Computational Engineering Mechanics 2

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Project

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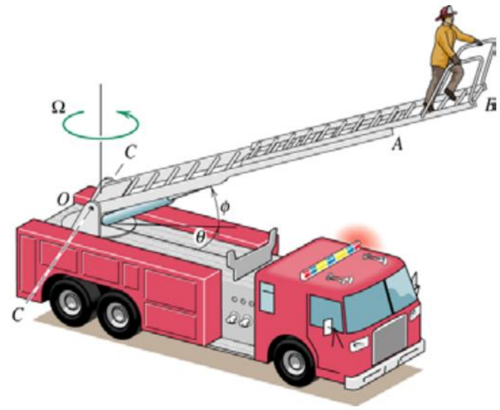
Q1):The base structure of the firetruck ladder rotates about a vertical axis through O with a constant angular velocity θ = Ω. At the same time, the ladder unit OB elevates at a constant rate ϕ = Ψ, and section AB of the ladder extends from within section OA at the constant rate R =λ. Find general expressions for the components of acceleration of point B in spherical coordinates if, at time t = 0, θ = 0, ϕ = 0, and AB = 0. Express your answers in terms of Ω, Ψ, Λ, R_0 and t, where R_0 = OA and is constant. Plot the components of acceleration of B as a function of time for the case Ω =5 deg/s, Ψ = 14 deg/s, λ= 1 m/s, and R0 = 4.5 m. Let t vary between 0 and the time at which ϕ = 90°.



Sol):

The components of acceleration in spherical coordinates are,

$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2 \cos^2 \phi$$

$$a_\theta = \frac{\cos\phi}{R}\frac{d}{dt}\left(R^2\dot{\theta}\right) - 2R\dot{\theta}\dot{\phi}\sin\phi$$

$$a_\phi = \frac{1}{R}\frac{d}{dt}\left(R^2\dot{\phi}\right) + R\dot{\theta}^2 \sin\phi\cos\phi$$

The components may be obtained as functions of time by substituting,

$$R = R0 + \Lambda t \text{ , } \theta = \Omega t \text{ and } \phi = \Psi t$$

Differentiation and substitution will be performed in MATLAB. The results are,

$$a_R = (R_0 + \lambda t)\left(\Psi^2 - \Omega^2 \cos^2(\Psi t)\right)$$

$$a_\theta = 2\Omega\lambda\cos(\Psi t) - 2\Omega\Psi(R_0 + \lambda t)\sin(\Psi t)$$

$$a_\phi = 2\Psi\lambda + (R_0 + \lambda t)\Omega^2 \sin(\Psi t)\cos(\Psi t)$$

--------------- Script #1 ---------------

```
% acceleration symbolically
% O = Omega; P = Phi; L = Lambda
syms O P L t R0
R = R0+L*t;
theta = O*t;
phi = P*t;
a_R = diff(R,t,2)-R*diff(phi,t)^2-R*diff(theta,t)^2 *cos(phi)^2
a_theta = cos(phi)/R*diff(R^2*diff(theta,t),t)-2*R*diff(theta,t)*diff(phi,t)*s
a_phi = 1/R*diff(R^2*diff(phi,t),t)+R*diff(theta,t)^2 *sin(phi)*cos(phi)
```

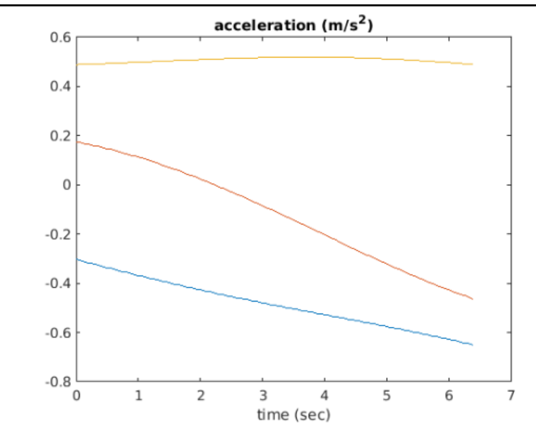
a_R = $(-R_0 - L\,t)\,\mathrm{O}^2\cos(P\,t)^2 + (-R_0 - L\,t)\,P^2$

a_theta = $2\,L\,\mathrm{O}\cos(P\,t) - \mathrm{O}\,P\sin(P\,t)\;(2\,R_0 + 2\,L\,t)$

