# Fourier Transform and its Applications in Image Restoration

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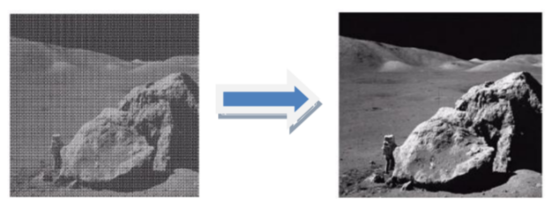
***Abstract-****Fourier Transform (FT) has been widely used as an image processing tool for analysis, filtering, reconstruction, and compression of images. The relevance of FT is considered in the image reconstruction process. During the process of image acquisition, sometimes images are degraded by several factors. The process of recovering such degraded or corrupted images is called Image Restoration. Restoration process improves the appearance of the image. The degraded image is the convolution of the original image, degraded function, and additive noise. Various methods available for image restoration such as inverse filter, Wiener filter etc. In this paper, description and comparison of restoration techniques are mentioned and various spatial domain filters are discussed which are used to remove noise from the images.*

## INTRODUCTION

Restoration attempts to recover an image that has been degraded by using a priori knowledge of the degradation phenomenon. Thus, restoration techniques are oriented toward modeling the degradation and applying the inverse process. Generally, main causes of degradation are blur, noise, and motion. Restoration of the image is a very big challenge in the field of image processing. To restore the image we must have knowledge of degradation.



**(a)** **(b)**

In the figure, there are two images shown. Figure (a) is imperfect due to various reasons described in this paper. Figure (b) shows the clear image which is obtained by different types of restoration techniques.

An image degradation model is developed as an operator ***H*** that, together with an additive noise term, operates on an input image ***F***$(x,y)$ to produce a degraded image ***G***$(x,y)$.

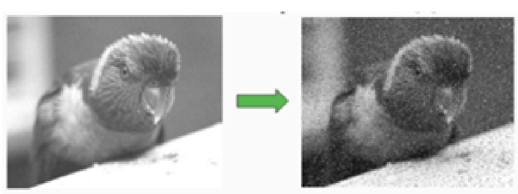
Given **G**($x$,**y**), some knowledge about ***H*** and some knowledge about the additive noise term ***N***($x$, $y$), the objective of restoration is to obtain an estimate ***f ˆ(x,y)*** of the original image. We want the estimate to be as close as possible to the original image and the more we know about ***H*** and ***N***, the closer ***f ˆ(x,y)*** will be to ***F***($x$,$y$).

$$G(x,y) = H(x,y) * F(x,y) + N(x, y)$$

# NOISE

Noise is always present in digital images during image acquisition, coding, transmission, and processing steps. It is very difficult to remove noise from digital images without knowing filtering techniques. Noise filters are selected by analysis of the behaviour of the noise.

Image noise is a random variation of brightness or color information in the images. External sources cause degradation in image signal. Images containing multiplicative noise have the characteristic that the brightness is proportional to noise. But mostly it is additive.



**Gaussian Noise**

Gaussian noise is an image noise having a probability density function (PDF) equal to that of the normal distribution (Gaussian distribution).

The probability density function p of a Gaussian Random variable is given by

$$p_G(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$$

Where z represents the gray level, u represents the mean gray value and sigma the standard deviation. Pixels of sensed image Quantized to a number of discrete levels. That is why it is also called quantization noise.

**Salt and Pepper Noise**

Salt and pepper noise, also known as impulse noise. This noise can be caused by the sharp and sudden disturbances in image signal. It presents itself as minute white and black pixels spread over the image.

**Poisson Noise**

Poisson noise or Shot noise is a type of noise which can be modeled by a Poisson process. Poisson noise originates from the discrete nature of electric charge. It also occurs in photon counting in optical devices, where Poisson noise is associated with the particle nature of light.

**Multiplicative Noise**

Multiplicative noise is an unwanted random signal that gets multiplied into some relevant signal during capture, transmission, or other processing of images. An important example is the speckle noise commonly observed in radar imagery.

