MSc Project Report

Optimal sizing of supercapacitors for Hybrid Electric Vehicles

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ABSTRACT

Supercapacitors are becoming an important energy source in the power, energy, and automotive industry. The characteristics of SC, such as long life, low maintenance and high-power energy density are attracting researchers in automotive field to pursue efficient transportation and sustainability for future. The energy efficiency of any Hybrid vehicle is depending on its energy storage system. This project focuses on optimization of supercapacitor bank for a hybrid electric vehicle(HEV) to minimise mass & cost. Applications of SC's are discussed for constant power demand mode. A numerical iterative method is implemented to find minimum number of capacitors required for application and validated with MATLAB script code and named as non-ideal circuit model and this model is enhanced by adding additional capacitor in parallel and sized by using MATLAB script code and named as two branch model. Voltage leakages effect is observed for two branch model. Lastly differences of two models and applications are discussed furtherly in this report by validating with experimental data.

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TABLE OF CONTENTS

GLOSSARY

SC's- Supercapacitors

EV's – Electric Vehicles

HEV's - Hybrid Electric Vehicles

ESS - Energy Storage System

EPS - Electrical Power Subsystems

OBCDH - Onboard Computer and Data Handling

ADCS - Attitude Determination and Control Subsystem

DOD - Depth of discharge

KERS - Kinetic Energy Recovery System

DMU - Diesel Multiple Units

PSO - Particle Swarm Optimization

ICE – Internal Combustion Engine

P-HEV – Plugin Hybrid Electric Vehicle

LIST OF FIGURES

NOMENCLATURE

C- Capacitance

Q- positive charge

V- Voltage

A- surface area of each electrode

D- Distance between electrodes

ε0- dielectric constant of free space

ε1- dielectric constant of insulating material between electrodes

E(t)- Internal voltage at time t

V(t)- terminal voltage at time t

R_{ESR}*- Equivalent Series Resistance*

P- Power

I(t)- current at time t

C0- Capacitance of independent capacitor

C1- Capacitance of dependent capacitor

F-Farads

C_{cell}*- Capacitance of single cell*

n- number of cells

1. INTRODUCTION

In the current world, humans are dependent on transportation for their day-to-day activities. Since the emergence of technology, usage of transportation is upsurged and will relentlessly upsurge. As increasing in transportation technology, the pollution caused by the automobiles is persistently increasing. As this became global interest to control emissions by moving to EV's, HEV's. The function of machines in modification of humans and objects has significantly increased over the evaluation of technology. No vehicle on earth, powered by external composition runs without engine or a motor, the type or kind of motors or engines can be change depends on application. The substance that drives the engine is process of extinction because of decreasing of fossil fuels and other political issues. This is where EV's with Li-ion batteries technology is still ruling over a decade. From the figure 1, the usage of EV's is relatively increased over the decade. However, the production of lithium ore is slowly decreasing over the time. And the drawbacks such as high maintenance and cost of EV battery is still worry for end users.

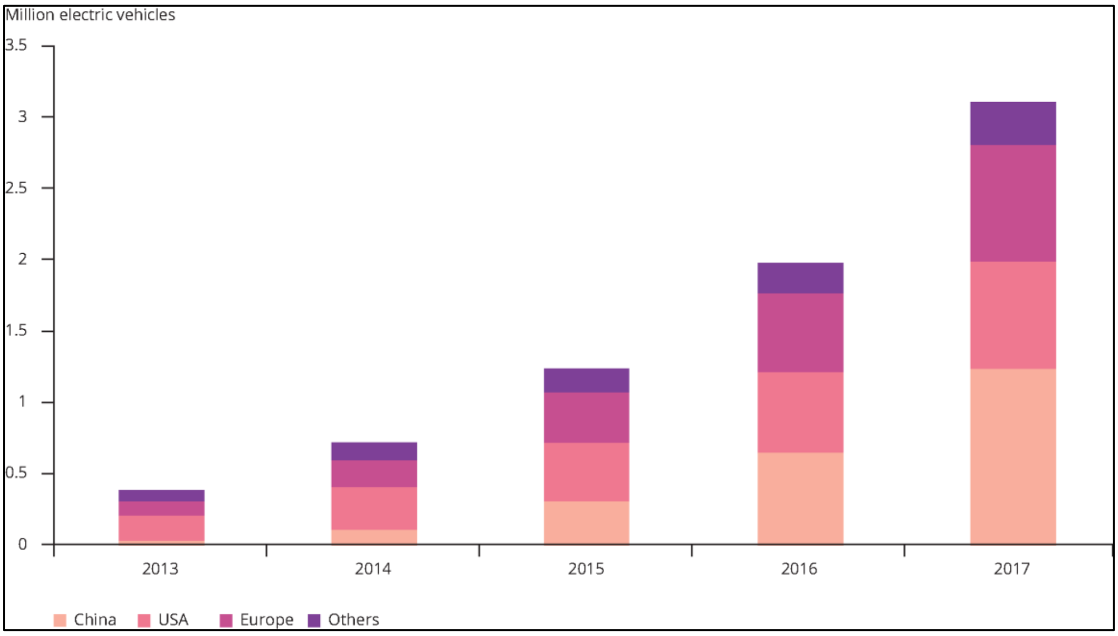

Figure- 1-1 Usage of EV's on global chart[1]

Hence automobile researchers are in search of other substance which overcomes the drawbacks of the batteries, and the result is ultracapacitors/supercapacitor. Specifically called as electrochemical capacitors. Because of their high power and energy densities these capacitors are 1000's of times greater than high-capacity electrolytic capacitors currently available in market [2]

1.1 SUPERCAPACITOR MECHANISM

The structure of supercapacitor is based on govern of electrical 2 layer (anode Al foil) – active carbon – separator – active carbon – cathode (Al foil) and results in higher capacity. The carbon particles on carbon nanotubes have surface area of 2000 m^2/g because of its significant surface area[2].

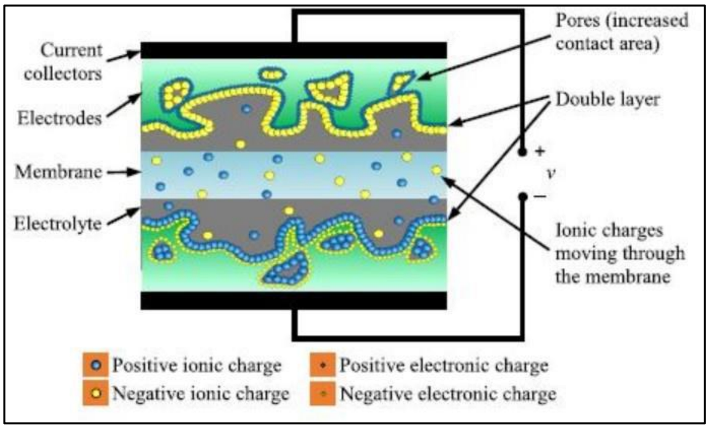

Figure 1-1 Sectional view of supercapacitor[2]**.**

From the figure 1-1, SC's are consisting of 2 porous electrodes quenched in an electrolyte. The energy is created by using electric field between electrodes and electrolyte, and positive charges are coloured in blue and negative charges in yellow. Ionic charges in membrane shows bigger because of their size and results in performance of supercapacitor. An ion-conductive membrane is the centre part of the SC schematic and separates two electrodes that shown as green colour. And membrane allows ion throughout while taking care not to allow short circuits between the electrodes.

1.2 ENERGY STORAGE DOMAINS

Supercapacitors or ultracapacitors are mainly known for to fill the gap between conventional capacitors and rechargeable batteries by compensating each other weaknesses. Transport sector is responsible for over quarter (27%) of global emissions which is directly proportional to increase in global warming[3]. To minimise emissions, many technologies have been introduced to increase thermal and mechanical efficiencies in SI and CI engines, though there is reduction in emissions which is not significant enough to minimise effect on global warming this is where hybridization of energies plays key role. Recent advances in supercapacitor technology have made SC's a viable option for use in hybrid energy storage system. Specifically, SC's can become integral part in development of hybrid energy storage systems, where they can be used as secondary sources of energy for batteries, compressed air, or fuel cells.