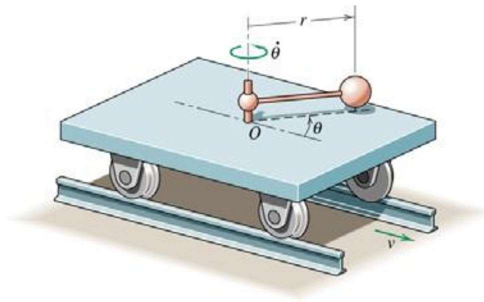
a_phi = $\cos(P\,t)\sin(P\,t)\;(R_0 + L\,t)\,\mathrm{O}^2 + 2\,L\,P$

--------------- Script #2 ---------------

```
% This script plots the components of the acceleration
% as functions of time
% O = Omega; P = Phi; L = Lambda
L = 1; O = 5*pi/180;
P = 14*pi/180; R0 = 4.5;
tf = pi/2/P; % time at which phi=pi/2
t=0:0.05:tf;
a_R = -(R0+L*t)*P^2-(R0+L*t).*O^2.*cos(P*t).^2;
a_theta = 2*cos(P*t)*O*L-(2*R0+2*L*t)*O*P.*sin(P*t);
a_phi = 2*P*L+(R0+L*t)*O^2.*sin(P*t).*cos(P*t);
plot(t,a_R,t,a_theta,t,a_phi)
xlabel('time (sec)')
title('acceleration (m/s^2)')
```

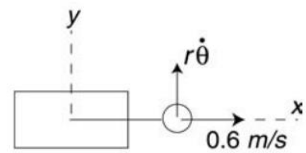


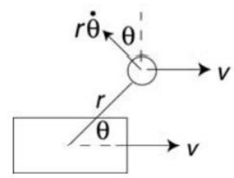
Q2):The small car, which has a mass of 20 kg, rolls freely on the horizontal track and carries the 5- kg sphere mounted on the light rotating rod with *r* = 0.4 m. A geared motor drive maintains a constant angular speed $\dot{\theta}$ = 4 rad/s of the rod. If the car has a velocity *v* = 0.6 m/s when θ = 0, plot *v* as a function of θ for one revolution of the rod. Also plot the absolute position of the sphere for two revolutions of the rod. Neglect the mass of the wheels and any friction



Sol):

Since $\Sigma F_x = 0$ we have conservation of momentum in the *x* direction. The diagram to the right shows the system at θ = 0 and at an arbitrary angle θ. From the relative velocity equation, the velocity of the sphere is the vector sum of the velocity of the car (*v*) and the velocity of the sphere relative to the car ($r\dot{\theta}$)





$$(G_x)_{\theta=0} = 20(0.6) + 5(0.6) = 15 \ \text{N}\bullet\text{s}$$

$$(G_x)_\theta = 20v + 5\left(v - r\dot{\theta}\sin\theta\right) = 25v - 8\sin\theta$$

Setting $(G_x)_{\theta=0} = (G_x)_\theta$ and solving yields,

$v = 0.6 + 0.32 \sin\theta$

Now let time *t* = 0 be the time when θ = 0 and place an *x*-*y* coordinate system at the center of the car as shown in the diagram so that *x*(*t*) is the position of the center of the car. Since *v* = d*x*/d*t* and θ = 4*t* we have,

$$x = \int_0^t v\,dt = \int_0^t \left(0.6 + 0.32\sin(4t)\right)dt = 0.6t + 0.08\left(1 - \cos(4t)\right)$$

The *x* and *y* components of the sphere can now be determined as,λ

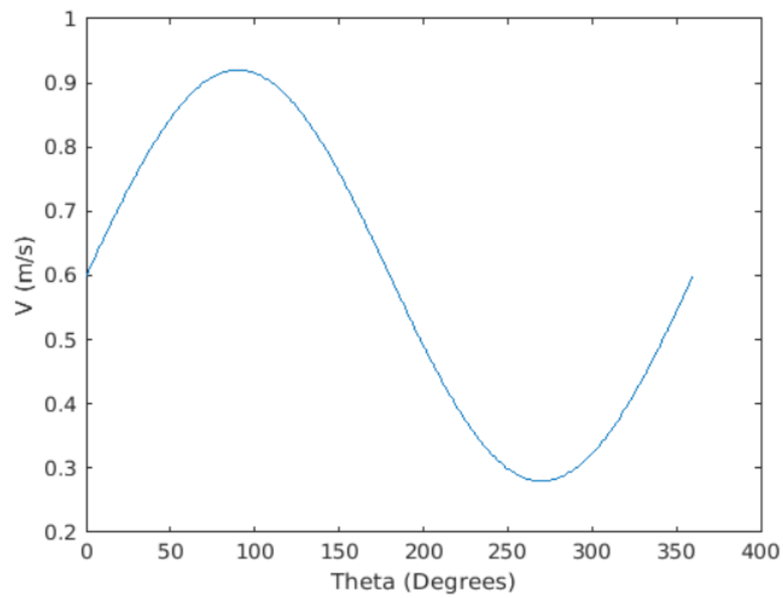
$$x_s = x + r\cos\theta = 0.08 + 0.6t + 0.32\cos(4t)$$

