

IE 535 Project

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Model 5 and Model 22 (extra credit) are solved.

Model 5

x_{ij} : Weight of food i for type j .

$i \in \{1, 2, 3, 4\}$

$j \in \{1, 2\}$

Objective

$$\min -0.895x_{11} - 1.305x_{21} - 1.265x_{31} - 1.125x_{41} - 0.37x_{12} - 0.78x_{22} - 0.74x_{32} - 0.6x_{42}$$

s.t

$$43.419x_{11} + 6.56x_{21} + 11.41x_{31} + 25.96x_{41} \geq 0$$

$$11.075x_{11} + 35.785x_{21} + 23.435x_{31} + 37.685x_{41} \leq 0$$

$$3.579x_{11} + 0.879x_{21} + 1.779x_{31} + 2.679x_{41} \geq 0$$

$$11.075x_{12} + 35.785x_{22} + 23.435x_{32} + 37.685x_{42} \geq 0$$

$$x_{11} + x_{12} \leq 1500$$

$$x_{21} + x_{22} \leq 500$$

$$x_{31} + x_{32} \leq 1000$$

$$x_{41} + x_{42} \leq 2000$$

$$x_{ij} \geq 0 \forall i, j$$

Code

```
1 import numpy as np
2
3
4
5 def Phase_1(A, b, z):
6     m, n = A.shape
7     I = np.identity(m)
8     # Adds artificial variables to all the constraints i.e "checks" for the identity part
9     A_new = np.hstack((A, I))
10    cb = np.array([1 for _ in range(m)])
11    z_new = cb@IOA      # C_N is zero in 2 phase objective
```

```

12 padding = np.array([0 for _ in range(m)])
13 z_new = np.hstack((z_new, padding))
14 xb1 = np.array([(n+i+1) for i in range(m)])
15 z_o = cb@I@b
16 # building the tableau for 2 phase
17 tb, m_2, n_2 = build_tableau(A_new, b, -z_new, xb1, z_o)
18 # Phase 1 of Simplex
19 tb, z_star, sol, xb = Simplex(tb, m_2, n_2, opt_flag= False)
20 if z_star != 0:
21     print("Infeasible")
22     return (0, 0, 0, 0)
23 # Removing the artificial variables
24 artf_idx = list(range(n+m+2))
25 del artf_idx[-(m+1):-1]
26 tb = tb[:, artf_idx]
27 # Redundent Constraints
28 l = [np.where(xb==i)[0] for i in xb1]
29 for i in l:
30     tb = np.delete(tb, i+2, axis = 0)
31     xb = np.delete(xb, i, axis=0)
32     sol = np.delete(sol, i, axis = 0)
33 m = xb.shape[0]
34 # Convert the 2 phase z row to regular z row
35 tb[0, :] = np.array([i for i in range(n+2)])
36 nbv_idx = np.array(list(range(1, n+1)))
37 nbv_idx = np.setdiff1d(nbv_idx, xb)
38 B_invN = tb[2:, nbv_idx]
39 cb = z[(xb-1).astype(int)]
40 cn = z[(nbv_idx-1).astype(int)]
41 tb[1, nbv_idx.astype(int)] = cb@B_invN - cn
42 tb[1, xb.astype(int)] = 0
43 tb[1, -1] = cb@sol
44 # Phase II Simplex
45 f_tb, f_z_star, f_sol, f_xb = Simplex(tb, m, n)
46 return f_tb, f_z_star, f_sol, f_xb
47
48
49
50 def Optimality_check(z, xb):
51     #Bland's rule: argmax picks the smallest index for a tie
52     idx = np.argmax(z)
53     if z[idx] == 0:
54         return True, idx+1
55     return False, idx+1
56
57 def ratio_test(b, pivot_col, xb):
58     ix = 0
59     flag2 = False
60     if pivot_col[np.argmax(pivot_col)] <= 0:
61         flag2 = True
62         return flag2, ix+2
63     pivot_col[pivot_col<0] = 0
64     np.seterr(divide='ignore')
65     t = b/pivot_col
66     t[np.isnan(t)] = np.Inf
67     # t[t<0] = np.Inf

```

```

68     ix = np.argmin(t)
69     #Bland's rule: picks the smallest index for a tie
70     bld_rule = (t==t[ix])
71     if sum(bld_rule) > 1:
72         ties = xb[bld_rule]
73         min_idx = np.argmin(ties)
74         ix = np.where(xb==ties[min_idx])[0][0]
75     return flag2, ix+2
76
77
78
79 def build_tableau(A, b, z, xb, z_o = 0):
80     m, n = A.shape
81     tableau = np.zeros((m+2, n+2))
82     tableau[0,:] = np.array(range(n+2))
83     tableau[1, 1:n+1] = -z
84     tableau[1, -1] = z_o
85     tableau[2:(m+2), 1:n+1] = A
86     tableau[2:(m+2), -1] = b
87     tableau[2:, 0] = xb
88     return tableau, m, n
89
90 def Simplex(tableau, m, n, opt_flag = True):
91     i = 0
92     while 1:
93         eps = 1e-6
94         tableau[np.abs(tableau) < eps] = 0
95         z = tableau[1, 1:n+1]
96         xb = tableau[2:, 0]
97         b = tableau[2:(m+2), -1]
98         flag1, idx = Optimality_check(z, xb)
99         if flag1 == True:
100             if opt_flag == True:
101                 print("Optimal Solution.")
102                 break
103             pivot_col = np.copy(tableau[2:, idx])
104             flag2, ix = ratio_test(b, pivot_col, xb)
105             if flag2 == True:
106                 print("Unbounded.")
107                 break
108             pivot = tableau[ix, idx]
109             tableau[ix, 1:] = tableau[ix, 1:]/pivot
110             npvt = list(range(1, m+2))
111             npvt.remove(ix)
112             pivot_col_ex_pivot = tableau[npvt, idx]
113             pivot_col_ex_pivot.shape = (m,1)
114             pivot_row = tableau[ix, 1:]
115             tableau[npvt, 1:] = tableau[npvt, 1:] - (pivot_col_ex_pivot * pivot_row) #broadcasting works
116             tableau[ix, 0] = idx
117             i = i+1
118             print("tableau for iteration {0}: \n".format(i),tableau)
119
120
121     z_star = tableau[1, -1]
122     sol = b
123     xb = tableau[2:, 0]

```

```

124
125     return tableau, z_star, sol, xb
126
127
128
129
130 if __name__ == '__main__':
131
132     # TEST CASES
133
134     # Regular problem for testing
135     # P = [[1, 1, -4, 0, 0, 0],
136     #      [1, 1, 2, 1, 0, 0],
137     #      [1, 1, -1, 0, 1, 0],
138     #      [-1, 1, 1, 0, 0, 1]]
139
140     # b = [9, 2, 4]
141
142     # xb = [4, 5, 6]
143
144     # Two phase problem for testing
145     # P = [[6, 3, 0, 0, 0],
146     #      [1, 1, -1, 0, 0],
147     #      [2, -1, 0, -1, 0],
148     #      [0, 3, 0, 0, 1]]
149
150     # b = [1, 1, 2]
151
152     # xb = [4, 5, 6]
153
154     # P = [[-3, -2, -1, 0, 0],
155     #      [3, -3, 2, 1, 0],
156     #      [-1, 2, 1, 0, 1]]
157
158     # b = [3, 6]
159
160     # xb = [4, 5]
161
162     # Edge case with 0/-ve in ratio test
163     # P = [[-1, -3, 0, 0, 0],
164     #      [1, -2, 1, 0, 0],
165     #      [-2, 1, 0, 1, 0],
166     #      [5, 3, 0, 0, 1]]
167
168     # b = [0, 4, 15]
169
170     # xb = [3, 4, 5]
171
172     # Redundent constraint problem
173
174     # P = [[19, 17, 23, 21, 25],
175     #      [60, 25, 45, 20, 50],
176     #      [10, 15, 45, 50, 40],
177     #      [30, 60, 10, 30, 10],
178     #      [1, 1, 1, 1, 1]]
179