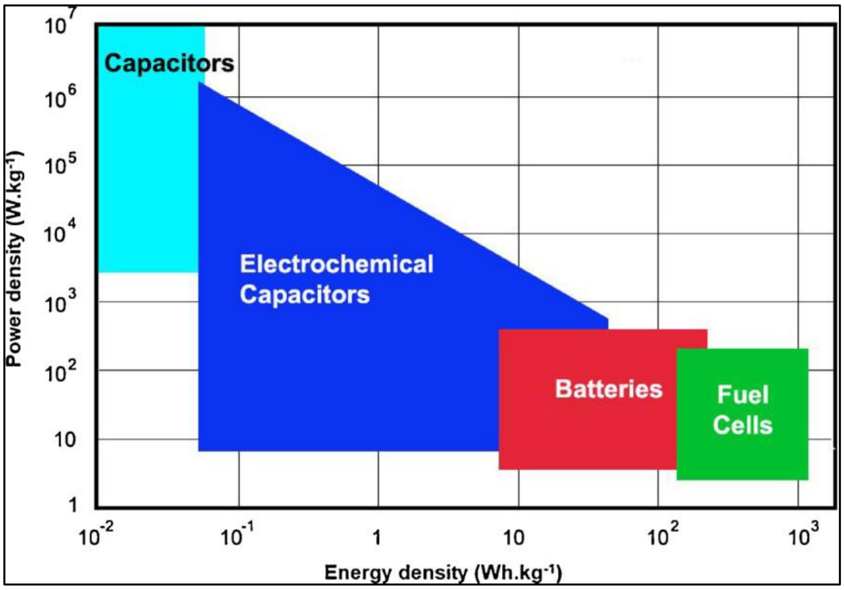

Figure 1-2 Ragone plot of the energy storage domains[4]

Supercapacitors are mainly known for providing high power over short durations and are effective solutions when quick supplement energies are needed. Other properties such as long cycle life, temperature range and rapid discharging and charging can be considered depends on application and eco-friendly as they known for easy disposal. As they can recharge quickly after deep discharge can be used as a most effective in regenerative applications. Sizing of SC's is vital function for Electric vehicles where under sizing leads to decrease the performance and oversizing increases the overall cost and mass of vehicle. Thus, critical sizing of SCs in energy storage system of EV's is necessary. Methodology for existing articles is concentrated on advantages such as overall vehicle efficiency (minimising weight of energy system), enhancement of SC's and battery performance for Lithium ion & SC's hybrid vehicle. However, important parameters to optimise sizing of SC's are electrical load as a constant power over discharge period, iterate number of cells for minimum mass, cell voltage and capacitance, Equivalent series resistance impact, aging effects, optimizing energy density & efficiency and validation with drive cycle data. Mostly two common types of sizing techniques are used to attain minimum mass, volume and cost namely rule based or optimization-based algorithms[5].

Applications such as electricity grids, renewable energy, and transportation (busses) SC's gained popularity as an energy storage system where SC's can be used alone or in combination with other power sources[5].

1.3 CHARGE AND DISCHARGE PROFILES OF SC's

Selection and design of SC's needs to focus on factors that are affecting overall performance of vehicle, factors such as mass & cost of the cell array, peak voltage, maximum allowable discharge percentage, peak current flow, time constant, capacitance of single cell and number of cells required[6].

The relation between voltage and current of SC is vice-versa, means increase in current over time leads to voltage drop as shown in figure 1-3.

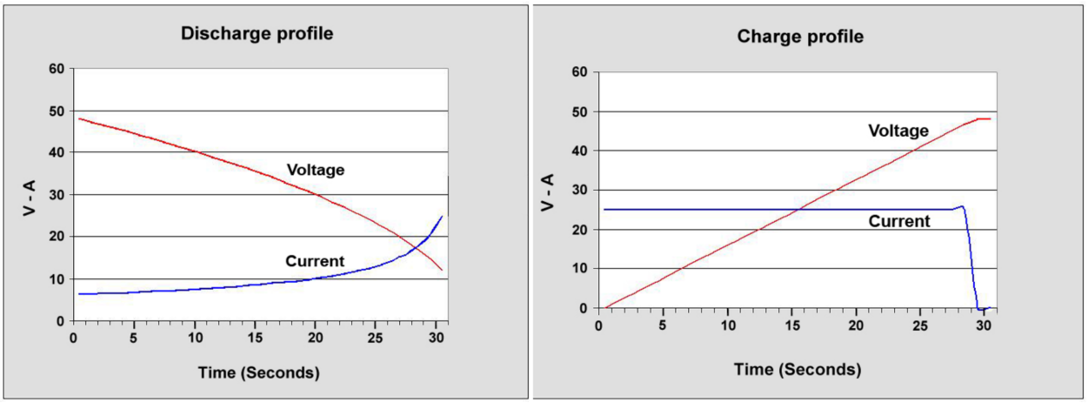

Figure 1-3 Discharge and charge profiles of SC's[7]

This project aims on optimal sizing of supercapacitors for hybrid electrical vehicle under different modes of power demand.

MATLAB script code is used to validate results from mathematical model.

1.4 ABOUT MATLAB

Matrix Laboratory is a high-level programming language and interface designed for numerical computing, data visualization & analysis. Interfaces such as Scripting environment, Data types, Functions, Plotting & Visualization, toolboxes, Guide development, SIMULINK, Parallel computing, and Interoperability. MATLAB has its own syntax programming language which is designed to solve numerical and matrix computation problems and supports procedural & object-oriented programming. Data types such as arrays, matrices, character, and structure, mostly used in physics, engineering and other scientifical fields. Typically, scripting environment written in *.matlabcode* format.

1.5 AIM AND OBJECTIVES

1.5.1 AIM

To investigate and model the characteristics of supercapacitors, with non-ideal equivalent circuit model for supercapacitor bank under constant power demand and employ numerical iterative method to determine number of cells are required for non-ideal supercapacitor model and enhance initial model by incorporating a branch term to account for leakage effects.

The goal is to compute the optimum number of supercapacitors corresponding to different models. Additionally, a MATLAB script code will be developed to validate numerical results and ensure accuracy of the proposed models.

1.5.2 OBJECTIVES

- Survey supercapacitor characteristics and models.
- Create a non-ideal equivalent circuit model of a supercapacitor bank under constant power demand.
- Resolve non-ideal supercapacitor model employing a numerical iterative method and calculate optimum number of cells.
- Correct initial (non-ideal) supercapacitor model by including a term branch.
- Generate a MATLAB/Simulink model to validate numerical results with experimental data.

2. LITERATURE REVIEW

2.1 Taxonomy of supercapacitors

Conventional capacitors include of two conducting electrodes set apart with insulating dielectric material. Capacitance is defined as the positive charge to the applied voltage. And capacitance directly proportional to area and inversely proportional to distance.

$$C = \frac{Q}{V}$$ ***Equation 2-1***

$$C \ \alpha \frac{A}{D}$$ ***Equation 2-2***

$$C = \varepsilon 0 \varepsilon r \frac{A}{D}$$ ***Equation 2-3***

Among all properties, energy and power densities stands out, thus energy stored in capacitor is directly proportional to its capacitance.

$E \alpha C$

$$E = \frac{1}{2} C V^2$$ ***Equation 2-4***

Internal components such as current collectors, dielectric materials and electrodes exhibits resistance and commonly known as Equivalent Series Resistance ESR. And hence maximum power is known as,

$$P_{maximum} = \frac{V^2}{4R_{ESR}}$$ ***Equation 2-5***

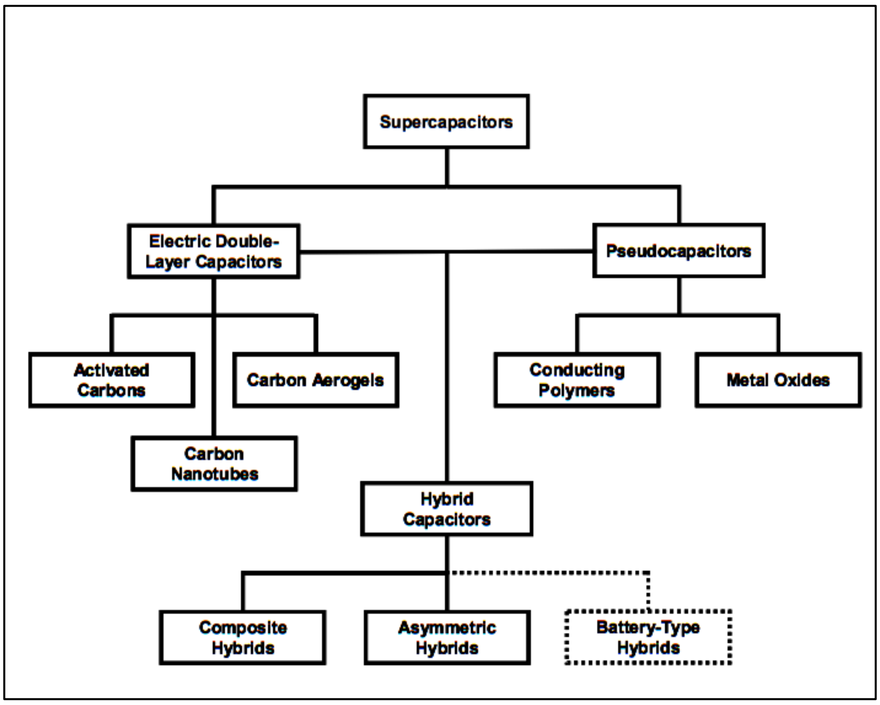

Figure 2-1 Taxonomy of supercapacitors[8]

2.2 Ideal and non-ideal supercapacitor models

To minimise mass of supercapacitor pack, H. Douglas & P. Pillay [6]did a study on ideal circuit with Four ideal circuit elements shown in figure 2-1, C capacitance, L inductor and R_s & R_p series and parallel resistors respectively and R_s is also known as equivalent series resistance (ESR). In parallel capacitors leakage of current can be found from milliamps to tens of milliamps in big SC's. However, inductance can be neglected for constant current discharging/charging applications[9].