$$y_s = r\sin\theta = 0.4\sin(4t)$$

The absolute position of the sphere can be obtained by plotting y_s versus x_s. The time required for two revolutions of the arm is 4π/4 = π seconds.

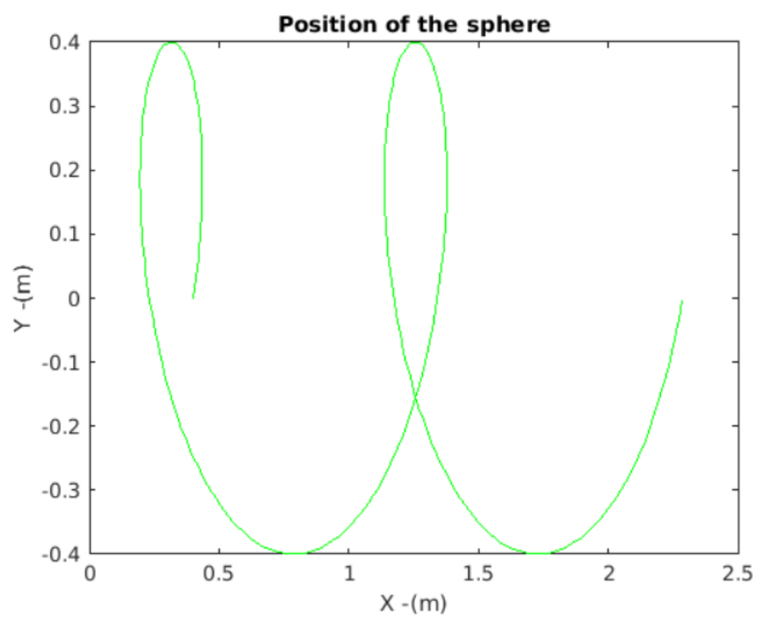
--------------- Script #1 ---------------

```
%  plots v as a function of theta
theta = 0:0.02:2*pi;
v = 0.6 + 0.32*sin(theta);
plot(theta*180/pi, v)
xlabel('Theta (Degrees)')
ylabel('V (m/s)')
```

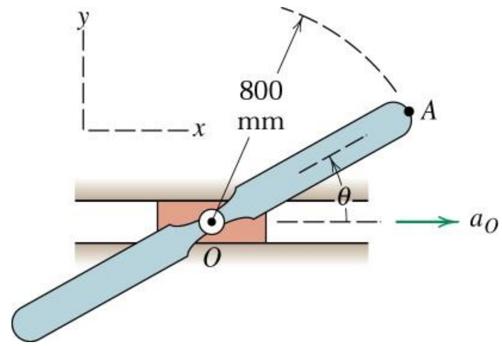


--------------- Script #2 ---------------

```
% This script plots the position of the sphere
t = 0:0.01:pi;
xs = 0.08+0.6*t+0.32*cos(4*t);
ys = 0.4*sin(4*t);
plot(xs, ys,"G")
xlabel('X -(m)')
ylabel('Y -(m)')
title('Position of the sphere')
```



Q3):The two rotor blades of radius r = 800-mm rotate counterclockwise with a constant angular velocity ω about the shaft at O mounted in the sliding block. The acceleration of the block is a_O. Determine the magnitude of the acceleration of the tip A of the blade in terms of r, ω, a_O, and θ. Plot the acceleration of A as a function of θ for one revolution if a_O = 3 m/s. Consider three