```

```

180     # b = [40, 35, 25, 1]
181
182     # degenerate LP
183
184     # P = [[-1, -1, -1, 0, 0],
185     #      [1, 1, 0, 1, 0],
186     #      [0, -1, 1, 0, 1]]
187
188     # b = [8, 0]
189
190     # PROJECT MODELS
191
192     # Model 5
193     #['x11', 'x21', 'x31', 'x41', 'x12', 'x22', 'x32', 'x42', 's1', 's2', 's3', 's4', 's5', 's6', 's7', 's8']
194
195     #P = [[-0.895, -1.305, -1.265, -1.125, -0.37, -0.78, -0.74, -0.6, 0, 0, 0, 0, 0, 0, 0, #0, 0],
196     #     [43.419, 6.56, 11.41, 25.96, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0],
197     #     [11.075, 35.785, 23.435, 37.685, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0],
198     #     [3.579, 0.879, 1.779, 2.679, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0],
199     #     [0, 0, 0, 0, 11.075, 35.785, 23.435, 37.685, 0, 0, 0, -1, 0, 0, 0, 0],
200     #     [1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0],
201     #     [0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0],
202     #     [0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0],
203     #     [0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0]]
204
205     #b = [0, 0, 0, 0, 1500, 500, 1000, 2000]
206
207     # EXTRA CREDIT: MODEL 22
208     #['x111, x121, x131, x141, x151, x211, x221, x231, x241, x251, x311, x321, x331, x341, x351, x112, x122, x132, x142, x152, x212, x222, x232, x242, x252, x312, x322, x332, x342, x352, x113, x123, x133, x143, x153, x213, x223, x233, x243, x253, x313, x323, x333, x343, x353, x114, x124, x134, x144, x154, x214, x224, x234, x244, x254, x314, x324, x334, x344, x354, x115, x125, x135, x145, x155, x215, x225, x235, x245, x255, x315, x325, x335, x345, x355, x116, x126, x136, x146, x156, x216, x226, x236, x246, x256, x316, x326, x336, x346, x356, x117, x127, x137, x147, x157, x217, x227, x237, x247, x257, x317, x327, x337, x347, x357, x118, x128, x138, x148, x158, x218, x228, x238, x248, x258, x318, x328, x338, x348, x358, x119, x129, x139, x149, x159, x219, x229, x239, x249, x259, x319, x329, x339, x349, x359, x120, x130, x140, x150, x220, x230, x240, x250, x320, x330, x340, x350, x121, x131, x141, x151, x221, x231, x241, x251, x321, x331, x341, x351, x122, x132, x142, x152, x222, x232, x242, x252, x322, x332, x342, x352, x123, x133, x143, x153, x223, x233, x243, x253, x323, x333, x343, x353, x124, x134, x144, x154, x224, x234, x244, x254, x324, x334, x344, x354, x125, x135, x145, x155, x225, x235, x245, x255, x325, x335, x345, x355, x126, x136, x146, x156, x226, x236, x246, x256, x326, x336, x346, x356, x127, x137, x147, x157, x227, x237, x247, x257, x327, x337, x347, x357, x128, x138, x148, x158, x228, x238, x248, x258, x328, x338, x348, x358, x129, x139, x149, x159, x229, x239, x249, x259, x329, x339, x349, x359, x130, x140, x150, x230, x240, x250, x330, x340, x350, x131, x141, x151, x231, x241, x251, x331, x341, x351, x132, x142, x152, x232, x242, x252, x332, x342, x352, x133, x143, x153, x233, x243, x253, x333, x343, x353, x134, x144, x154, x234, x244, x254, x334, x344, x354, x135, x145, x155, x235, x245, x255, x335, x345, x355, x136, x146, x156, x236, x246, x256, x336, x346, x356, x137, x147, x157, x237, x247, x257, x337, x347, x357, x138, x148, x158, x238, x248, x258, x338, x348, x358, x139, x149, x159, x239, x249, x259, x339, x349, x359, x140, x150, x240, x250, x340, x350, x141, x151, x241, x251, x341, x351, x142, x152, x242, x252, x342, x352, x143, x153, x243, x253, x343, x353, x144, x154, x244, x254, x344, x354, x145, x155, x245, x255, x345, x355, x146, x156, x246, x256, x346, x356, x147, x157, x247, x257, x347, x357, x148, x158, x248, x258, x348, x358, x149, x159, x249, x259, x349, x359, x150, x250, x350, x151, x251, x351, x152, x252, x352, x153, x253, x353, x154, x254, x354, x155, x255, x355, x156, x256, x356, x157, x257, x357, x158, x258, x358, x159, x259, x359, x160, x260, x360, x161, x261, x361, x162, x262, x362, x163, x263, x363, x164, x264, x364, x165, x265, x365, x166, x266, x366, x167, x267, x367, x168, x268, x368, x169, x269, x369, x170, x270, x370, x171, x271, x371, x172, x272, x372, x173, x273, x373, x174, x274, x374, x175, x275, x375, x176, x276, x376, x177, x277, x377, x178, x278, x378, x179, x279, x379, x180, x280, x380, x181, x281, x381, x182, x282, x382, x183, x283, x383, x184, x284, x384, x185, x285, x385, x186, x286, x386, x187, x287, x387, x188, x288, x388, x189, x289, x389, x190, x290, x390, x191, x291, x391, x192, x292, x392, x193, x293, x393, x194, x294, x394, x195, x295, x395, x196, x296, x396, x197, x297, x397, x198, x298, x398, x199, x299, x399, x200, x300, x400, x401, x402, x403, x404, x405, x406, x407, x408, x409, x410, x411, x412, x413, x414, x415, x416, x417, x418, x419, x420, x421, x422, x423, x424, x425, x426, x427, x428, x429, x430, x431, x432, x433, x434, x435, x436, x437, x438, x439, x440, x441, x442, x443, x444, x445, x446, x447, x448, x449, x450, x451, x452, x453, x454, x455, x456, x457, x458, x459, x460, x461, x462, x463, x464, x465, x466, x467, x468, x469, x470, x471, x472, x473, x474, x475, x476, x477, x478, x479, x480, x481, x482, x483, x484, x485, x486, x487, x488, x489, x490, x491, x492, x493, x494, x495, x496, x497, x498, x499, x500]
209     P = [[61, 72, 45, 55, 66, 69, 78, 60, 49, 56, 59, 66, 63, 61, 47, 58.5, 68.3, 47.8, 0, 63.5, 65.3, 74.8, 55.0, 49.0,
210     57.5, 0, 61.3, 63.5, 58.8, 50.0, 0, 0, 0, 0],
211         [1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0],
212         [0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0],
213         [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 0],
214         [1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0],
215         [0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0],
216         [0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0],
217         [0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0],
218         [0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0],
219         [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 27.5, 30.3, 23.8, 0, 28.5, 29.3, 31.8, 27.0, 25.0, 26.5, 0, 28.3,
220         27.5, 26.8, 24.0, 0, 0, 0, 1],
221         [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
222         [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0]]
223
224     b = [10, 20, 15, 7, 11, 9, 10, 8, 675000, 0, 0]
225
226     P = np.array(P)
227     b = np.array(b)
228     z = P[0, :]
229     A = P[1:, :]
230     # xb = np.array(xb)
231
232     print("Input matrix A:{0}\nb:{1}".format(A, b.T))
233
234     # A, b, z have there usual meaning. Input has to be in standard form (minimization)
235     # with slack variables. For the initial basis program automatically adds artificial