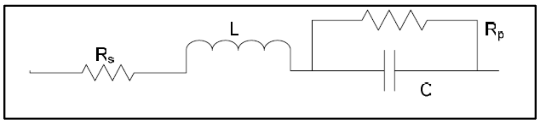

Figure 2-2 First order model of an SC[6]

To overcome current leakage in parallel circuits for higher voltage ratings, many SC's are required to connect in series to form a string of supercapacitors. More likely as a regular capacitor, SC's are also subjected to deviations in capacitance, while connected in series the deviations can be larger as 20% from the overall cell capacitance value, when capacitance differs cell voltage also vary, this leads to exceeding voltages of cells by being overcharged, to eliminate overcharge a passive voltage balancing circuit is needed to monitor cell voltages.

And analysed about a circuit without ESR and with ESR for a constant power application by chosen maximum current based on rating of the individual cell. The ratio of final over initial voltage is known as discharge ratio. However, its common to choose the value of energy being used for the application is 50% or 75%, its always depends on the maximum voltage and capacitance of cell. For constant power applications, high currents are needed to run application. To avoid under sizing or oversizing an alternative method is needed where ESR, cables and connectors are considered.

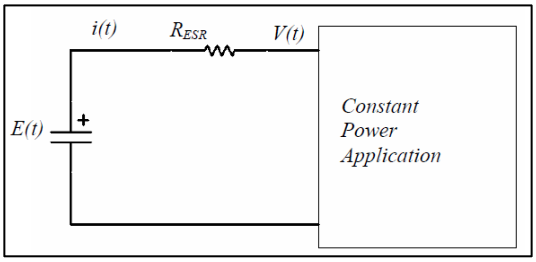

Figure 2-3 Equivalent circuit included ESR[6]

The alternative approach required a numerical method to find number of SC's required for a constant power application, having maximum current for time t, where E(t) internal voltage, V(t) is terminal voltage and assumed initial voltage based on maximum rating of the capacitor to find final internal voltage later used iterative approach to find internal and external voltages trajectories and as well for current profile for time t where application needed constant power to drive.

About power electronics effects, the choice of operating current and voltage is influenced by interface of power electronics. Upon specifying maximum operating current to be the specified rated current of the SC's results in minimum SC's and therefore minimum mass.

2.3 Ideal, two branch and dynamic supercapacitors models

Another study by Sakshi Bansal, Praveen Nambisan, Pankaj Saha, and Munmun Kahnra [5], analysed effect on ESS in EV by modelling and unit cell selection, five distinct maxwell capacitors were evaluated and used two methods to find number of SC i. analytical method ii. Optimal sizing method.

However, analytical method is introduced because there is regenerative power due to braking system. The sizing of ESS is not performed only by considering maximum load power demands.

Urban drive cycle for a bus is considered to perform analytical method. By considering 3 models as shown in Figure 2-3.

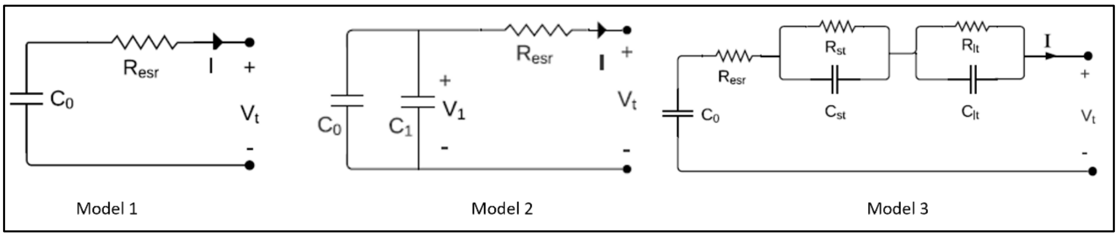

Figure 2-4 Rint-Capacity, two branch, Dynamic models[5]

It is observed that SC modelling on optimal sizing of ESS in as SC driven bus and selection on SC cell has effect of investment cost. The estimation of energy storage system pack can be obtained by analytical method without any high computing machine however this method just obtains rough value which might leads to wrong selection of capacitor and underperforming as well raise in the cost, this is where optimal sizing method fills the gap. In optimal sizing method Particle swarm optimization (PSO) technique is used. PSO is an Integer nonlinear programming method where user can

give a constraint at any interval due to its feasibility such as nonlinear characteristic, both integer and continuous problems can be solved. The metaphor behind this PSO is inspired by a swarm of flying birds on the food hunt.
In PSO algorithm, unit selection and sizing of SCM can satsify desired objective such as,i. Input variables-Power demand, Cost and mass of cell ii.Optimization of varibales-number of capacitors in series and parallel iii.Objective fuction-minimise cost and mass of ESS iv.Equality constraints-traction and regerative v.Inequality constraints- Current, voltage and power. And conclude that model 3 is effective for give uncertain driving conditions this indicates that model 3 is best for ESS sizing for EV's which also implies reliability and economic sizng. Regarding selection of cell, for mass and voltage 350F maxwell is feasible, for cost 650F maxwell is economical. Besides cost and mass, performance is also one of the factor to be considered, where 1200F maxwell capacitor is reliable and affordable which is also having lowest deviation in optimal capacity compared to 100F,350F,1000F,1200F and 3000F maxwell capacitors.
Also robostness study is followed by authors where model 3 has no significant effect on ESS sizing though there is bit of uncertainty in RC branch parameter. However C_0 has more noticeable effect on ESS sizing than equivalent series resistance. Thus C_0 need as additional attention for identifcation and results in obtaining accurate and oprimized reliable size of energy storage system in EV.

Selection of architecture of hybrid electric vehicle is major step to use the energy from supercapacitors in most effective way. However, energy storage management plays crucial role in distributing power for different devices wherever devices demand.
System architecture: Three kinds of architectures are currently using in HEV namely series, parallel, series - parallel. However, parallel, and series-parallel are commonly used for passenger cars and lightweight vehicles, series architecture is known to produce more compared to other two, this leads to series is ideal for heavy vehicles[10]

S. Marin-coca, E. Robias-Millan & S. Pindado discussed analytical modelling and sizing of SC's in spacecraft hybrid energy storage systems. It is known that aerospace applications, particularly micro satellites, have boundaries on mass and volume. Thus, most of the efforts must be made to reduced mass and volume of Electrical Power Subsystems (EPS). EPS is the most crucial part of the spacecraft cause it is responsible for providing continuous power from all the spacecraft active subsystems which is required to collect Onboard Computer and Data Handling (OBCDH), & to communicate to ground by using Attitude Determination and Control Subsystem (ADCS). Hence sizing of SC's need to results in without effecting reliability, performance and avoiding

the increase in complexity & cost of the EPS. However, nowadays, Li-ion technology is most used source of energy producer. Though it has higher specific energy maximum Depth of discharge (DOD) to ensure long life cycles is biggest drawback cause all the stored energy is not being used. And having low power density results in charging and discharging currents are limited, and performance is effected by low temperatures. All these limitations are biggest issues during eclipse. On one side, since there is no solar power supply batteries has to takeover to supply overall required power to run satellite this restrict the power consumption of different subsystems to be limited. And must avoid high consumed payloads. On the other side, due to eclipse temperatures drops significantly hence, batteries are connected in parallel to surpass maximum current at the expense of raise in mass and volume.
Therefore, EPS capabilities are oversized to supply high power and to keep DoD to increase life of batteries.

To overcome all these challenges with batteries, researchers found implementation of SC's in EPS cause these are high-capacitance capacitors and can be used for applications between regular capacitors and batteries, briefly shown in table 2-1.

Source	Energy density (Wh/kg)	Power density (W/kg)
Regular capacitors	$10^{-2} – 10^{-1}$	$10^{4} – 10^{7}$
Batteries	$10^{0} – 10^{3}$	$10^{0} – 10^{2}$
Supercapacitors	$10^{-1} – 10^{0}$	$10^{0} – 10^{4}$

Table 2-1 Energy & Power densities of capacitors, batteries & SC's[11]

For all the above reasons in [11] they developed a practical model which is accurate of SC's that matches the overall behaviour by using numerical expressions and provided a procedure for SC sizing and methodology is purely for application during eclipse for both constant power and constant current applications in parallel series configuration. And concluded around 36% of mass is reduced from the first approximation and 2 to 32% of increase in volume relativity to the battery system [11] and suggested this field should focus on accuracy by considering battery efficiency and power converter losses.