# Blur

Blur is a sharp image space caused by camera or subject movement, inaccurate focusing, or the utilization of associate aperture that gives shallow depth of field. The Blur effects area unit filters that sleek transitions and reduce contrast by averaging the pixels next to onerous edges of outlined lines and areas where there are similar colour transitions.

**Average Blur**

The Average blur is one in all many tools you'll use to get rid of noise and specks in a picture. Use it once noise is over the complete image. this type of blurring may be distribution in horizontal and vertical direction and might be circular averaging by radius R that is evaluated by the formula: $R = \sqrt{g2 + f2}$ Where: g is that the horizontal size blurring direction and f is vertical blurring size direction and R is that the radius size of the circular average blurring.

**Gaussian Blur**

Blur impact may be a filter that blends a particular variety of pixels incrementally, following a Gaussian curve. The blurring is dense within the center and feathers at the sting. Apply mathematician Blur to a picture once you need additional management over the Blur impact.

Both grayscale and colour pictures will contain loads of noise or random variation in brightness or among pixels. The pixels in these pictures have a high variance, that simply means that there's loads of variation at intervals between teams of pixels. As a result of a photograph being two-dimensional, mathematician blur uses 2 mathematical operations (one for the coordinate axis and one for the y) to make a 3rd function, additionally referred to as a convolution.

**Motion Blur**

The Motion Blur impact may be a filter that produces the image seem to be moving by adding a blur in an exceedingly specific direction. The motion may be controlled by angle or direction (0 to 360 degrees or –90 to +90) and/or by distance or intensity in pixels (0 to 999).

**Defocus blur**

Defocus blur is one kind of blur degradation that results from defocus and improper depth of focus. For scenes with multiple depth layers, however, solely the layer on a focal plane can target the camera detector, which results in others being out of focus.

Focusing distance (distance between the camera and also the subject) the number of defocus changes supported the camera settings, like "aperture" and "focal length," and conjointly supported the space between the camera and also the subject. Background defocus will increase as you progress the camera nearer to the topic.

# Literature Review

The project is based on the usage of fourier transform in image processing and to restore the images from degradation caused by noise, blur and motion. Noise and blur are the main causes of image degradation.

Image noise is a random variation of brightness or color information in the images. Different types of image noises are Gaussian, Salt-and-Pepper, Poisson, Multiplicative etc.

Blur is a sharp image space  that gives shallow depth of field. Different types of blurs are Average, Gaussian, Motion, Defocus etc.

These degradations are removed using some filtering methods like mean filtering, median filtering, inverse filtering etc.

Mean filtering deals with the mean of intensities at a particular region. Maximum and Minimum filters are used to find the brightest and darkest points in an image.

Median filter uses the idea of median instead of mean for a neighborhood of pixels.

The blurring function of the corrupted image is known or can be developed by using an inverse filter. Wiener removes the additive noise and inverts the blurring at the same time.

Adaptive filters are commonly used in image processing to enhance or restore data by removing noise without significantly blurring the structures in the image.

Morphological operators — dilate, erode, open, and close — can be applied through image filtering to grow or shrink image regions, as well as to remove or fill-in image region boundary pixels.

The matlab codes for all the above filters are also given.

# Dataset

Index of /Matlab/images/MATLAB_DEMO_IMAGES

# Methods

- arithmetic mean filtering and geometric mean filtering
- Inverse Filtering
- median filtering
- maximum and minimum filtering
- adaptive filtering
- Wiener filtering
- Filtering with morphological operators(Contrast Adjustment)

## Arithmetic Mean Filtering

An **arithmetic mean filter** calculates the mean intensity value of a localized region and the central pixel of that corresponding block is then replaced with the mean thus obtained. The block is now moved along the length and breadth of the image and each of the central pixels get replaced with their respective localized mean. Thus, if there are scattered salt and pepper pixels in the image, they will be blurred, providing a trivial illusion of noise removal.
Let Sxy represent the set of coordinates in a rectangular subimage window (neighborhood) of size m×n, centered on the point ( x,y ). The arithmetic mean filter computes the average value of the corrupted image, g (x,y), in the area defined by Sxy. The value of the restored image f ^ at point (x,y) is the arithmetic mean computed using the pixels in the region defined by Sxy.