```

```

236     # variables for all the constraints and every problem goes through two phases.
237     #
238     # The code that deals with redundancy is in "Phase_1" function
239     #
240     # Bland's rule is used to prevent cycling. Check Ratio_test, Optimality_check #functions
241
242     f_tb, f_z_star, f_sol, f_xb = Phase_1(A, b, z)
243
244     print("Final Optimal Tableau:", f_tb)
245
246     print("Optimal Objective function value:", f_z_star)
247     for i, j in zip(f_xb, f_sol):
248         print("Var:{0} = {1}".format(i, j))
249     print("Rest of the variables are non basic variables, therefore = 0")

```

Results

NOTE: Output for each iteration is in the Appendix!

$$\begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \\ x_{41} \\ x_{12} \\ x_{22} \\ x_{32} \\ x_{42} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1500 \\ 500 \\ 1000 \\ 2000 \\ 0 \\ 0 \\ 0 \\ 133310.0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$z^* = -2884.9999$$

```
1.00000000e+00 2.00000000e+05]]
Optimal Objective function value: -2884.999999999995
Var:1.0 = 0.0
Var:9.0 = 0.0
Var:7.0 = 1000.0
Var:5.0 = 1500.0
Var:6.0 = 500.0
Var:12.0 = 133310.0
Var:8.0 = 2000.0
Rest of the variables are non basic variables, therefore = 0
□
```

Figure 1: Optimal Solution

```

Optimal Solution.
Final Optimal Tableau: [[ 0.00000000e+00  1.00000000e+00  2.00000000e+00  3.00000000e+00
  4.00000000e+00  5.00000000e+00  6.00000000e+00  7.00000000e+00
  8.00000000e+00  9.00000000e+00  1.00000000e+01  1.10000000e+01
  1.20000000e+01  1.30000000e+01  1.40000000e+01  1.50000000e+01
  1.60000000e+01  1.70000000e+01]
[ 0.00000000e+00  0.00000000e+00 -1.17135440e+00 -5.85914221e-01
-1.26142212e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00 -4.74040632e-02  0.00000000e+00
  0.00000000e+00 -3.70000000e-01 -7.80000000e-01 -7.40000000e-01
-6.00000000e-01 -2.88500000e+03]
[ 1.00000000e+00  1.00000000e+00  3.23115124e+00  2.11602709e+00
 3.40270880e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  9.02934537e-02  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00]
[ 9.00000000e+00  0.00000000e+00  1.33733356e+02  8.04657801e+01
 1.21782214e+02  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  1.00000000e+00  3.92045147e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00]
[ 7.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
  0.00000000e+00  1.00000000e+03]
[ 5.00000000e+00  0.00000000e+00 -3.23115124e+00 -2.11602709e+00
-3.40270880e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00 -9.02934537e-02  0.00000000e+00
  0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  1.50000000e+03]
[ 6.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
  0.00000000e+00  5.00000000e+02]
[ 1.20000000e+01  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00 -1.00000000e+00  0.00000000e+00
  1.00000000e+00  1.10750000e+01  3.57850000e+01  2.34350000e+01
  3.76850000e+01  1.33310000e+05]
[ 8.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  2.00000000e+03]]

```

Figure 2: Optimal tableau

Commercial Solver (Octave gplk)

```

1  c = [-0.895, -1.305, -1.265, -1.125, -0.37, -0.78, -0.74, -0.6, 0, 0, 0, 0, 0, 0, 0, 0];
2
3  A = [43.419, 6.56, 11.41, 25.96, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0;
4        11.075, 35.785, 23.435, 37.685, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0;
5        3.579, 0.879, 1.779, 2.679, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0;
6        0, 0, 0, 0, 11.075, 35.785, 23.435, 37.685, 0, 0, 0, -1, 0, 0, 0, 0;
7        1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0;
8        0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0;
9        0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1;
10       0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1];
11
12  % A = [ 1, 1, 1;
13         10, 4, 5;
14         2, 2, 6];
15  b = [0, 0, 0, 0, 1500, 500, 1000, 2000]';
16  lb = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]';
17  ub = [];
18  ctype = "SSSSSSSS";
19  vartype = "CCCCCCCCCCCCCCCC";
20  s = 1;
21
22  param.msglev = 1;
23  param.itlim = 100;
24
25  [xmin, fmin, status, extra] = ...
26      glpk (c, A, b, lb, ub, ctype, vartype, s, param);
27
28  disp("Status"), disp(status);
29  disp("Optimal Objective:"), disp(fmin);
30  disp("Solution:"), disp(xmin);

```

$$\begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \\ x_{41} \\ x_{12} \\ x_{22} \\ x_{32} \\ x_{42} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1500 \\ 500 \\ 1000 \\ 2000 \\ 0 \\ 0 \\ 0 \\ 133310.0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$z^* = -2885$$

```
Status
0
Optimal Objective:
-2885
Solution:
0
0
0
0
1500
500
1000
2000
0
0
0
133310
0
0
0
0
```

Figure 3: Commercial Solver Output

Extra Credit: Model 22

$i \in \{1, 2, 3\}$

$j \in \{1, 2, 3, 4, 5\}$

$k \in \{1, 2\}$

r_{ij} : Unit cost by rail for i th source and j th market

s_{ij} : Unit cost by ship for i th source and j th market

I_{ij} : Uniform annual cost of investment for i th source and j th market

x_{ijk} : wood units in million board feet for i th source, j th market and k th transportation type

a_i : Availability of i th source

d_j : Demand for j th market

Objective

$$\min \sum_j \sum_i r_{ij} x_{ij1} + \sum_j \sum_i (s_{ij} + I_{ij}) x_{ij2}$$

s.t

$$\sum_k \sum_j x_{ijk} \leq a_i \quad \forall i$$

$$\sum_k \sum_i x_{ijk} = d_j \quad \forall j$$

$$\sum_j \sum_i I_{ij} x_{ij2} \leq 675000$$

$$x_{142} = 0$$

$$x_{312} = 0$$

$$x_{ijk} \geq 0$$

Results

NOTE: Output for each iteration is in the Appendix at the end!

$$x_{131} = 9.0, x_{241} = 10.0, x_{251} = 4.0, x_{351} = 4.0, x_{112} = 1.0, x_{212} = 6.0, x_{322} = 11, s3 = 674485.4$$

$$z^* = 2431.5999$$

```
Optimal Objective function value: 2431.5999999999999
Var:21.0 = 6.0
Var:27.0 = 11.0
Var:15.0 = 4.0
Var:16.0 = 1.0
Var:9.0 = 10.0
Var:32.0 = 0.0
Var:10.0 = 4.0
Var:3.0 = 9.0
Var:34.0 = 674485.40000000001
Var:19.0 = 0.0
Var:26.0 = 0.0
Rest of the variables are non basic variables, therefore = 0
□
```

Figure 4: Model 22 Optimal value

[illegible]

