}{\partial \lambda} = \frac{N}{\lambda} - \cancel{\left(\frac{\partial\lambda}{\partial\lambda}\right)} \sum_{i=1}^{N} x_i = 0$$

$$0 = \frac{N}{\lambda} - \sum_{i=1}^{N} x_i$$

$$\Rightarrow \boxed{\lambda = \frac{N}{\sum_{i=1}^{N} x_i}}$$

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Gaussian

Assuming N data points from a gaussian distribution of mean 'μ' and standard deviation 'σ'

$$\text{Pdf} = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

$$L(x_i;\mu,\sigma) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\log(L(x_i;\mu,\sigma)) = \sum_{i=1}^{N} \log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - \sum_{i=1}^{N} \frac{(x_i-\mu)^2}{2\sigma^2}$$

$$\frac{\partial(\log L)}{\partial \mu} = 0 \qquad \frac{\partial(\log L)}{\partial \sigma} = 0$$

$$\frac{1}{N}\sum_{i=1}^{N} x_i - \mu = 0 \qquad \sum_{i=1}^{N}\left(\frac{-N}{\sigma}\right) + \sum_{i=1}^{N}\frac{(x-\mu)^2}{\sigma^3} = 0$$

$$2)\ \boxed{\mu = \frac{1}{N}\sum_{i=1}^{N} x_i} \qquad \boxed{\sigma^2 = \frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}}$$

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Laplacian

$$P_{\mu,b} = \frac{1}{2b} e^{\left(-\frac{|x-\mu|}{b}\right)}$$

Taking N data samples from the above distribution

$$L(x; \mu, b) = \prod_{i=1}^{N} \frac{1}{2b} e^{-\frac{|x_i-\mu|}{b}}$$

$$\log L = \sum_{i=1}^{N} \left[\log\left(\frac{1}{2b}\right) - \frac{|x_i-\mu|}{b} \right]$$

$$\frac{\partial(\log L)}{\partial b} = 0$$

$$\sum_{i=1}^{N} -\frac{1}{b} + \frac{\sum_{i=1}^{N} |x_i-\mu|}{b^2} = 0$$

$$\boxed{b = \frac{\sum_{i=1}^{N} |x_i - \mu|}{N}}$$

$$\frac{\partial L}{\partial \mu} = 0$$

$$\frac{1}{b} \sum_{i=1}^{N} \frac{|x_i-\mu|}{(x_i-\mu)} = 0 \Rightarrow \sum_{i=1}^{N} \operatorname{sgn}(x_i-\mu) = 0$$

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If μ = median (x_i)

There will half samples less than μ

and remaining half greater than μ

Such that summation is '0'

Thus μ = median (x_i)

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