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s, t) \in S_{xy}} g(s, t)$$

where, s and t are the row and column coordinates of the pixels contained in the neighborhood Sxy. This operation can be implemented using a spatial kernel of size m×n in which all coefficients have values 1/mn. Here's what an arithmetic mean filter does to a noisy image:

The filter causes a certain amount of blurring(proportional to the window size) to the image thereby reducing the effects of noise. It can be used to reduce noise of different types but works best for Gaussian, uniform or Erlang noise.

## Geometric Mean Filtering

A **geometric mean filter** is a variation of the arithmetic mean filter primarily used on images with Gaussian noise.The geometric mean filter is a member of a set of nonlinear mean filters which are better at removing Gaussian type noise and preserving edge features than the arithmetic mean filter. It returns image details better than the arithmetic mean filter. It is based on the mathematical geometric mean. The output image G(x,y) of a geometric mean is given by :

$$\hat{f}(x,y) = \left[ \prod_{(r,c) \in S_{xy}} g(r,c) \right]^{\frac{1}{mn}}$$

Here, each restored pixel is given by the product of all the pixels in the subimage area, raised to the power of 1/mn.

## Median Filtering

Median filtering is usually used to reduce the noise in an image, similar to the mean filter. But it is better than the mean filter for preserving useful details in the image. It is very effective at removing noise and can also preserve the edges. It is very effective in removing salt and pepper noise.

**Working :** The median filter works by moving through the image through each pixel, and replaces each value with the median value of nearby pixels. The pattern of the nearby pixels is called the "window", which slides, pixel by pixel over the entire image. The median is calculated by first sorting all the pixel values from the window into numerical order, and then replacing the pixel being considered with the median pixel value.

**Example:**

| | | | | |
|---|---|---|---|---|
| 123 | 125 | 126 | 130 | 140 |
| 122 | 124 | 126 | 127 | 135 |
| 118 | 120 | 150 | 125 | 134 |
| 119 | 115 | 119 | 123 | 133 |
| 111 | 116 | 110 | 120 | 130 |

Consider the pixels 124, 126, 127, 120, 150, 125, 115, 119 and 123.

Median value = 124

Calculation : The central pixel value of 150 doesn't represent the surrounding pixels and it is replaced with the median value, 124. A 3×3 square cluster of pixels is used here. Larger number of pixels will produce more severe smoothing.

## Maximum and Minimum filters

**Maximum and Minimum filters** are used to find the brightest and darkest points in an image. The Minimum filter replaces the pixel with the darkest point. The Maximum filter replaces the pixel value with the brightest point. Max filter helps to find light colored pixels in an image but the Min filter helps to find dark points in the image. These are also known as erosion and dilation filters. These are morphological filters that work by considering a neighborhood around each pixel. From the neighbor pixels, the minimum or maximum value is found and stored as the corresponding resulting value. Each pixel in the image is replaced by the resulting value generated for its associated neighborhood.

$$\hat{f}(x, y) = \max_{(s,t)\in S_{xy}} \{g(s, t)\}$$

$$\hat{f}(x, y) = \min_{(s,t)\in S_{xy}} \{g(s, t)\}$$

The max and min filter is used to process image data in the spatial domain. They belong to order-statistic filters along with median filters. Earlier we have mentioned it is a type of Morphological image processing, It's a technique which is introducing operations to transform the images in a special way which takes image content into account. The most common morphological operations are minimum and also known as dilation whereas maximum are also known as erosion filters. The minimum filter is to extend the object boundaries, whereas the maximum filter is to erode the shapes on the image.

## Inverse Filtering

The blurring function of the corrupted image is known or can be developed then it has been proved as the quickest and easiest way to restore the distorted image. Blurring can be considered
as low pass filtering in an inverse filtering approach and use high pass filtering action to reconstruct the blurred image without much effort. Suppose first that the additive noise is negligible.
A problem arises if it becomes very small or zero for some point or for a whole region in the plane then in that region inverse filtering cannot be applied.