Figure 5: Model 22 Optimal Tableau

Commercial Solver (Octave gplk)

```
1  c = [-0.895, -1.305, -1.265, -1.125, -0.37, -0.78, -0.74, -0.6, 0, 0, 0, 0, 0, 0, 0, 0];
2
3  A = [43.419, 6.56, 11.41, 25.96, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0;
4       11.075, 35.785, 23.435, 37.685, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0;
5       3.579, 0.879, 1.779, 2.679, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0;
6       0, 0, 0, 0, 11.075, 35.785, 23.435, 37.685, 0, 0, 0, -1, 0, 0, 0, 0;
7       1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0;
8       0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0;
9       0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0;
10      0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1];
11
12  % A = [ 1, 1, 1;
13         10, 4, 5;
14         2, 2, 6];
15  b = [0, 0, 0, 0, 1500, 500, 1000, 2000]';
16  lb = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]';
17  ub = [];
18  ctype = "SSSSSSSS";
19  vartype = "CCCCCCCCCCCCCCCC";
20  s = 1;
21
22  param.msglev = 1;
23  param.itlim = 100;
24
25  [xmin, fmin, status, extra] = ...
26      glpk (c, A, b, lb, ub, ctype, vartype, s, param);
27
28  disp("Status"), disp(status);
29  disp("Optimal Objective:"), disp(fmin);
30  disp("Solution:"), disp(xmin);
```

$$x_{131} = 9.0, x_{241} = 10.0, x_{251} = 4.0, x_{351} = 4.0, x_{112} = 1.0, x_{212} = 6.0, x_{322} = 11, s3 = 674485.4$$

$$z^* = 2431.6$$

```

Status
0
Optimal Objective:
2431.6
Solution:
0
0
9.0000e+00
0
0
0
0
0
0
1.0000e+01
4.0000e+00
0
0
0
0
4.0000e+00
1.0000e+00
0
0
0
0
6.0000e+00
0
0
0
0
0
1.1000e+01
0
0
0
0
8.8818e-16
0
6.7449e+05

```

Figure 6: Solution of Commercial Solver

Appendix

Model 5 tableau Output

```
tableau for iteration 1:
[[ 0.00000000e+00  1.00000000e+00  2.00000000e+00  3.00000000e+00
  4.00000000e+00  5.00000000e+00  6.00000000e+00  7.00000000e+00
  8.00000000e+00  9.00000000e+00  1.00000000e+01  1.10000000e+01
  1.20000000e+01  1.30000000e+01  1.40000000e+01  1.50000000e+01
  1.60000000e+01  1.70000000e+01  1.80000000e+01  1.90000000e+01
  2.00000000e+01  2.10000000e+01  2.20000000e+01  2.30000000e+01
  2.40000000e+01  2.50000000e+01]
[ 0.00000000e+00 -5.35287240e+01  2.72114638e+01  8.03359784e+00
  0.00000000e+00  1.20750000e+01  3.67850000e+01  2.44350000e+01
  3.86850000e+01  1.59337442e+00  1.00000000e+00 -1.00000000e+00
 -1.00000000e+00  1.00000000e+00  1.00000000e+00  1.00000000e+00
  1.00000000e+00 -2.59337442e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  5.00000000e+03]
[ 4.00000000e+00  1.67253467e+00  2.52696456e-01  4.39522342e-01
  1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00 -3.85208012e-02  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  3.85208012e-02  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00]
[ 1.80000000e+01 -5.19544690e+01  2.62621341e+01  6.87160054e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  1.45165639e+00  1.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00 -1.45165639e+00  1.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00]
[ 1.90000000e+01 -9.01720378e-01  2.02026194e-01  6.01519646e-01
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  1.03197227e-01  0.00000000e+00 -1.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00 -1.03197227e-01  0.00000000e+00  1.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00]
[ 2.00000000e+01  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  1.10750000e+01  3.57850000e+01  2.34350000e+01
  3.76850000e+01  0.00000000e+00  0.00000000e+00  0.00000000e+00
 -1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00]
[ 2.10000000e+01  1.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  1.50000000e+03]
[ 2.20000000e+01  0.00000000e+00  1.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
  0.00000000e+00  5.00000000e+02]
[ 2.30000000e+01  0.00000000e+00  0.00000000e+00  1.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
  0.00000000e+00  1.00000000e+03]
[ 2.40000000e+01 -1.67253467e+00 -2.52696456e-01 -4.39522342e-01
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  1.00000000e+00  3.85208012e-02  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  1.00000000e+00 -3.85208012e-02  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  1.00000000e+00  2.00000000e+03]]
```

Figure 7: Phase 1 itr 1

```

1.0000000e+00 2.0000000e+03]]
tableau for iteration 2:
[[ 0.0000000e+00 1.0000000e+00 2.0000000e+00 3.0000000e+00
 4.0000000e+00 5.0000000e+00 6.0000000e+00 7.0000000e+00
 8.0000000e+00 9.0000000e+00 1.0000000e+01 1.1000000e+01
 1.2000000e+01 1.3000000e+01 1.4000000e+01 1.5000000e+01
 1.6000000e+01 1.7000000e+01 1.8000000e+01 1.9000000e+01
 2.0000000e+01 2.1000000e+01 2.2000000e+01 2.3000000e+01
 2.4000000e+01 2.5000000e+01]
[ 0.0000000e+00 -5.35287240e+01 2.72114638e+01 8.03359784e+00
 0.0000000e+00 7.06116492e-01 5.04179382e-02 3.78134536e-01
 0.0000000e+00 1.59337442e+00 1.0000000e+00 -1.0000000e+00
 2.65357569e-02 1.0000000e+00 1.0000000e+00 1.0000000e+00
 1.0000000e+00 -2.59337442e+00 0.0000000e+00 0.0000000e+00
 -1.02653576e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 5.0000000e+03]
[ 4.0000000e+00 1.67253467e+00 2.52696456e-01 4.39522342e-01
 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 -3.85208012e-02 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 3.85208012e-02 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00]
[ 1.8000000e+01 -5.19544690e+01 2.62621341e+01 6.87160054e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 1.45165639e+00 1.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 -1.45165639e+00 1.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00]
[ 1.9000000e+01 -9.01728378e-01 2.02026194e-01 6.01519646e-01
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 1.03197227e-01 0.0000000e+00 -1.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 -1.03197227e-01 0.0000000e+00 1.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00]
[ 8.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 2.93883508e-01 9.49582062e-01 6.21865464e-01
 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 -2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00]
[ 2.1000000e+01 1.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 1.5000000e+03]
[ 2.2000000e+01 0.0000000e+00 1.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
 0.0000000e+00 5.0000000e+02]
[ 2.3000000e+01 0.0000000e+00 0.0000000e+00 1.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
 0.0000000e+00 1.0000000e+03]
[ 2.4000000e+01 -1.67253467e+00 -2.52696456e-01 -4.39522342e-01
 0.0000000e+00 -2.93883508e-01 -9.49582062e-01 -6.21865464e-01
 0.0000000e+00 3.85208012e-02 0.0000000e+00 0.0000000e+00
 2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
 1.0000000e+00 -3.85208012e-02 0.0000000e+00 0.0000000e+00
 -2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
 1.0000000e+00 2.0000000e+03]]