2.4 Types of Hybrid levels:

In hybrid, different level functionality is provided from internal combustion engine and electric motors, however, in this project battery and SC are considered.

Level1: Micro hybrid, where limited power from motor and acts like an alternator, where ICE responsible for propulsion. Due to rapid dynamic of EM, it employs stop-and-go function results in ICE stops where vehicle stands still, for instance, at traffic signal light.

Level2: Mild hybrid, in addition to level1 stop-and-go function, in level2 EM boost the ICE during acceleration and deceleration (braking) by providing supplement torque. However, EM cannot start the vehicle.

Level3: Full hybrid, drawbacks from level2, will be complemented here where EM able to propel the vehicle alone, this results in zero emissions where in urban areas vehicle can switch to EV more and ensures no emissions, can be switched to ICE mode while driving in rural areas, lead to increase fuel economy to 20% to 50% where in level2 fuel economy is 10% to 20%.

Level4: Plug-in hybrid, where fuel economy reached to 100% by complementing ICE completely, only battery and/or SC used to drive EM of the vehicle. This results on electrical grid loads, where P-HEV demand peak power to charge them. However early studies proven that demanded power can be produced without any additional power plants by implementing vehicle to grid (V2G) network infrastructure[10]

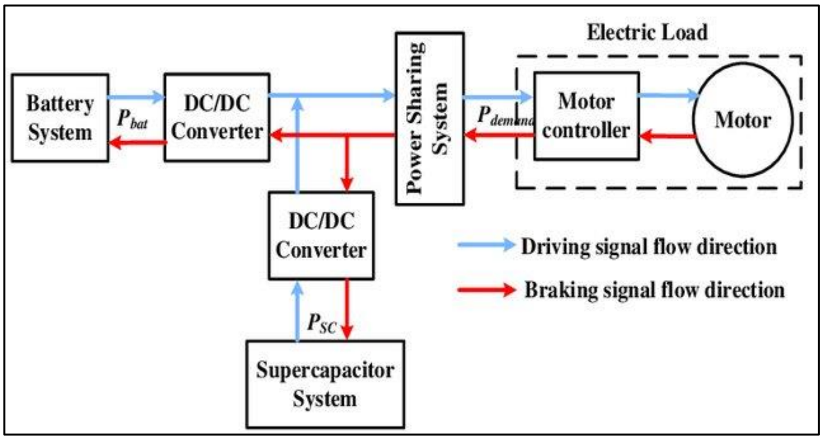

Figure 2-5 Configuration of battery/SC hybrid system [12]

2.5 Applications of Supercapacitors

Though SC's are relatively new technology compared to conventional capacitors, the primary application is to recharge batteries to avoid interruptions and knows as uninterruptable power supplies. SC's are widely used to cover the gap at peak power requirements for instance, to start backup generators and provide power for short term till generator acts.

However, SC's are also known as energy harvesters, cause SC's can store energy from all kinds of renewable sources and, it can be operated in peak +/- temperatures and can communicate by using sensor technology.

In automotive and defence sectors, SC's are known for emerging technologies where vehicles regenerative power can be used to recharge battery pack of HEV and can provide peak powers during accelerations whenever control unit demanded. Hence, overall efficiency of vehicle can be increased.

Light weight goods, UK based Innervated Vehicle Engineering aims on zero emission by 2030 implemented a van as shown in figure 6-1-1 , skeleton technologies 51V 177F SC pack is used.

Figure 2-6 SC pack implemented LGV[13]

Rail industry also interested, as SC's are absorbing energy from braking systems and its already implemented on street trams in Spain.

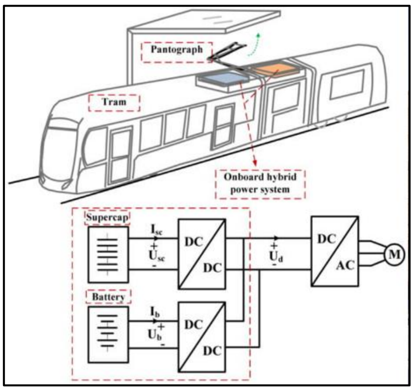

Figure 2-7 Supercapacitor application – Tram in Spain[14]

Kinetic Energy Recovery System (KERS) for Diesel Multiple Units (DMU) trains recovers 20% of electricity[15] when train is being used for long distances or full stop commuter.

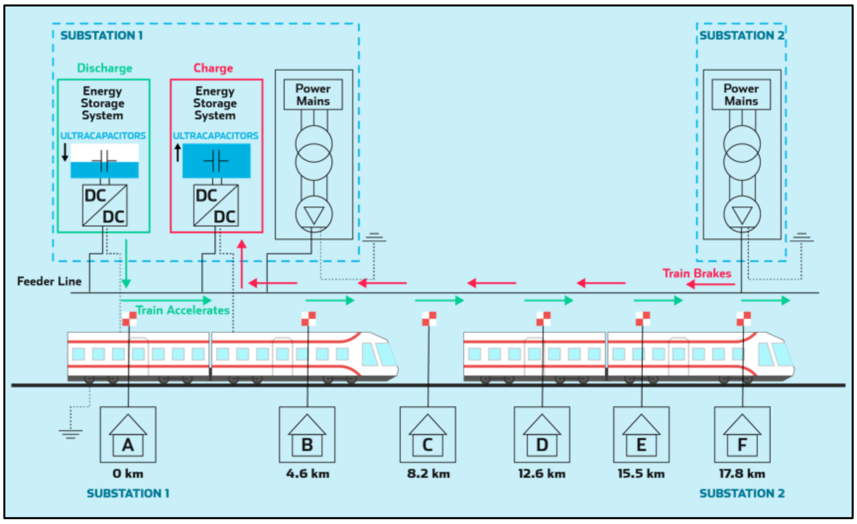

Figure 2-8 Fast charging implementation results in 1490Mwh saved energy/year [15]

Safety and dependability is one of the most important factors in extreme weather conditions -40^{0}C. one of the most common issue in DMU train fails to start, and results in longer delay time and costly downtimes. The main issue to not start of a train engine is the temperatures of the batteries as they are limited to certain level. Supercapacitor starting system is significantly smaller in size and results in minimise the mass and volume of ESS compared to typical lead-acid batteries starting system, where SC's are 6 times less in space and 30 times lower weight. Fast charging is one of the biggest advantages for the train as it can charge in seconds which is available both for onboard and wayside ESS which is enough for the train to travel to the next couple or so stations.

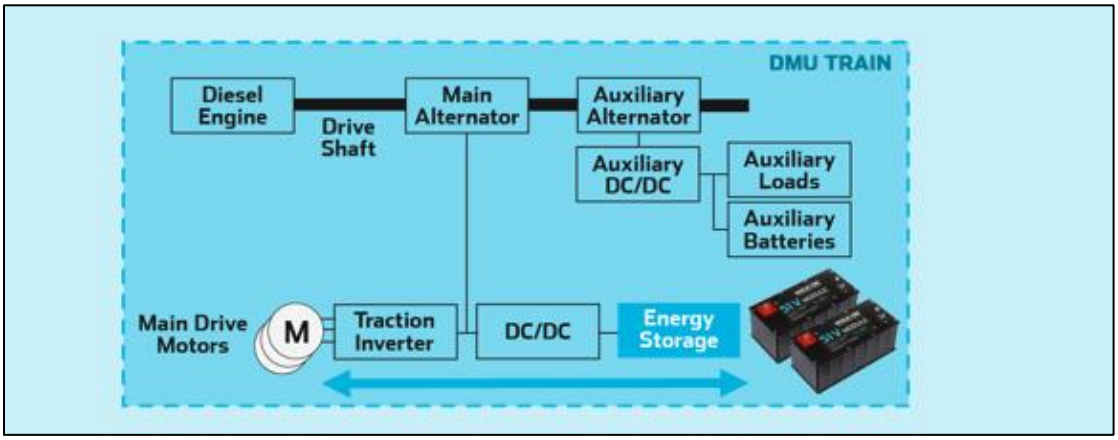

Figure 2-9 On board ESS to provide electric range to pass non-diesel areas[15]

2.6 Maxwell 2700 Supercapacitor

Figure 2-10 Maxwell 2700F, 2.5V SC[16]