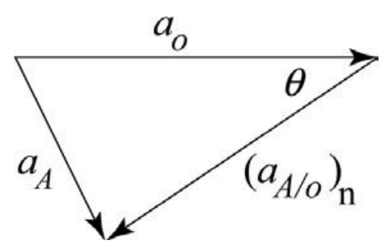
<u>cases: ω = 2, 4, and 6 rad/s.</u>

<u>Sol):</u>

The acceleration of A relative to O is

$$\vec{a}_A = \vec{a}_O + (\vec{a}_{A/O})_n + (\vec{a}_{A/O})_t$$

The acceleration of O is to the right while the normal relative acceleration must point from A towards O. Since ω is constant, the tangential relative acceleration will be zero. These considerations lead to the vector diagram shown to the right. Using the law of cosines,

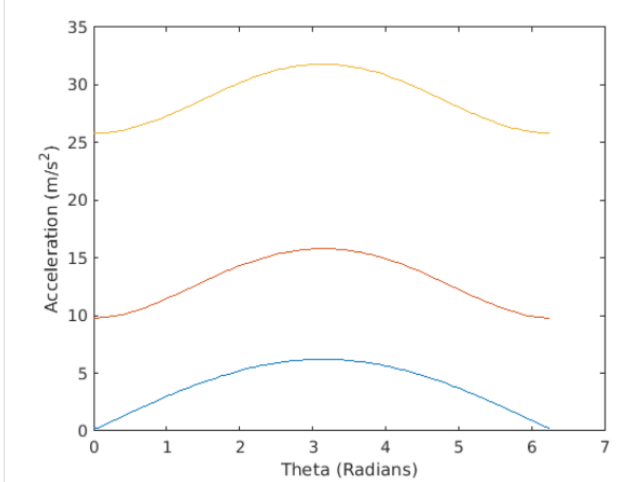


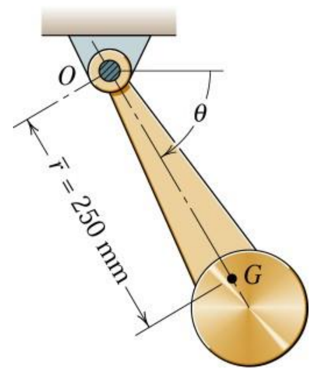
$$a_A = \sqrt{a_O^2 + (a_{A/O})_n^2 - 2a_O(a_{A/O})_n\cos\theta}$$

$$a_A = \sqrt{a_O^2 + (r\omega^2)^2 - 2a_O(r\omega^2)\cos\theta}$$

--------------- Script # ---------------

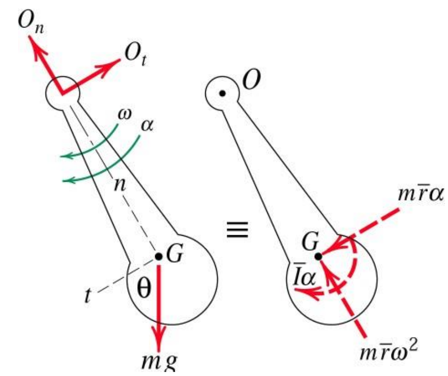
```
aA = inline('sqrt(3^2+0.8^2*omega^4-2*3*0.8*omega^2*cos(theta))');
th = 0:.05:2*pi;
plot(th,aA(2,th),th,aA(4,th),th,aA(6,th))
xlabel('Theta (Radians)')
ylabel('Acceleration (m/s^2)')
```





Q4):The pendulum has a mass of 7.5 kg with a mass center at *G* and a radius of gyration about the pivot *O* of 295 mm. If the pendulum is released from rest when θ = 0, plot the total force supported by the bearing at *O* along with its normal and tangential components as a function of θ. Let θ range between 0 and 180°.

Sol):



The free body and mass acceleration diagrams are identical to those in the sample problem. The main difference in approach is that we will obtain results at an arbitrary angle θ rather than at 60°

$$[\Sigma M_O = I_O\alpha] \qquad mg\bar{r}\cos\theta = mk_0^2\alpha \qquad \alpha = \frac{g\bar{r}}{k_0^2}\cos\theta$$

$$[\omega d\omega = \alpha d\theta] \qquad \int_0^{\omega}\omega d\omega = \frac{g\bar{r}}{k_0^2}\int_0^{\theta}\cos\theta d\theta = \frac{g\bar{r}}{k_0^2}\sin\theta$$

$$\omega^2 = \frac{2g\bar{r}}{k_0^2}\sin\theta$$

$$[\Sigma F_n = m\bar{r}\omega^2] \qquad O_n - mg\sin\theta = m\bar{r}\omega^2$$

$$[\Sigma F_t = m\bar{r}\alpha] \qquad -O_t + mg\cos\theta = m\bar{r}\alpha$$

After substituting for α and ω we have,

$$O_n = mg\left(1 + 2\frac{\bar{r}^2}{k_0^2}\right)\sin\theta \qquad\qquad O_t = mg\left(1 - \frac{\bar{r}^2}{k_0^2}\right)\cos\theta$$