```

Figure 8: Phase 1 itr 2


```

1.00000000e+00 2.00000000e+03]]
tableau for iteration 3:
[[ 0.00000000e+00 1.00000000e+00 2.00000000e+00 3.00000000e+00
 4.00000000e+00 5.00000000e+00 6.00000000e+00 7.00000000e+00
 8.00000000e+00 9.00000000e+00 1.00000000e+01 1.10000000e+01
 1.20000000e+01 1.30000000e+01 1.40000000e+01 1.50000000e+01
 1.60000000e+01 1.70000000e+01 1.80000000e+01 1.90000000e+01
 2.00000000e+01 2.10000000e+01 2.20000000e+01 2.30000000e+01
 2.40000000e+01 2.50000000e+01]
[ 0.00000000e+00 -2.3634600e+02 0.00000000e+00 -3.92960976e+01
-1.07684390e+02 7.06116492e-01 5.04179382e-02 3.78134536e-01
0.00000000e+00 5.74146341e+00 1.00000000e+00 -1.00000000e+00
2.65357569e-02 1.00000000e+00 1.00000000e+00 1.00000000e+00
1.00000000e+00 -6.74146341e+00 0.00000000e+00 0.00000000e+00
-1.02653576e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 5.00000000e+03]
[ 2.00000000e+00 6.61875000e+00 1.00000000e+00 1.73932927e+00
3.95731707e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 -1.52439024e-01 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 1.52439024e-01 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00]
[ 1.80000000e+01 -2.25776969e+02 0.00000000e+00 -3.88068979e+01
-1.03927591e+02 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 5.45503049e+00 1.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 -5.45503049e+00 1.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00]
[ 1.90000000e+01 -2.23888125e+00 0.00000000e+00 2.50129573e-01
-7.99481707e-01 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 1.33993902e-01 0.00000000e+00 -1.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 -1.33993902e-01 0.00000000e+00 1.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00]
[ 8.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 2.93883508e-01 9.49582062e-01 6.2185464e-01
1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
-2.65357569e-02 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
2.65357569e-02 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00]
[ 2.10000000e+01 1.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 1.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 1.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 1.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 1.50000000e+03]
[ 2.20000000e+01 -6.61875000e+00 0.00000000e+00 -1.73932927e+00
-3.95731707e+00 0.00000000e+00 1.00000000e+00 0.00000000e+00
0.00000000e+00 1.52439024e-01 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 1.00000000e+00 0.00000000e+00
0.00000000e+00 -1.52439024e-01 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 1.00000000e+00 0.00000000e+00
0.00000000e+00 5.00000000e+02]
[ 2.30000000e+01 0.00000000e+00 0.00000000e+00 1.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00
0.00000000e+00 1.00000000e+03]
[ 2.40000000e+01 0.00000000e+00 0.00000000e+00 0.00000000e+00
1.00000000e+00 -2.93883508e-01 -9.49582062e-01 -6.2185464e-01
0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
2.65357569e-02 0.00000000e+00 0.00000000e+00 0.00000000e+00
1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
-2.65357569e-02 0.00000000e+00 0.00000000e+00 0.00000000e+00
1.00000000e+00 2.00000000e+03]]

```

Figure 9: Phase 1 itr 3

```

1.0000000e+00 2.0000000e+03]]
tableau for iteration 4:
[[ 0.0000000e+00 1.0000000e+00 2.0000000e+00 3.0000000e+00
 4.0000000e+00 5.0000000e+00 6.0000000e+00 7.0000000e+00
 8.0000000e+00 9.0000000e+00 1.0000000e+01 1.1000000e+01
 1.2000000e+01 1.3000000e+01 1.4000000e+01 1.5000000e+01
 1.6000000e+01 1.7000000e+01 1.8000000e+01 1.9000000e+01
 2.0000000e+01 2.1000000e+01 2.2000000e+01 2.3000000e+01
 2.4000000e+01 2.5000000e+01]
[ 0.0000000e+00 3.9747352e+00 0.0000000e+00 1.5484742e+00
 1.7002347e+00 7.0611649e-01 5.0417938e-02 3.7813453e-01
 0.0000000e+00 0.0000000e+00 -5.2508034e-02 -1.0000000e+00
 2.6535756e-02 1.0000000e+00 1.0000000e+00 1.0000000e+00
 1.0000000e+00 -1.0000000e+00 -1.0525080e+00 0.0000000e+00
 -1.0265357e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 5.0000000e+03]
[ 2.0000000e+00 3.0948721e-01 1.0000000e+00 6.5488331e-01
 1.0530948e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 2.7944669e-02 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 2.7944669e-02 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00]
[ 9.0000000e+00 -4.1388763e+01 0.0000000e+00 -7.1139653e+00
 -1.9051697e+01 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 1.0000000e+00 1.8331703e-01 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 -1.0000000e+00 1.8331703e-01 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00]
[ 1.9000000e+01 3.3069074e+00 0.0000000e+00 1.2033575e+00
 1.7533296e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 -2.4563364e-02 -1.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 -2.4563364e-02 1.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00]
[ 8.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 2.9388350e-01 9.4958206e-01 6.2186546e-01
 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 -2.6535756e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 2.6535756e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00]
[ 2.1000000e+01 1.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 1.5000000e+03]
[ 2.2000000e+01 -3.0948721e-01 0.0000000e+00 -6.5488331e-01
 -1.0530948e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 -2.7944669e-02 0.0000000e+00
 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 -2.7944669e-02 0.0000000e+00
 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
 0.0000000e+00 5.0000000e+02]
[ 2.3000000e+01 0.0000000e+00 0.0000000e+00 1.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
 0.0000000e+00 1.0000000e+03]
[ 2.4000000e+01 0.0000000e+00 0.0000000e+00 0.0000000e+00
 1.0000000e+00 -2.9388350e-01 -9.4958206e-01 -6.2186546e-01
 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 2.6535756e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 -2.6535756e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
 1.0000000e+00 2.0000000e+03]]
tableau for iteration 5:

```

Figure 10: Phase 1 itr 4

```

tableau for iteration 5:
[[ 0.0000000e+00 1.0000000e+00 2.0000000e+00 3.0000000e+00
 4.0000000e+00 5.0000000e+00 6.0000000e+00 7.0000000e+00
 8.0000000e+00 9.0000000e+00 1.0000000e+01 1.1000000e+01
 1.2000000e+01 1.3000000e+01 1.4000000e+01 1.5000000e+01
 1.6000000e+01 1.7000000e+01 1.8000000e+01 1.9000000e+01
 2.0000000e+01 2.1000000e+01 2.2000000e+01 2.3000000e+01
 2.4000000e+01 2.5000000e+01]
[ 0.0000000e+00 0.0000000e+00 -1.29164415e+01 -6.91028804e+00
-1.19028036e+01 7.06116492e-01 5.04179382e-02 3.78134536e-01
0.0000000e+00 0.0000000e+00 -4.13453725e-01 -1.0000000e+00
2.65357569e-02 1.0000000e+00 1.0000000e+00 1.0000000e+00
1.0000000e+00 -1.0000000e+00 -1.41345372e+00 0.0000000e+00
-1.02653576e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 5.0000000e+03]
[ 1.0000000e+00 1.0000000e+00 3.23115124e+00 2.11602709e+00
3.40278880e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 9.02934537e-02 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 9.02934537e-02 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 9.0000000e+00 0.0000000e+00 1.33733356e+02 8.04657801e+01
1.21782214e+02 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 1.0000000e+00 3.92845147e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 -1.0000000e+00 3.92845147e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 1.9000000e+01 0.0000000e+00 -1.06852903e+01 -5.79426095e+00
-9.49929481e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 -3.23160271e-01 -1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 -3.23160271e-01 1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 8.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 2.93883508e-01 9.49582062e-01 6.21865464e-01
1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
-2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 2.1000000e+01 0.0000000e+00 -3.23115124e+00 -2.11602709e+00
-3.40278880e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 -9.02934537e-02 0.0000000e+00
0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 -9.02934537e-02 0.0000000e+00
0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 1.5000000e+03]
[ 2.2000000e+01 0.0000000e+00 1.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 2.3000000e+01 0.0000000e+00 0.0000000e+00 1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 2.4000000e+01 0.0000000e+00 0.0000000e+00 0.0000000e+00
1.0000000e+00 -2.93883508e-01 -9.49582062e-01 -6.21865464e-01
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
-2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
1.0000000e+00 2.0000000e+03]]

