EE5603: Concentration Inequalities, Spring 2019 (12)

Indian Institute of Technology Hyderabad

HW 1, 50 points. Assigned: Sunday 27.01.2019.

Due: Thursday 31.01.2019 at 11:59 pm.

1. The $\chi^2(n)$ random variable is defined as $\chi^2(n) = \sum_{i=1}^{n} X_i^2$ where X_i are independent standard normal random variables. Empirically check for sub-Gaussianity of $\chi^2(n)$ as a function of n. (5)

2. We saw a few examples of sub-Gaussian random variables in the class. Demonstrate the following with a Python script:

   (a) Sub-Gaussianity of a Gaussian random variable with zero mean and variance σ^2. (5)

   (b) Sub-Gaussianity of a Uniform random variable with range $[-a, a]$. (5)

   (c) Non sub-Gaussianity of a Laplacian random variable with zero mean and variance $2b^2$. (5)

   (d) Non sub-Gaussianity of a centered heavy tailed random variable of your choice. (5)

   (e) Sub-Gaussianity of a sum of bounded random variables with zero mean. (5)

   As discussed in class, choose the variance of the "reference" Gaussian appropriately.

3. Recall the definition of the Cramer's transform from the quiz. Find the Cramer's transform of a centered Bernoulli random variable with parameter p. (5)

4. Bennett's inequality: we proved in class that for centered random variables $X_1, X_2, \ldots, X_n$ that satisfy $X_i \leq b$ for some $b > 0$ almost surely for all $i \leq n$, and have finite variance with $v = \sum_{i=1}^{n} X_i^2$ and $S = \sum_{i=1}^{n} (X_i - E[X_i])$, for all $\lambda > 0$, $\log \mathbb{E} e^{\lambda S} \leq n \log(1 + \frac{v}{nb^2}\phi(\lambda b)) \leq \frac{v}{b^2}\phi(\lambda b)$. Here $\phi(u) = e^u - u - 1$. Now show the following tail bound for any $t > 0$: $P(S \geq t) \leq \exp(-\frac{v}{b^2} h(\frac{bt}{v}))$ where $h(u) = (1+u)\log(1+u) - u$ for $u > 0$. (5)

5. Empirically compare (using a Python script) the sharpness/tightness of the tail bound due to Bennett's inequality with the Hoeffding's inequality and the Chernoff's inequality. Show appropriate plots to demonstrate your comparisons. (10)