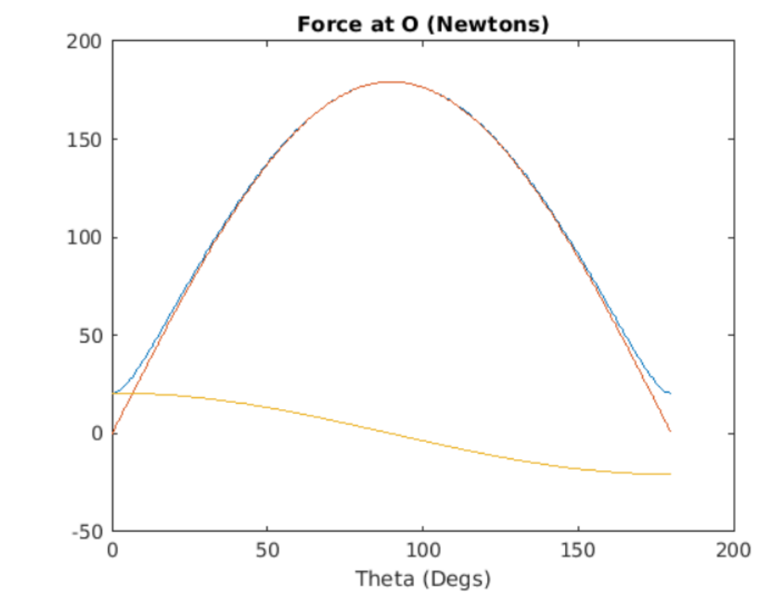
The magnitude of the force at O is,

$$O = \sqrt{(O_n)^2 + (O_t)^2}$$

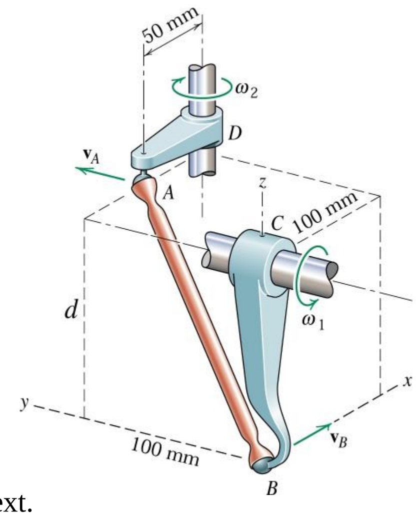
After substituting $\bar{r}$ = 0.25 m, k_0 = 0.295 m, m = 7.5 kg, and g = 9.81 m/s^2 all forces will be functions of θ only.

--------------- Script # ---------------

```
%  plots the force at O and its
% components as functions of theta
rb = 0.25;
k0 = 0.295;
mg = 7.5*9.81;
theta = 0:0.01:pi;
On = mg*(1+2*(rb/k0)^2)*sin(theta);
Ot = mg*(1-(rb/k0)^2)*cos(theta);
O=sqrt(On.^2+Ot.^2);
td=180*theta/pi; % converts to degrees
plot(td,O,td,On,td,Ot)
xlabel('Theta (Degs)')
title('Force at O (Newtons)')
```



Q5):Crank *CB* rotates about the horizontal axis with an angular velocity ω_1 = 6 rad/s, which is constant for a short interval of motion that includes the position shown. Link *AB* has a ball-and-socket fitting on each end and connects crank *DA* with *CB*. Let the length of crank *CB* be *d* mm (instead of 100 mm as in the sample problem in your text) and plot ω_2 and ω_n as a function of *d* for $0 \le d \le 200$ mm. ω_2 is the angular velocity of crank *DA* while ω_n is the angular velocity of link *AB*.