```

Figure 11: Phase 1 itr 5

```

tableau for iteration 6:
[[ 0.0000000e+00 1.0000000e+00 2.0000000e+00 3.0000000e+00
 4.0000000e+00 5.0000000e+00 6.0000000e+00 7.0000000e+00
 8.0000000e+00 9.0000000e+00 1.0000000e+01 1.1000000e+01
 1.2000000e+01 1.3000000e+01 1.4000000e+01 1.5000000e+01
 1.6000000e+01 1.7000000e+01 1.8000000e+01 1.9000000e+01
 2.0000000e+01 2.1000000e+01 2.2000000e+01 2.3000000e+01
 2.4000000e+01 2.5000000e+01]
[ 0.0000000e+00 0.0000000e+00 -9.68529029e+00 -4.79426095e+00
-8.49929481e+00 -2.93883508e-01 5.04179382e-02 3.78134536e-01
0.0000000e+00 0.0000000e+00 -3.23160271e-01 -1.0000000e+00
2.65357569e-02 0.0000000e+00 1.0000000e+00 1.0000000e+00
1.0000000e+00 -1.0000000e+00 -1.32316027e+00 0.0000000e+00
-1.02653576e+00 -1.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 3.5000000e+03]
[ 1.0000000e+00 1.0000000e+00 3.23115124e+00 2.11602789e+00
3.40270880e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 9.02934537e-02 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 9.02934537e-02 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 9.0000000e+00 0.0000000e+00 1.33733356e+02 8.04657801e+01
1.21782214e+02 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 1.0000000e+00 3.92045147e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 -1.0000000e+00 3.92045147e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 1.9000000e+01 0.0000000e+00 -1.06852903e+01 -5.79426095e+00
-9.49929481e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 -3.23160271e-01 -1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 -3.23160271e-01 1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 8.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 2.93883508e-01 9.49582062e-01 6.21865464e-01
1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
-2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 1.3000000e+01 0.0000000e+00 -3.23115124e+00 -2.11602789e+00
-3.40270880e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 -9.02934537e-02 0.0000000e+00
0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 -9.02934537e-02 0.0000000e+00
0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 1.5000000e+03]
[ 2.2000000e+01 0.0000000e+00 1.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
0.0000000e+00 5.0000000e+02]
[ 2.3000000e+01 0.0000000e+00 0.0000000e+00 1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
0.0000000e+00 1.0000000e+03]
[ 2.4000000e+01 0.0000000e+00 0.0000000e+00 0.0000000e+00
1.0000000e+00 -2.93883508e-01 -9.49582062e-01 -6.21865464e-01
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
-2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
1.0000000e+00 2.0000000e+03]]

```

Figure 12: Phase 1 itr 6

```

tableau for iteration 7:
[[ 0.0000000e+00 1.0000000e+00 2.0000000e+00 3.0000000e+00
 4.0000000e+00 5.0000000e+00 6.0000000e+00 7.0000000e+00
 8.0000000e+00 9.0000000e+00 1.0000000e+01 1.1000000e+01
 1.2000000e+01 1.3000000e+01 1.4000000e+01 1.5000000e+01
 1.6000000e+01 1.7000000e+01 1.8000000e+01 1.9000000e+01
 2.0000000e+01 2.1000000e+01 2.2000000e+01 2.3000000e+01
 2.4000000e+01 2.5000000e+01]
[ 0.0000000e+00 0.0000000e+00 -1.06852903e+01 -4.79426095e+00
-8.49929481e+00 -2.93883508e-01 -9.49582062e-01 3.78134536e-01
0.0000000e+00 0.0000000e+00 -3.23160271e-01 -1.0000000e+00
2.65357569e-02 0.0000000e+00 0.0000000e+00 1.0000000e+00
1.0000000e+00 -1.0000000e+00 -1.32316027e+00 0.0000000e+00
-1.02653576e+00 -1.0000000e+00 -1.0000000e+00 0.0000000e+00
0.0000000e+00 3.0000000e+03]
[ 1.0000000e+00 1.0000000e+00 3.23115124e+00 2.11602709e+00
3.40270880e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 9.02934537e-02 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 9.02934537e-02 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 9.0000000e+00 0.0000000e+00 1.33733356e+02 8.04657801e+01
1.21782214e+02 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 1.0000000e+00 3.92045147e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 -1.0000000e+00 3.92045147e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 1.9000000e+01 0.0000000e+00 -1.06852903e+01 -5.79426095e+00
-9.49929481e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 -3.23160271e-01 -1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 -3.23160271e-01 1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 8.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 2.93883508e-01 9.49582062e-01 6.21865464e-01
1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
-2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00]
[ 1.3000000e+01 0.0000000e+00 -3.23115124e+00 -2.11602709e+00
-3.40270880e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 -9.02934537e-02 0.0000000e+00
0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 -9.02934537e-02 0.0000000e+00
0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 1.5000000e+03]
[ 1.4000000e+01 0.0000000e+00 1.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00
0.0000000e+00 5.0000000e+02]
[ 2.3000000e+01 0.0000000e+00 0.0000000e+00 1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00
0.0000000e+00 1.0000000e+03]
[ 2.4000000e+01 0.0000000e+00 0.0000000e+00 0.0000000e+00
1.0000000e+00 -2.93883508e-01 -9.49582062e-01 -6.21865464e-01
0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
-2.65357569e-02 0.0000000e+00 0.0000000e+00 0.0000000e+00
1.0000000e+00 2.0000000e+03]]