Let **f** be the original image, **h** the blurring kernel, and **g** the blurred image. The idea in inverse filtering is to recover the original image from the blurred image.
From the convolution theorem, the DFT of the blurred image is the product of the DFT of the original image and the DFT of the blurring kernel. Thus, dividing the DFT of the blurred image by the DFT of the kernel, we can recover the original image. Then the inverse frequency filter, R(u), to be applied is 1/H(u). We are assuming no noise is present in the system. Observe the difficulties we encounter when H(u) is very small or equal to zero.

$$F(u) = \frac{1}{H(u)} G(u)$$

$$F(u) = R(u) G(u)$$

$$R(u) = \frac{1}{H(u)}$$

The formulas correspond to the 1D case and can be easily extended to the 2D domain.
**P(u,v) = H(u,v) Q(u,v)**, where P(u,v) is the degraded image.
**H(u,v)** is the degradation transfer function.
**Q(u,v)** is the original image.
The inverse filtering process is then
**Q(u,v) = P(u,v) / H(u,v)**.

## Wiener Filtering

There is a method referred to as Wiener filtering that's used in image restoration. This method assumes that if noise is there within the system, then it's considered to be additive white Gaussian noise (AWGN).It removes the additive noise and inverts the blurring at the same time. The Wiener filtering is perfect in terms of the mean square error. In alternative words, it minimizes the general mean square error within the method of inverse filtering and noise smoothing. The Wiener filtering may be a linear estimation of the original image .Wiener filters play a central role in an exceedingly wide selection of applications like linear prediction, echo cancellation, signal restoration, channel deed and system identification. The Wiener filter coefficients are calculated to minimize the common square distance between the filter output
and a desired signal.
The inverse filter of a blurred image could be a high pass filter. The parameter K of the Wiener filter is said to be the low frequency side of the Wiener filter. The Wiener filter behaves as a bandpass filter, wherever the high pass filter is due to the inverse filter and also the lowpass filter to the parameter K. Observe that once K=0, the Wiener filter becomes the inverse filter.
(u,v) is a dataset..

$$\widehat{F}(u,v) = W(u,v)\ G(u,v)$$

$$W(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + K(u,v)}$$

where

$K(u,v) = S_\eta(u,v)/S_f(u,v)$

$S_f(u,v) = |F(u,v)|^2$ power spectral density of $f(x,y)$

$S_\eta(u,v) = |N(u,v)|^2$ power spectral density of $\eta(x,y)$

## Adaptive Filtering

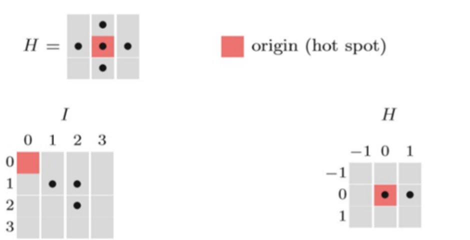
Adaptive filters find widespread use in countering the effects of so-called "speckle" noise, which afflicts coherent imaging systems like SAR and ultrasound. With these imaging techniques, scattered waves interfere with one another to contaminate an acquired image with multiplicative speckle noise. Various statistical models of speckle noise exist, with one of the more common being

$$g(x,y) = f(x,y) + f(x,y)n(x,y)$$

where g is the corrupted image and n(x,y) is drawn from a zero-mean Gaussian distribution with a given standard deviation. It is clear from the above model that speckle noise is dependent on the magnitude of the signal f, and in fact this type of noise is a serious impediment on the interpretability of image data because it degrades both spatial and contrast resolution.

## Filtering with morphological operators

Morphological operators dilate, erode, open, and close can be applied through the image filtering to grow or shrink image regions, and also to remove or fill-in image region boundary pixels. The main principle of morphological filters is shrink and let grow. revenue using average filters to round off the large assemblies and to detach the small structures and in the grow method, remaining structures are growing back by the same quantity.
These are the key elements of this process.