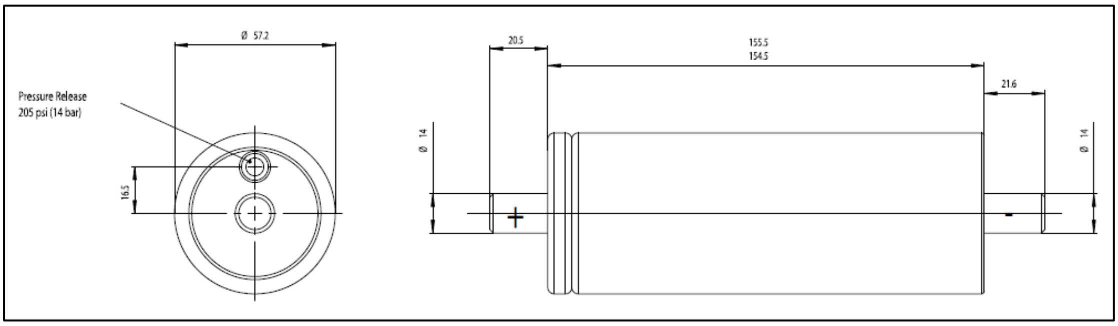

Figure 2-11 2D drawing of Maxwell 2700F, 2.5V SC[16]

Physical Characteristics
Dimensions 155mm X 57mm, tolerance ± 1mm

Table 2-2 dimensions of 2700F, 2.5V capacitor[16]

Advantages such as, compared to regular capacitors, SC delivers 100x of energy and 10x of power and optimized for individual applications through its capacity to repeatedly charge and discharge. Designed for smaller applications however, if connected in series or parallel depends on applications it produces required energy and power.

Available performance	**Lead acid battery**	**Ultracapacitor**	**Conventional Capacitor**
Charge time	60 to 300 minutes	0.30 to 30 seconds	10^{-3} to 10^{-6} seconds
Discharge time	30 to 180 minutes	0.30 to 30 seconds	10^{-3} to 10^{-6} seconds
Cycle life	<1000	<10,000	<100,000
Charge/discharge efficiency	70% to 85%	85% to 98%	>95%

Table 2-3 Battery vs supercapacitor vs conventional capacitor[16]

Below specification of 2700F, 2.5V SC in table 2-4, stands out typical capacitors as its available in cylindrical shape, allows critical information and functions to design and energy & power storage capability so that cycle remain available during outages, dips, and sags in the main hundreds of thousands of charges and discharges without power source. In addition, it can increase battery lifetime by reliving them from burst power performance degradation. And reducing cost and weight of the system.

Specifications	
Capacitance	2700 Farads
Capacitance tolerance	±5%
Voltage	Continuous: 2.5V Peak: 2.7V
Series resistance	DC: 0.7mΩ tolerance±25% 1kHz: 0.5mΩ tolerance±25%
Current	Rated: 500Ampers
Stored energy	8400 Joules
Leakage current	5mA (72h, 25°C)
Weight	600grams
Volume	With terminals 0.51L Without terminals 0.40L
Temperature	Operating -40°C to 65°C Storage -40°C to 85°C
High temperature performance	65°C Capacitance 2700F ±5% Resistance 0.7mΩ ±25% for DC
Low temperature performance	-40°C Capacitance 2700F ±5% Resistance 1.1mΩ ±25% for DC
Lifetime at 25°C	10 years ΔC >20%, ESR < 200% of initial value
Cyclability	>500,000 ΔC >20%, ESR < 200% of initial value

Table 2-4 Specifications of 2700F, 2.5V supercapacitor[16]

3. METHODOLOGY

In this chapter sizing, and optimal sizing of supercapacitors methodology for non-ideal circuit and two branch circuit models discussed in detail.

3.1 *Sizing of supercapacitors*

Sakshi Bansal and co, proposed an analytical method to size supercapacitors for EV's. In existing theories, all proposed models are as per SC banks available in the market. To overcome issue of regenerative power caused from braking. The analytical method calculates capacitance of single cell and number of cells required in series and parallel to achieve maximum power demand.

Total energy required to drive vehicle defined as power demand at all times.

$E_{required} = sum(P_{demand}(t))$ **Equation 3-1-1**

All the EV's runs on both motoring and regenerative, hence taking average power would be ideal.

$P_{average} = \frac{E_{required}}{T_{total}}$ **Equation 3-1-2**

T_{total} is drive cycle time.

Current flow from SC module is calculate from the relation for voltage and current of supercapacitor, where voltage is vice-versa to current.

$I_{out\_DCmaximum} = \frac{P_{average}}{V_{minimum}}$ **Equation 3-1-3**

$I_{out\_DCminimum} = \frac{P_{average}}{V_{maximum}}$ **Equation 3-1-4**

$I_{out\_avg} = \frac{I_{out\_DCmaximum}+I_{out\_DCminimum}}{2}$ **Equation 3-1-5**

Total capacity required to drive vehicle is defined as,

$C_{total} = \frac{I_{out\_avg}+T_{traction}}{V_{maximum}-V_{minimum}}$ **Equation 3-1-6**

$T_{traction}$ is the time duration during which the Supercapacitor module supplies the power to wheels to keep the vehicle running.

Number of cells required in series can be calculated as,

$N_{series} = \frac{V_{maximum}}{V_{cell}}$ **Equation 3-1-7**

Our requirement to calculate unit cell capacitance can be calculated by,

$C_{cell} = (N_{series} * C_{total})/N_{Parallel}$ **Equation 3-1-8**

Where, $N_{Parallel}$ is a match with commercially available. If returns a value of C_{cell} that is not commercially available, equation 3-1-8 can be re-written as,

$N_{Parallel} = (N_{series} * C_{total})/C_{cell}$ **Equation 3-1-9**

3.2 *Optimal sizing of non-ideal supercapacitor module*

A numerical iterative method used to find number of capacitors for a constant power required application as shown in Figure 3-1

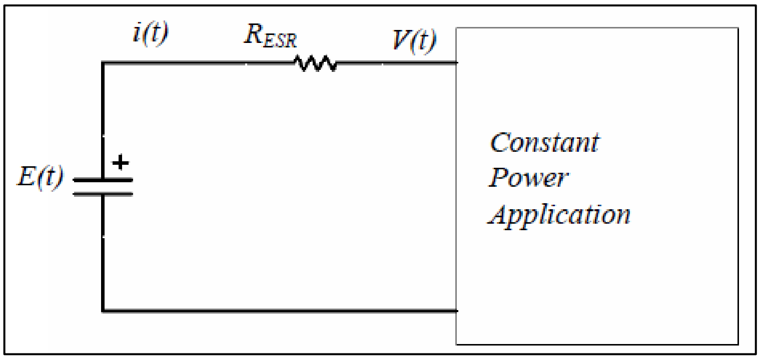

Figure 3-1 Equivalent circuit including ESR[6]

Where at time t, E(t) is internal capacitor voltage,

V(t) is terminal voltage,

I(t) is current,

R_{ESR} is the equivalent series resistance.

By using Euler's iterative approach, considering power as constant and current as maximum at final stage. For that initial voltage value for internal voltage is required to get final target minimum voltage, maximum current is chose based on supercapacitor rating, final voltage of internal E(t) can be determined by adding voltage drop across the equivalent series resistance. The final voltage E(t) is used as target voltage to employ numerical method.

The minimum amount of voltage can be found if the final value of E(t) is less than the targeted value for and so on, till final value is equal to targeted voltage. When requirements are achieved minimum number of cells can be found, it is used to determine the initial current and it can be used to show the current wave form by using current equation 3-2-5.

However, this model is only used for constant power demand, for real time power demands this model is not applicable, for that two-branch simplified model is discussed in methodology 3.3.

$$\frac{dE(t)}{dt} = -CE(t) \pm \frac{\sqrt{C^2E(t)^2-4RC^2p}}{2RC^2}$$ **Equation 3-2-1**

Where, p is constant power,

C is total capacitance,

R is equivalent series resistance

From the relation $P = iV$, to determine terminal voltages.

$$V(t) = \frac{P}{i(t)}$$ **Equation 3-2-2**

To determine, internal voltages at time t,

$$E(t) = i(t)Resr + V(t)$$ **Equation 3-2-3**

As, the circuits is in series, to determine the total capacitance required for constant power application can be determined as follows,

$$C = \frac{Ccell}{n}$$ **Equation 3-2-4**

where,

C_{cell} is the capacitance for single cell,

n is the total number of cells.

For constant current applications,

$$\frac{di(t)}{dt} = \frac{i(t)}{C\left(R-\frac{p}{i(t)^2}\right)}$$ **Equation 3-2-5**

Where, i(t) can be calculated from below equation,

Current at time t can be calculated by,

$$i(t) = -C\frac{dE(t)}{dt}$$ **Equation 3-2-6**

where, $\frac{dE(t)}{dt}$ can be calculated from equation 3-2-1.

From this method we can find overall mass of the supercapacitor bank by choosing accurate choice of maximum operating supercapacitor current only for constant power demand.