```

Figure 13: Phase 1 itr 7

```

1.00000000e+00 2.00000000e+03]]
tableau for iteration 8:
[[ 0.00000000e+00 1.00000000e+00 2.00000000e+00 3.00000000e+00
 4.00000000e+00 5.00000000e+00 6.00000000e+00 7.00000000e+00
 8.00000000e+00 9.00000000e+00 1.00000000e+01 1.10000000e+01
 1.20000000e+01 1.30000000e+01 1.40000000e+01 1.50000000e+01
 1.60000000e+01 1.70000000e+01 1.80000000e+01 1.90000000e+01
 2.00000000e+01 2.10000000e+01 2.20000000e+01 2.30000000e+01
 2.40000000e+01 2.50000000e+01]
[ 0.00000000e+00 0.00000000e+00 -1.06852903e+01 -5.79426095e+00
-8.49929481e+00 -2.93883508e-01 -9.49582062e-01 -6.21865464e-01
 0.00000000e+00 0.00000000e+00 -3.23160271e-01 -1.00000000e+00
 2.65357569e-02 0.00000000e+00 0.00000000e+00 0.00000000e+00
 1.00000000e+00 -1.00000000e+00 -1.32116027e+00 0.00000000e+00
-1.02653576e+00 -1.00000000e+00 -1.00000000e+00 -1.00000000e+00
 0.00000000e+00 2.00000000e+03]
[ 1.00000000e+00 1.00000000e+00 3.23115124e+00 2.11602709e+00
 3.40270880e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 9.02934537e-02 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 9.02934537e-02 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00]
[ 9.00000000e+00 0.00000000e+00 1.33733356e+02 8.04657801e+01
 1.21782214e+02 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 1.00000000e+00 3.92045147e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 -1.00000000e+00 3.92045147e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00]
[ 1.90000000e+01 0.00000000e+00 -1.06852903e+01 -5.79426095e+00
-9.49929481e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 -3.23160271e-01 -1.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 -3.23160271e-01 1.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00]
[ 8.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 2.93883508e-01 9.49582062e-01 6.21865464e-01
 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
-2.65357569e-02 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 2.65357569e-02 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00]
[ 1.30000000e+01 0.00000000e+00 -3.23115124e+00 -2.11602709e+00
-3.40270880e+00 1.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 -9.02934537e-02 0.00000000e+00
 0.00000000e+00 1.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 1.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 1.50000000e+03]
[ 1.40000000e+01 0.00000000e+00 1.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 1.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 1.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 1.00000000e+00 0.00000000e+00
 0.00000000e+00 5.00000000e+02]
[ 1.50000000e+01 0.00000000e+00 0.00000000e+00 1.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00
 0.00000000e+00 1.00000000e+03]
[ 2.40000000e+01 0.00000000e+00 0.00000000e+00 0.00000000e+00
 1.00000000e+00 -2.93883508e-01 -9.49582062e-01 -6.21865464e-01
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 2.65357569e-02 0.00000000e+00 0.00000000e+00 0.00000000e+00
 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
-2.65357569e-02 0.00000000e+00 0.00000000e+00 0.00000000e+00
 1.00000000e+00 2.00000000e+03]]

```

Figure 14: Phase 1 itr 8

```

tableau for iteration 9:
[[ 0.00000000e+00 1.00000000e+00 2.00000000e+00 3.00000000e+00
 4.00000000e+00 5.00000000e+00 6.00000000e+00 7.00000000e+00
 8.00000000e+00 9.00000000e+00 1.00000000e+01 1.10000000e+01
 1.20000000e+01 1.30000000e+01 1.40000000e+01 1.50000000e+01
 1.60000000e+01 1.70000000e+01 1.80000000e+01 1.90000000e+01
 2.00000000e+01 2.10000000e+01 2.20000000e+01 2.30000000e+01
 2.40000000e+01 2.50000000e+01]
[ 0.00000000e+00 0.00000000e+00 -1.06852903e+01 -5.79426095e+00
-9.49929481e+00 1.44328993e-15 -2.66453526e-15 2.33146835e-15
 0.00000000e+00 0.00000000e+00 -3.23160271e-01 -1.00000000e+00
-3.81639165e-17 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 -1.00000000e+00 -1.32316027e+00 0.00000000e+00
-1.00000000e+00 -1.00000000e+00 -1.00000000e+00 -1.00000000e+00
-1.00000000e+00 0.00000000e+00]
[ 1.00000000e+00 1.00000000e+00 3.23115124e+00 2.11602709e+00
 3.40270880e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 9.02934537e-02 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 9.02934537e-02 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00]
[ 9.00000000e+00 0.00000000e+00 1.33733356e+02 8.04657801e+01
 1.21782214e+02 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 1.00000000e+00 3.92045147e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 -1.00000000e+00 3.92045147e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00]
[ 1.90000000e+01 0.00000000e+00 -1.06852903e+01 -5.79426095e+00
-9.49929481e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 -3.23160271e-01 -1.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 -3.23160271e-01 1.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00]
[ 8.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 2.93883508e-01 9.49582062e-01 6.21865464e-01
 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
-2.65357569e-02 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 2.65357569e-02 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00]
[ 1.30000000e+01 0.00000000e+00 -3.23115124e+00 -2.11602709e+00
-3.40270880e+00 1.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 -9.02934537e-02 0.00000000e+00
 0.00000000e+00 1.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 -9.02934537e-02 0.00000000e+00
 0.00000000e+00 1.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 1.50000000e+03]
[ 1.40000000e+01 0.00000000e+00 1.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 1.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 1.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00]
[ 1.50000000e+01 0.00000000e+00 0.00000000e+00 1.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00]
[ 1.60000000e+01 0.00000000e+00 0.00000000e+00 0.00000000e+00
 1.00000000e+00 -2.93883508e-01 -9.49582062e-01 -6.21865464e-01
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 2.65357569e-02 0.00000000e+00 0.00000000e+00 0.00000000e+00
 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
-2.65357569e-02 0.00000000e+00 0.00000000e+00 0.00000000e+00
 1.00000000e+00 2.00000000e+03]]

```

Figure 15: Phase 1 itr 9

```

tableau for iteration 1:
[[ 0.00000000e+00  1.00000000e+00  2.00000000e+00  3.00000000e+00
  4.00000000e+00  5.00000000e+00  6.00000000e+00  7.00000000e+00
  8.00000000e+00  9.00000000e+00  1.00000000e+01  1.10000000e+01
  1.20000000e+01  1.30000000e+01  1.40000000e+01  1.50000000e+01
  1.60000000e+01  1.70000000e+01]
[ 0.00000000e+00  0.00000000e+00 -1.58688036e+00 -6.28844244e-01
-1.92042438e+00  2.02880307e-02 -3.49972264e-01  0.00000000e+00
-5.89967997e-01  0.00000000e+00 -8.08126411e-02  0.00000000e+00
 3.15767015e-02  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00]
[ 1.00000000e+00  1.00000000e+00  3.23115124e+00  2.11602709e+00
 3.40270880e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  9.02934537e-02  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00]
[ 9.00000000e+00  0.00000000e+00  1.33733356e+02  8.04657801e+01
 1.21782214e+02  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  1.00000000e+00  3.92045147e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00]
[ 7.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  4.72583742e-01  1.52698955e+00  1.00000000e+00
 1.60806486e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
-4.26712183e-02  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00]
[ 1.30000000e+01  0.00000000e+00 -3.23115124e+00 -2.11602709e+00
-3.40270880e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00 -9.02934537e-02  0.00000000e+00
 0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  1.50000000e+03]
[ 1.40000000e+01  0.00000000e+00  1.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
 0.00000000e+00  5.00000000e+02]
[ 1.50000000e+01  0.00000000e+00  0.00000000e+00  1.00000000e+00
 0.00000000e+00 -4.72583742e-01 -1.52698955e+00  0.00000000e+00
-1.60806486e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 4.26712183e-02  0.00000000e+00  0.00000000e+00  1.00000000e+00
 0.00000000e+00  1.00000000e+03]
[ 1.60000000e+01  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  2.00000000e+03]]