Sol):

Our analysis will follow closely that in the sample problem in your text.

$$\mathbf{v}_A = \mathbf{v}_B + \omega_n \times \mathbf{r}_{A/B}$$

where $\mathbf{v}_A = 50\omega_2\mathbf{j}$ $\quad \mathbf{v}_B = 6d\mathbf{i}$ $\quad \mathbf{r}_{A/B} = 50\mathbf{i} + 100\mathbf{j} + d\mathbf{k}$

Substitution into the velocity equation gives

$$50\omega_2\mathbf{j} = 6d\mathbf{i} +$$

$$50\omega_2\mathbf{j} = 6d\mathbf{i} + \begin{vmatrix} \mathbf{I} & \mathbf{j} & \mathbf{k} \\ \omega_{nx} & \omega_{nz} & \omega_{nz} \\ 50 & 100 & d \end{vmatrix}$$

Expanding the determinant and equating the **i**, **j**, and **k** components yields the following three equations

$$d\left(6+\omega_{ny}\right) - 100\omega_{nz} = 0$$

$$50\left(\omega_2 - \omega_{nz}\right) + d\omega_{nx} = 0$$

$$2\omega_{nx} - \omega_{ny} = 0$$

At this point we have three equations with four unknowns. As explained in the sample problem in your text, the fourth equation comes by requiring ω_n to be normal to $\mathbf{v}_{A/B}$

$$\omega_n \bullet \mathbf{r}_{A/B} = 50\omega_{nx} + 100\omega_{ny} + d\omega_{nz} = 0$$

These four equations will be solved simultaneously for ω_2, ω_{nx}, ω_{ny}, *and* ω_{nz}. Once this is done,

$$\omega_n = \sqrt{\omega_{nx}^2 + \omega_{ny}^2 + \omega_{nz}^2}$$

--------------- Script 1 # ---------------

```
% solves four equations for
% omega_2 (w) and the three components of
% omega_n (x, y, and z). The magnitude of
% omega_n is then found from its components.
syms w x y z d
eqn1 = d*(6+y)-100*z;
eqn2 = 50*(w-z)+d*x;
eqn3 = 2*x-y;
eqn4 = 50*x+100*y+d*z;
[w,x,y,z]=solve(eqn1,eqn2,eqn3,eqn4)
omega_n = sqrt(x^2+y^2+z^2);
omega_n = simplify(omega_n)
```

w =

$$\frac{3\,d}{50}$$

x =

$$-\frac{3\,d^2}{d^2+12500}$$

y =

$$-\frac{6\,d^2}{d^2+12500}$$

z =

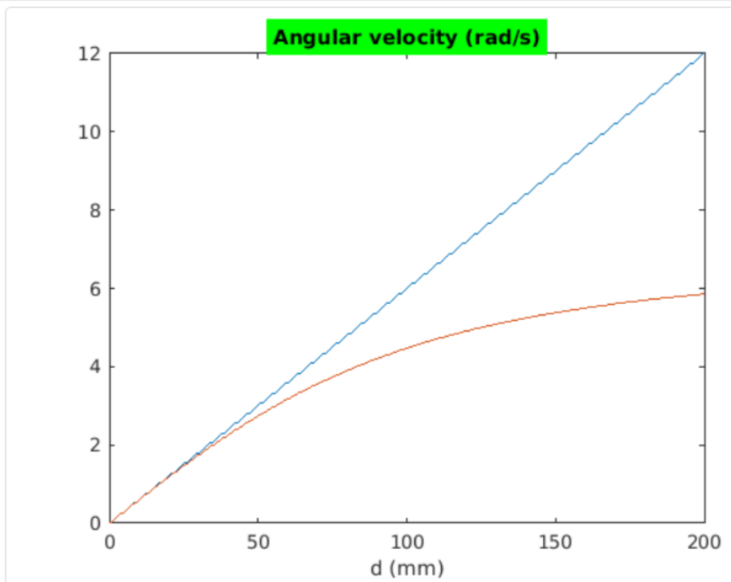
$$\frac{750\,d}{d^2+12500}$$

omega_n =

$$3\,\sqrt{5}\,\sqrt{\frac{d^2}{d^2+12500}}$$

--------------- Script 2 # ---------------

```
%  plots omega_2 and the magnitude of
% omega_n as functions of d
d = 0:0.5:200;
omega_2 = 3/50*d;
omega_n = 3*5^(1/2)*(d.^2./(d.^2+12500)).^(1/2);
plot(d, omega_2, d, omega_n)
xlabel('D (mm)')
title('Angular velocity (rad/s)','BackgroundColor','G')
```



References:

- **Meriam, et al. Engineering Mechanics. John Wiley & Sons, 2020. Solving dynamics problems with matlab. (2018, January). Retrieved from**
- MathWorks - Makers of MATLAB and Simulink - MATLAB & Simulink