For the model, supercapacitors are connected in series as shown in below figure 3-2.

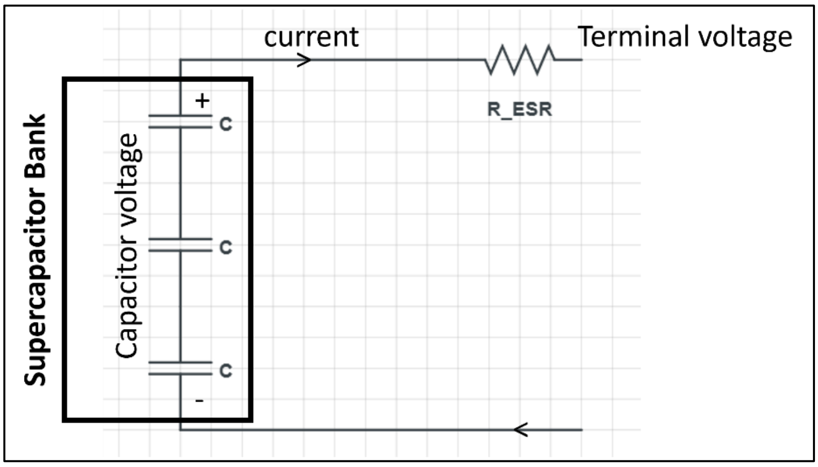

Figure 3-2 Non-ideal ESR model supercapacitor bank

3.3 Optimal sizing of two branch supercapacitor module

In this sub section, an additional supercapacitor is added to initial non-ideal module shown in figure 3-2, and determination number of cells required in series and parallel is carried out, as per existing studies energy storage systems are dependent on energy storage devices which are commercially available. However, C_0 is fixed capacitor and C_1 is variable or additional capacitor, shows behaviour of SC's discharge/charge over time period.in milliseconds to seconds. as there is regenerative power from braking systems, a modified numerical method is required to compute capacitance of a unit cell to build energy storage system for EV by also determining number of series and parallel capacitors required.

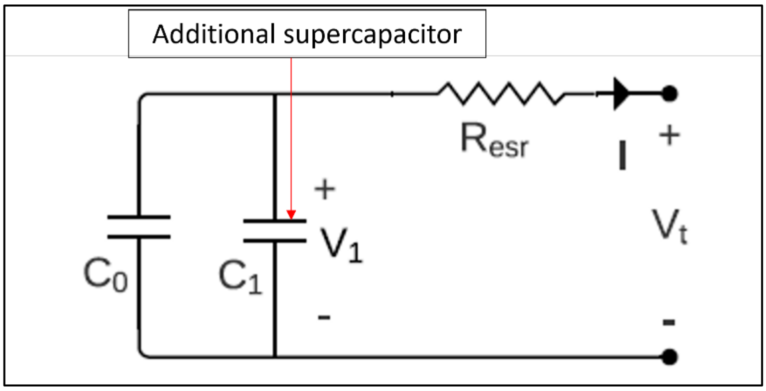

Figure 3-3 Two branch model[5]

Hence, model consists of two capacitors where C_0 represents immediate accumulation of charge at active electrode's surface. However, C_1 is not a independent capacitor it is depended on C_0 as C_1 is a voltage differential capacitor and interpret non-linear voltage dependent behaviour of capacitance and terminal voltage.

From electrolyte, electrodes, metal connectors and active electrodes contains carbon particles resistance and represented with R_{esr}. The relationship between complexity and accuracy can be identified from this model for vehicular applications.

Mathematical representation for additional SC's term model is,

Terminal voltage is represented as,m

$$V_t = V_1 - R_{ESR}I \quad \textbf{Equation 3-3-1}$$

V(t) is terminal voltage,

Voltage of C_1 at time t can be determined from below equation,

$$\frac{dV_1}{dt} = -\frac{I}{C_0+C_1} \quad \textbf{Equation 3-3-2}$$

Capacitance of additional capacitor can be represented as,

$$C_1 = K_1V_1 \quad \textbf{Equation 3-3-3}$$

V_1 voltage at C_1,

C_0 and C_1 are capacitors,

K_1 is constant or (F/V)

However, from above equations only terminal voltage can be achieved. To achieve V(t) we must need to know the internal voltage of the cell in parallel. To calculate internal voltage in parallel S. Marín-Coca, E. Robias-Millan, S. Pindado proposed third degree polynomial for constant power and constant current(load). In this project we are focused on constant power application. Our required output is constant power if current (i)>0 expressed as function of the charging power, P>0 then voltage of SC is E_{cell}. From equation 3-2-2, it can be modified as ,

$$i = \frac{P}{E_{Cell}} \quad \textbf{Equation 3-3-4}$$

By substituting equation 3-3-4 in equation 3-3-3 results in

$$V(t) = E_{cell} - \frac{PR_{ESR}}{E_{cell}} \quad \textbf{Equation 3-3-5}$$

Derivative of above equation is,

$$V'(t) = (1 + \frac{PR_{ESR}}{E_{cell}^2})E_{cell}' \quad \textbf{Equation 3-3-6}$$

From the relation, where current in the capacitor defined as change in charge rate through capacitor, mathematically $i = q'$ **Equation 3-3-7**

From the definition , charge stored in the capacitor is equal to its capacitance and voltage across the capacitor.

At given voltage its defined as $q = c(v)v$ **Equation 3-3-8**

Where *c(v)* is defined as linear function of capacitor voltage, defined by manufacturers minimum capacitance.

$c(v) = C_0 + Kv$ **Equation 3-3-9**

Where k is the slope of the linear function.

After substituting and equation 3-3-9 in 3-3-8 & 3-3-7 and deriving results in,

$i = (C_0 + 2K)v'$ **Equation 3-3-10**

Replacing *i, v(t), v'(t)* by equation 3-3-4, 3-3-5 and 3-3-5 in equation 3-3-10 leads to,

$\frac{P}{E_{cell}} = [C_0 + 2K(E_{cell} - \frac{PR_{ESR}}{E_{cell}})]((1 + \frac{PR_{ESR}}{E_{cell}^2})E_{cell}{}')$ **Equation 3-3-11**

Where, E_{Cell} is voltage one cell connected(E_1) in parallel to another which is dependent on primary cell (E_0) from figure 3-2.

Hence $E_{cell} = E_0 + E_1$.

In this report E_{cell} is considered as one cell. Detailed in figure 3-3.

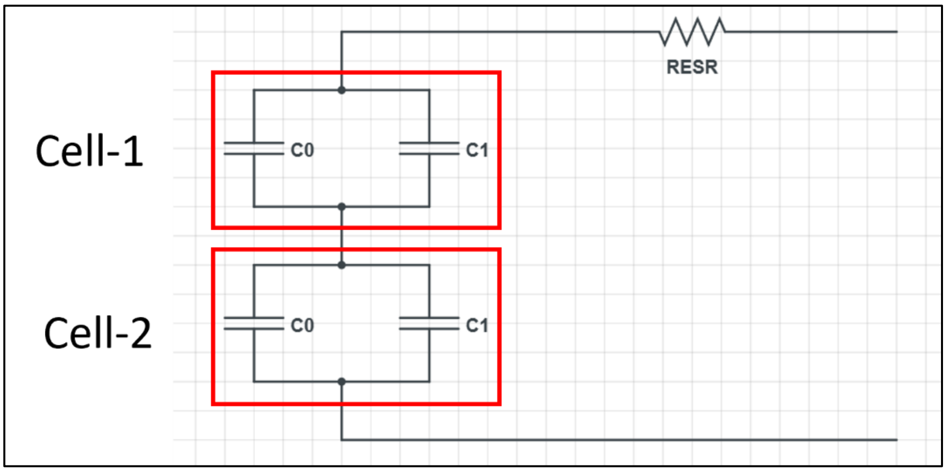

Figure 3-4 Two branch parallel-series model supercapacitor

After achieving, results for non-ideal and two branch models from analytical and script code, they are employed to compare them with implicit data from [17].

4. RESULTS AND DISCUSSIONS

In this section, we delve into a comprehensive discussion of the results from investigation of methodology section.

4.1 Applications

In this sub section, applications for both non-ideal and two branch models are discussed. For non-ideal model optimal sizing of supercapacitor bank for the high-power acceleration demands. An example power profile in figure 4-1 represents vehicles acceleration needs. The SC bank must be sized to meet this profile, while minimizing mass and maintaining current and voltages. The hybrid energy management strategy controls the power flow between the SC's, batteries and drivetrain depends on the driving mode. The careful sizing of the SC's bank is the key to maximizing performance.