$I \equiv \mathcal{Q}_I = \{(1,1),(2,1),(2,2)\}$ $\quad H \equiv \mathcal{Q}_H = \{(0,0),(1,0)\}$

The structuring element of a binary filter

- Dilation and Erosion
- Composite Operation

**The structuring element**

In this filter, each element in the matrix is known as a “structuring element” instead of a coefficient matrix from the linear filter. The structuring elements will only contain the values 0 and 1. Also the main spot of this filter is the dark shade element.
The binary image is taken as sets of 2-D coordinate points. It is known as ***‘Point Set’*** Q and the point set contains the coordinate pair p = (u,v) of all pixels. Some operations of a point set are the same as the operations in others images. To invert a binary image is a complement operation and combination of two binary images uses a union operator. Shifting binary image “I” by using some coordinate vector ‘d' by adding vector “d” to

the point “p”. Or reflection of binary image ‘I’ by multiplying -1 to ‘p’ point.

**Dilation and Erosion**

***Dilation*** is a morphological operator in which works for the continue process as we saw before as well and the equation can be defined as

$$I \oplus H \equiv \{(\boldsymbol{p}+\boldsymbol{q}) \mid \text{for every } \boldsymbol{p} \in I, \boldsymbol{q} \in H\}.$$

***Erosion*** is a morphological operator which works for the shrink process as as we saw before as well and the equation can be defined as

$$I \ominus H \equiv \{\boldsymbol{p} \in \mathbb{Z}^2 \mid (\boldsymbol{p}+\boldsymbol{q}) \in I, \text{ for every } \boldsymbol{q} \in H\}.$$

Properties of dilation and erosion are
Commutative: only in dilation and
The opening and closing also are two in the sense that opening the foreground is the same as closing the background.

$$I \circ H = \overline{(\bar{I} \bullet H)} \quad \text{and} \quad I \bullet H = \overline{(\bar{I} \circ H)}.$$

The Morphological Filter can also convert to gray-scale image, but in a different way. It is a generalization with MIN and MAX operators. We will see the following outline.

**Outlines**

Structuring Elements
Dilation and Erosion
Opening and Closing

**Structuring Element**

In the gray-scale syllable structure, structuring elements are clear as real-value 2D functions instead of point groups.

$$H(i,j) \in \mathbb{R}, \qquad \text{for } (i,j) \in \mathbb{Z}^2.$$

The worth in H can be negative or zero value. But in contrast to linear difficulty,

Associative: only in dilation. The erosion and dilation are ***duels***, for a dilation of the foreground can be shown by an erosion of background and subsequent of the output in two different ways but work at same time

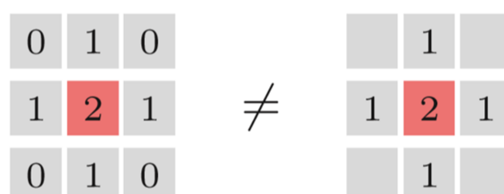
**Composite Operation**

In this process, dilation, and erosion work combined in composite operation. These are common ways to show the order of the two operations, opening and closing. Opening can be told as an erosion similarly by dilation and closing work in quite opposite ways.

$$I \circ H = (I \ominus H) \oplus H.$$

$$I \bullet H = (I \oplus H) \ominus H.$$

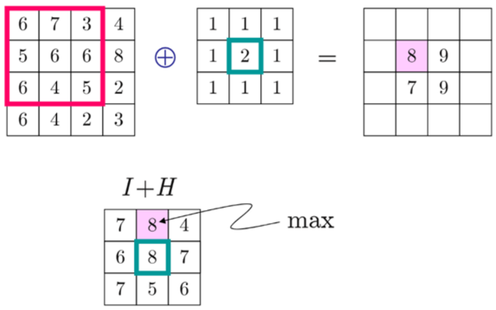
zero elements are used to calculate the result. And if we do not want to use the elements in some position, we  can put no element in that position.




**Dilation and Erosion**The outcome of dilation and erosion in gray-scale
Intended for erosion, the result is the lowest value of the alteration.
These operations can cause the negative value, so we need to clamp the result after calculation.