```

Figure 16: Phase 2 itr 1


```

1.00000000e+00 2.00000000e+00]]
tableau for iteration 2:
[[ 0.00000000e+00 1.00000000e+00 2.00000000e+00 3.00000000e+00
 4.00000000e+00 5.00000000e+00 6.00000000e+00 7.00000000e+00
 8.00000000e+00 9.00000000e+00 1.00000000e+01 1.10000000e+01
 1.20000000e+01 1.30000000e+01 1.40000000e+01 1.50000000e+01
 1.60000000e+01 1.70000000e+01]
[ 0.00000000e+00 0.00000000e+00 -1.58688036e+00 -1.36884424e+00
-1.92042438e+00 3.70000000e-01 7.80000000e-01 0.00000000e+00
 6.00000000e-01 0.00000000e+00 -8.08126411e-02 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 -7.40000000e-01
 0.00000000e+00 -7.40000000e+02]
[ 1.00000000e+00 1.00000000e+00 3.23115124e+00 2.11602709e+00
 3.40270880e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 9.02934537e-02 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00]
[ 9.00000000e+00 0.00000000e+00 1.33733356e+02 8.04657801e+01
 1.21782214e+02 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 1.00000000e+00 3.92045147e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00]
[ 7.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00
 0.00000000e+00 1.00000000e+03]
[ 1.30000000e+01 0.00000000e+00 -3.23115124e+00 -2.11602709e+00
-3.40270880e+00 1.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 -9.02934537e-02 0.00000000e+00
 0.00000000e+00 1.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 1.50000000e+03]
[ 1.40000000e+01 0.00000000e+00 1.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 1.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 1.00000000e+00 0.00000000e+00
 0.00000000e+00 5.00000000e+02]
[ 1.20000000e+01 0.00000000e+00 0.00000000e+00 2.34350000e+01
 0.00000000e+00 -1.10750000e+01 -3.57850000e+01 0.00000000e+00
-3.76850000e+01 0.00000000e+00 0.00000000e+00 0.00000000e+00
 1.00000000e+00 0.00000000e+00 0.00000000e+00 2.34350000e+01
 0.00000000e+00 2.34350000e+04]
[ 1.60000000e+01 0.00000000e+00 0.00000000e+00 0.00000000e+00
 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 1.00000000e+00 2.00000000e+03]]

```

Figure 17: Phase 2 itr 2

```

tableau for iteration 3:
[[ 0.00000000e+00  1.00000000e+00  2.00000000e+00  3.00000000e+00
  4.00000000e+00  5.00000000e+00  6.00000000e+00  7.00000000e+00
  8.00000000e+00  9.00000000e+00  1.00000000e+01  1.10000000e+01
  1.20000000e+01  1.30000000e+01  1.40000000e+01  1.50000000e+01
  1.60000000e+01  1.70000000e+01]
[ 0.00000000e+00  0.00000000e+00 -2.36688036e+00 -1.36884424e+00
-1.92042438e+00  3.70000000e-01  0.00000000e+00  0.00000000e+00
 6.00000000e-01  0.00000000e+00 -8.08126411e-02  0.00000000e+00
 0.00000000e+00  0.00000000e+00 -7.80000000e-01 -7.40000000e-01
 0.00000000e+00 -1.13000000e+03]
[ 1.00000000e+00  1.00000000e+00  3.23115124e+00  2.11602709e+00
 3.40270880e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  9.02934537e-02  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00]
[ 9.00000000e+00  0.00000000e+00  1.33733356e+02  8.04657801e+01
 1.21782214e+02  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  1.00000000e+00  3.92045147e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00]
[ 7.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
 0.00000000e+00  1.00000000e+03]
[ 1.30000000e+01  0.00000000e+00 -3.23115124e+00 -2.11602709e+00
-3.40270880e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00 -9.02934537e-02  0.00000000e+00
 0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  1.50000000e+03]
[ 6.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
 0.00000000e+00  5.00000000e+02]
[ 1.20000000e+01  0.00000000e+00  3.57850000e+01  2.34350000e+01
 0.00000000e+00 -1.10750000e+01  0.00000000e+00  0.00000000e+00
-3.76850000e+01  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  0.00000000e+00  3.57850000e+01  2.34350000e+01
 0.00000000e+00  4.13275000e+04]
[ 1.60000000e+01  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  2.00000000e+03]]

```

Figure 18: Phase 2 itr 3

```

tableau for iteration 4:
[[ 0.00000000e+00  1.00000000e+00  2.00000000e+00  3.00000000e+00
  4.00000000e+00  5.00000000e+00  6.00000000e+00  7.00000000e+00
  8.00000000e+00  9.00000000e+00  1.00000000e+01  1.10000000e+01
  1.20000000e+01  1.30000000e+01  1.40000000e+01  1.50000000e+01
  1.60000000e+01  1.70000000e+01]
[ 0.00000000e+00  0.00000000e+00 -2.36688036e+00 -1.36884424e+00
-2.52042438e+00  3.70000000e-01  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00 -8.08126411e-02  0.00000000e+00
 0.00000000e+00  0.00000000e+00 -7.80000000e-01 -7.40000000e-01
-6.00000000e-01 -2.33000000e+03]
[ 1.00000000e+00  1.00000000e+00  3.23115124e+00  2.11602709e+00
 3.40270880e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  9.02934537e-02  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00]
[ 9.00000000e+00  0.00000000e+00  1.33733356e+02  8.04657801e+01
 1.21782214e+02  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  1.00000000e+00  3.92045147e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00]
[ 7.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
 0.00000000e+00  1.00000000e+03]
[ 1.30000000e+01  0.00000000e+00 -3.23115124e+00 -2.11602709e+00
-3.40270880e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00 -9.02934537e-02  0.00000000e+00
 0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  1.50000000e+03]
[ 6.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
 0.00000000e+00  5.00000000e+02]
[ 1.20000000e+01  0.00000000e+00  3.57850000e+01  2.34350000e+01
 3.76850000e+01 -1.10750000e+01  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  0.00000000e+00  3.57850000e+01  2.34350000e+01
 3.76850000e+01  1.16697500e+05]
[ 8.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  2.00000000e+03]]

```

Figure 19: Phase 2 itr 4

```

tableau for iteration 5:
[[ 0.00000000e+00  1.00000000e+00  2.00000000e+00  3.00000000e+00
  4.00000000e+00  5.00000000e+00  6.00000000e+00  7.00000000e+00
  8.00000000e+00  9.00000000e+00  1.00000000e+01  1.10000000e+01
  1.20000000e+01  1.30000000e+01  1.40000000e+01  1.50000000e+01
  1.60000000e+01  1.70000000e+01]
[ 0.00000000e+00  0.00000000e+00 -1.17135440e+00 -5.85914221e-01
-1.26142212e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00 -4.74040632e-02  0.00000000e+00
 0.00000000e+00 -3.70000000e-01 -7.80000000e-01 -7.40000000e-01
-6.00000000e-01 -2.88500000e+03]
[ 1.00000000e+00  1.00000000e+00  3.23115124e+00  2.11602709e+00
 3.40270880e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  9.02934537e-02  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00]
[ 9.00000000e+00  0.00000000e+00  1.33733356e+02  8.04657801e+01
 1.21782214e+02  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  1.00000000e+00  3.92045147e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00]
[ 7.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
 0.00000000e+00  1.00000000e+03]
[ 5.00000000e+00  0.00000000e+00 -3.23115124e+00 -2.11602709e+00
-3.40270880e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00 -9.02934537e-02  0.00000000e+00
 0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  1.50000000e+03]
[ 6.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
 0.00000000e+00  5.00000000e+02]
[ 1.20000000e+01  0.00000000e+00  8.52651283e-14  6.39488462e-14
 1.06581410e-13  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00 -1.00000000e+00  0.00000000e+00
 1.00000000e+00  1.10750000e+01  3.57850000e+01  2.34350000e+01
 3.76850000e+01  1.33310000e+05]
[ 8.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 1.00000000e+00  2.00000000e+03]]

```

Figure 20: Phase 2 itr 5

Model 22 tableau Output (Only Phase II shown considering space)

[illegible]

Figure 21: Extra credit Phase 2 itr 1