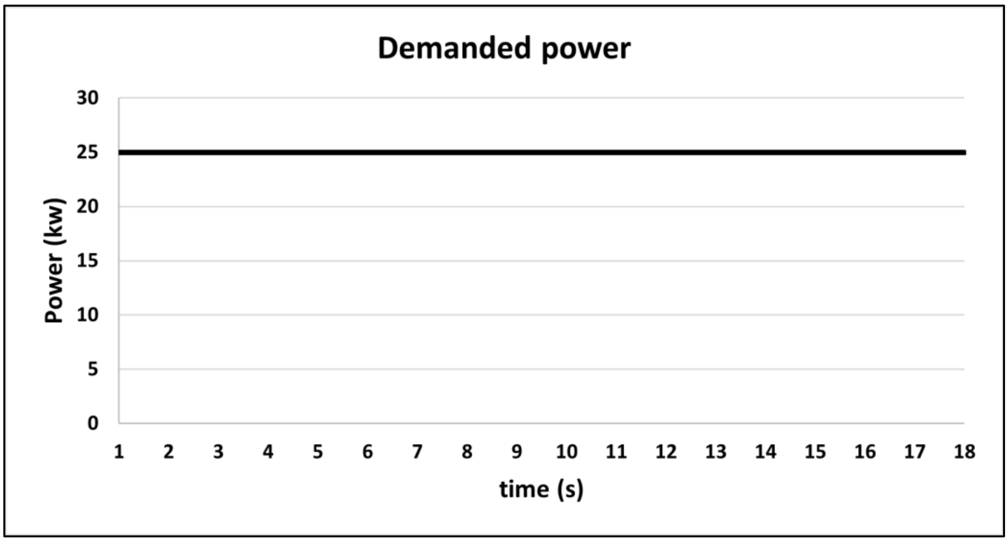

Figure 4-1 Power profile of 25kW for 18 seconds

In a Hybrid Electric Vehicle (HEV), a specialized **application for two-branch model** supercapacitors involves providing a power output of 200 Watts for a brief duration of 36 seconds or less than 36 seconds to assist in cold start scenarios. In cold weather conditions, starting an internal combustion engine can be challenging due to increased viscosity of engine oil and other factors. To address this, the integrated supercapacitor system is designed to swiftly discharge stored energy, delivering the immediate power boost required for a quick and reliable engine start. The high-power density of supercapacitors ensures their effectiveness, particularly in low-temperature conditions where traditional batteries may struggle. This cold start assistance system not only enhances the vehicle's reliability in adverse weather but also contributes to the extended lifespan of the overall battery system by efficiently managing high-power demands during critical start-up moments.

4.2 Optimization results of non-ideal model

From the methodology section 3.2, for non-ideal model analytical method particularly Euler's iterative method is employed in excel spreadsheet, and results are in figure 4-2 for internal and terminal voltages of the capacitor bank.

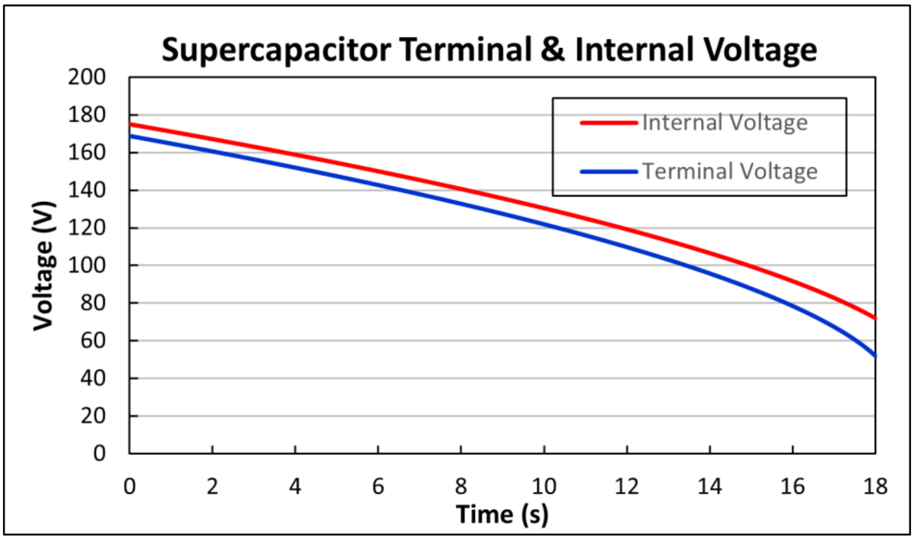

Figure 4-2 Internal and terminal voltages of SC bank.

Figure 4-2 shows the modelled internal capacitor voltage E(t) and terminal voltage V(t) over a time for a constant power discharge application shown in figure 4-1. The SC's is sized to deliver 25KW of power for 18 seconds.

The initial capacitor bank voltage E(t) is 175volts, based on 70 series connected supercapacitor cells with 2.5 cell voltage. As the constant power discharges the bank, E(t) decays exponentially from 175V to the minimum of 70V this represents the voltage discharge of 40%.

The terminal voltage V(t) starts at 168.78V which is 6.22 V less than due to the voltage drop across the ESR. As the discharge current increases, the voltage drop across the 70 cell's 0.0420 ohms ESR rises. This causes V(t) to diverge further below E(t).

At the end of the 18s discharge, V(t) drops to 51.75V as intended based on the designed maximum capacitor current 483.09Amps. the 20.29 V difference between E(t) and V(t) represents a 27.96% loss due to the SC's bank ESR at maximum current.

So, to reiterate figure 4-2 illustrates the modelled exponential voltage decay and growing divergence between internal and terminal voltage due to ESR during high current constant power discharge of SC's bank.

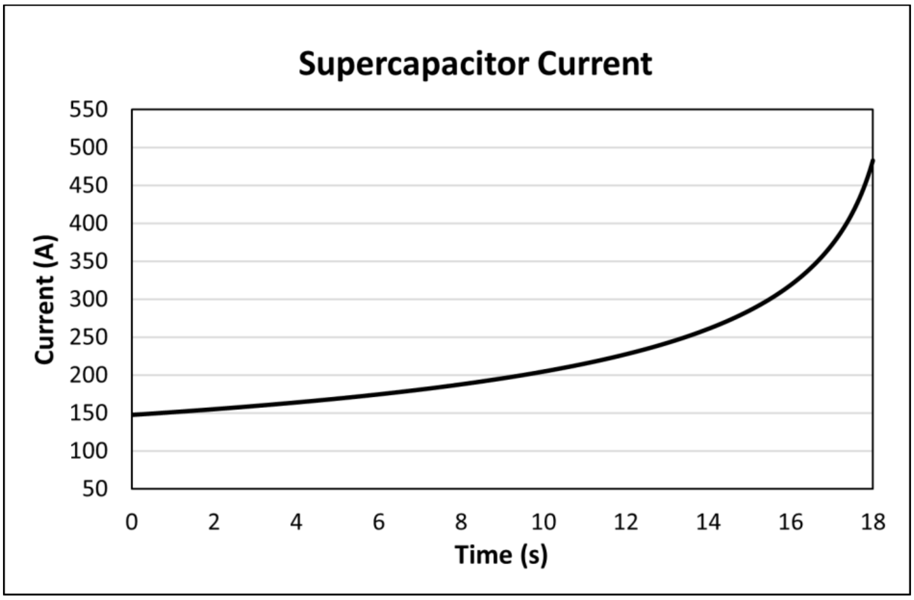

Figure 4-3 Current of SC bank

Figure 4-3 shows the modelled SC's discharge current i(t) over an 18 second constant power discharge period.

Initially, the discharge current i(t) starts at 148.12A at time 0s. This corresponds to the initial current required to supply the 25kW constant power load, based on the maximum capacitor terminal voltage V(t) of 168.78V.

As the capacitor bank discharges, the current i(t) increases exponentially from 148.12A up to 483.09A at 18s. This matches the capacitor voltage decrease shown in Figure 4-2, since i(t) = power / V(t).

The peak current of 483.09A is below the 500A maximum current rating from table 2-4 of the individual 2.7V, 2700F supercapacitor cells used in the bank.

In brief, Figure 4-3 shows the modelled ultracapacitor discharge current i(t) increasing exponentially from 148.12A to 483.09A over the 18s constant power discharge period, as the capacitor voltage drops. The peak current stays below the maximum rating of the individual cells.