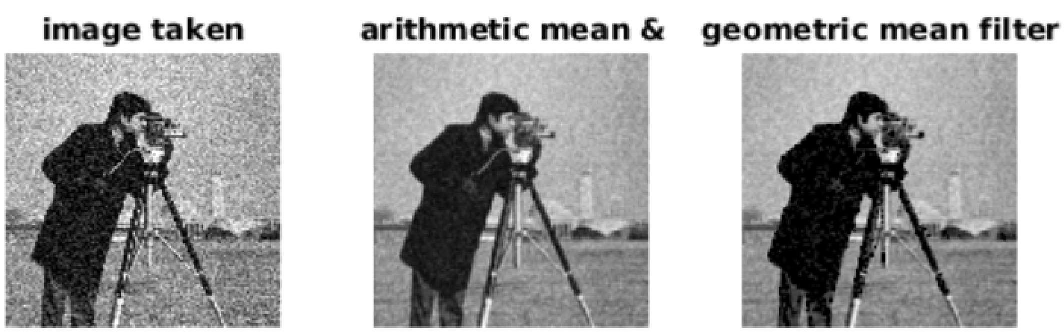
$$(I \oplus H)(u,v) = \max_{(i,j)\in H} \{I(u+i, v+j) + H(i,j)\}.$$

geomorphology is contributed from the extreme and lowest process. Intended for dilation, the result is the extreme value of the value in H added to the present sub-image.



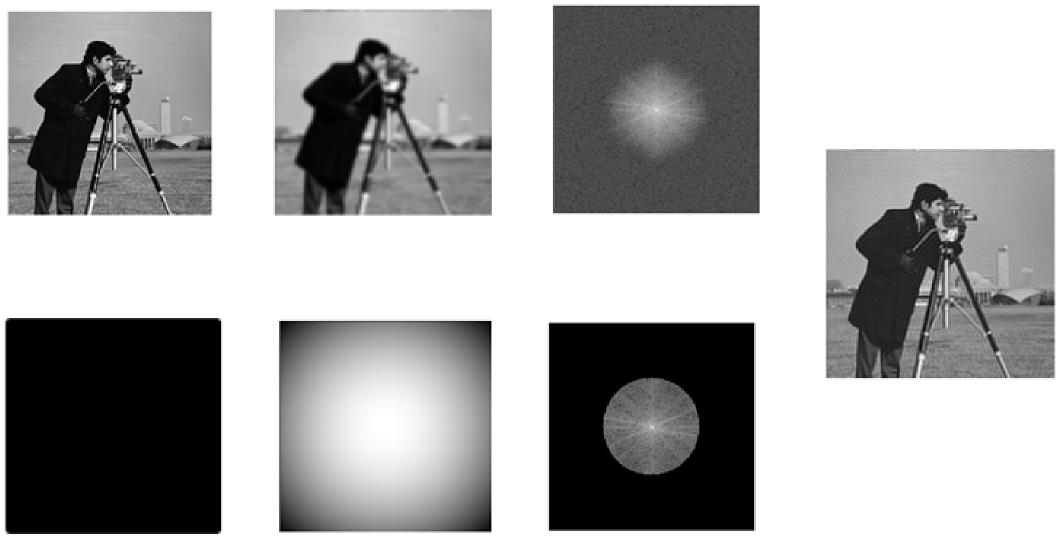

# Results & Discussion

- Arithmetic mean filtering and Geometric mean filtering

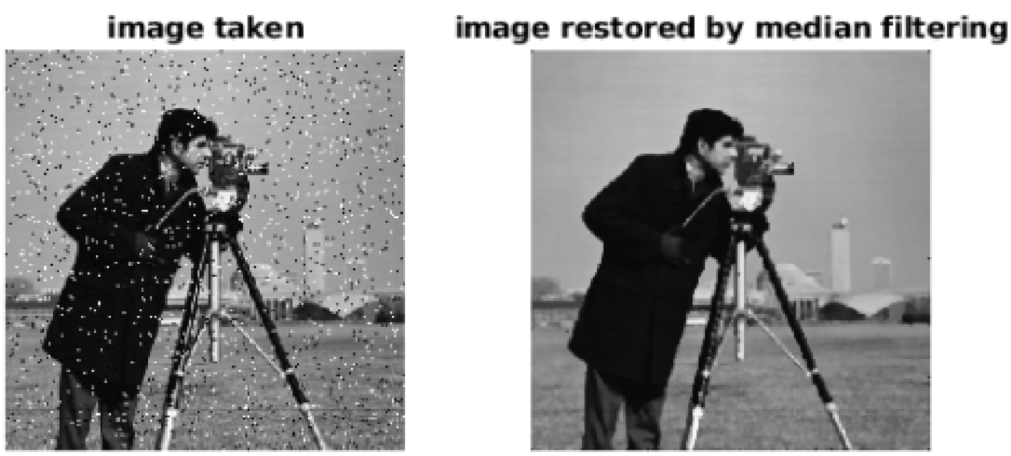



  - An arithmetic mean filter removes short tailed noise such as **uniform and Gaussian type noise** from the image at the cost of blurring the image. The arithmetic mean filter is defined as the average of all pixels within a local region of an image.
  - The geometric mean filter is most widely used to filter out **Gaussian noise**. In general it will help smooth the image with less data loss than an arithmetic mean filter.

- Inverse Filtering



- median filtering



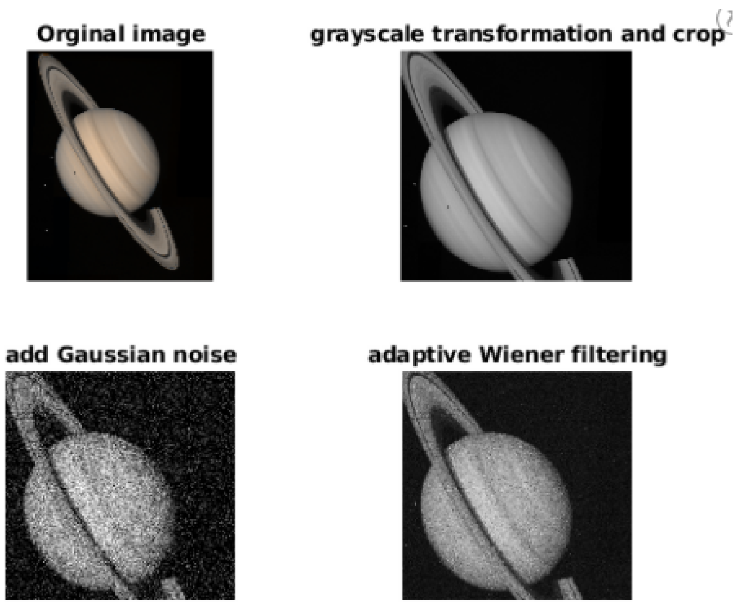

The image with the salt and pepper noise is restored into a clear image using the median filter.

- maximum and minimum filtering


MAXIMUM FILTERING

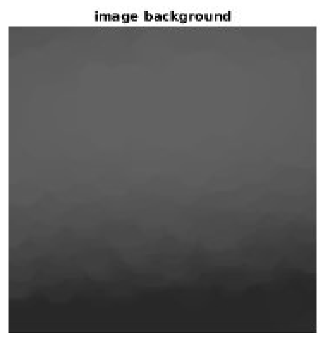

MINIMUM FILTERING

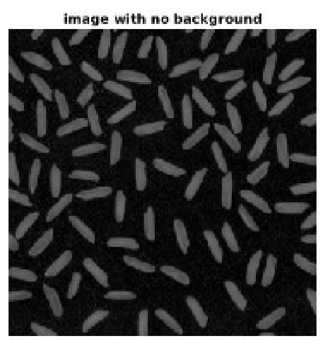
- adaptive filtering

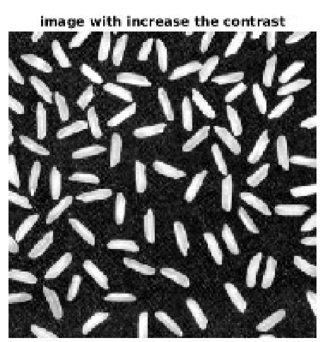


- Wiener filtering


- Filtering with morphological operators(Contrast Adjustment)



# Conclusion

The purpose of this paper was to apply image restoration techniques on degraded images using Fourier transformation. After a short review of the various filters used for image processing, we have achieved satisfactory results in restoring images using our proposed implementation of the filters. We have also performed an analysis to figure out which filter is best at denoising a specific noise in an image. In this paper noise model and blur model, blurring and deblurring techniques are elaborated. This discussion is useful for researchers to identify the restoration techniques based on noise and blur models.

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