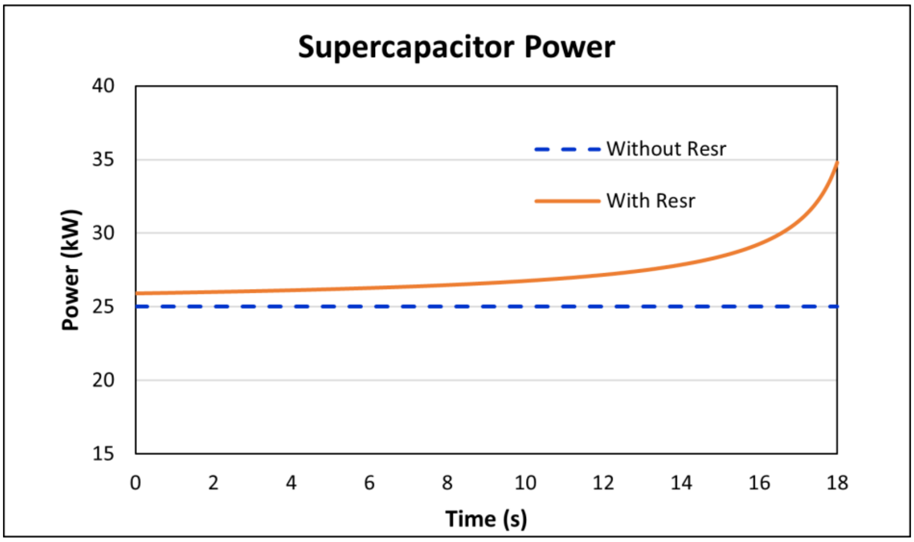

Figure 4-4 Power of SC bank

Figure 4-4 shows the modelled instantaneous power waveform of the ultracapacitor bank over an 18 second constant power discharge.

Initially, the power starts at 25kW, matching the specified constant discharge load.

As the capacitor bank discharges from 175V to 72.04V (Figure 4-2), the current increases from 148.12 A to 483.09A (Figure 4-3). The instantaneous power is the product of the instantaneous current and internal voltage.

At 18s, when the voltage is lowest and current highest, the power curve shows a peak of 34.80kW. This exceeds the 25kW load power due to the additional resistive power losses in the equivalent series resistance (ESR).

The model estimates the total ESR of the 70 cells bank to be 0.0420 Ohms. At 483.09A, this causes a 20.29V voltage drop. So, the "resistive loss power" at 18s is 20.29 V * 483.09A = 9.80 kW.

Adding the 25kW load power and 9.80KW resistive loss power gives the 34.80kW instantaneous power peak.

In summary, Figure 4-4 shows the supercapacitor bank power, including an increase at end of discharge due to ESR losses during maximum current. The peak power reaches 34.80kW due to a 9.80kW loss component.

4.3 Non-ideal model with MATLAB script

From the set of equations in methodology section 3-2, below MATLAB script code is implemented to compare the results with the analytical method modelling.

```
% Parameters
R = 0.6*10^-3; % Ohms - Resistance for one cell
C_cell = 2700; % Farads - Capacitance of the cell
n = 70; % Number of cells
C = C_cell/n; % Capacitance of an individual cell
p = 25e3; % Watts - Power
T = 18; % Seconds - Total simulation time
R_ESR = n * R; % Ohms - Equivalent Series Resistance

% Time points
t = 0:0.01:T; % Time vector with a step of 0.01 seconds

% Initial conditions
E0 = 2.5 * n; % Initial voltage across the cell

% Solve the differential equation for cell voltage
%from equation 3-2-1
dEdt = @(t, E) (-C * E + sqrt(C^2 * E.^2 - 4 * C^2 * R_ESR * p)) / (2 * R_ESR * C^2);
[t, E] = ode45(dEdt, [0, T], E0);

%Calculate current using the specified formula i(t) = -C * dE(t)/dt from equation 3-2-6
i = -C * gradient(E, t);

% Calculate power in kilowatts
power_kw = (E .* i) / 1000;

% Calculate terminal voltage
V = E - i * R_ESR;

% Plots
figure(1)
plot(t, E, t, V)
title('Supercapacitor Terminal & Internal Voltages')
legend('E(Internal Voltage)', 'V(Terminal Voltage)')
xlabel('Time (s)'); ylabel('Voltage (V)')

figure(2)
plot(t, i)
title('Supercapacitor Current')
xlabel('Time (s)'); ylabel('Current (A)')

figure(3)
plot(t, power_kw)
title('Supercapacitor Power')
xlabel('Time (s)'); ylabel('Power (kW)')
```

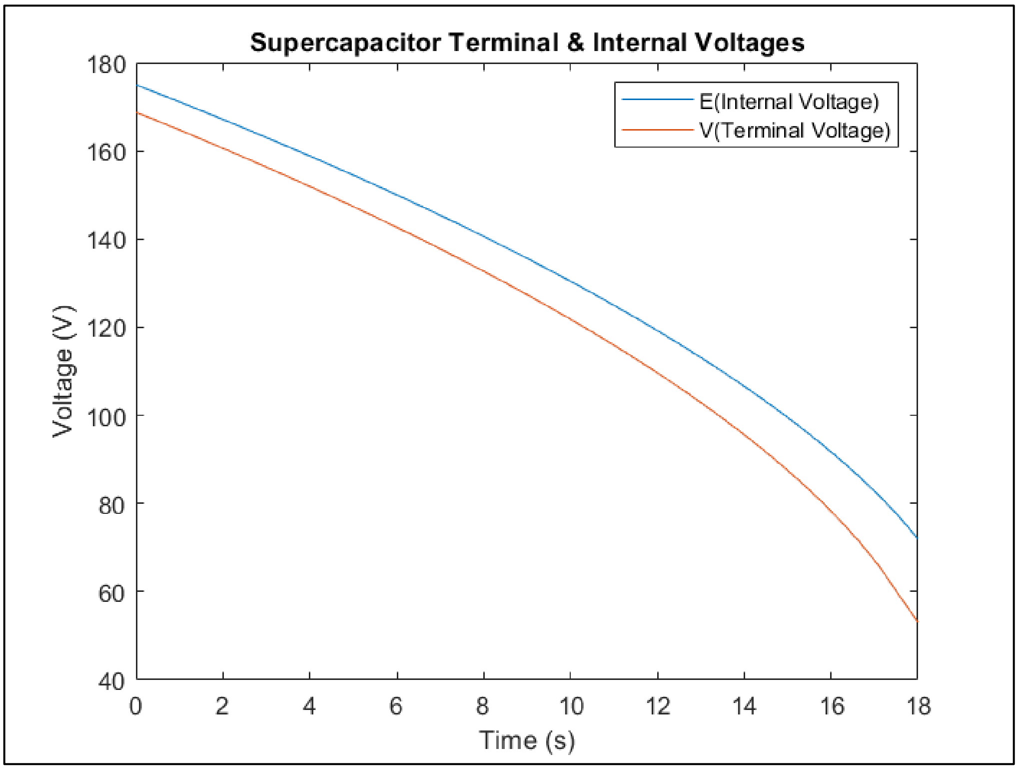

Figure 4-5 MATLAB results for internal and terminal voltages

The MATLAB simulation plots match the analytical results closely, validating the numerical model.

The internal voltage E(t) from MATLAB is 175 at 0s and 71.96V at 18s, very near the 175V at 0s and 72.04V 18s analytical result. And the difference is 0.08V at the end of the demanded time 18s.

The terminal voltage V(t) from MATLAB is 168.74 at 0s and 53.08V at 18s, very near the 168.78V at 0s and 51.75V 18s analytical result. And the difference is 1.33V at the end of the demanded time 18s.

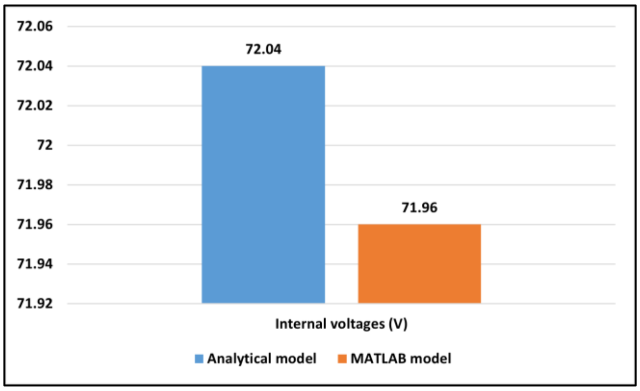

Figure 4-6 Internal Voltages at 18th second

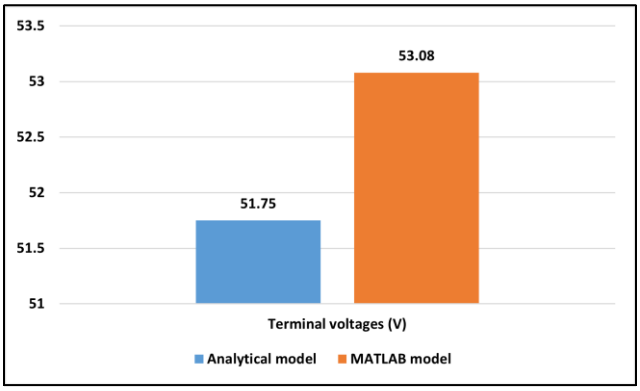

Figure 4-7 Terminal Voltages at 18th second

The reasons of the small voltage differences are:

1. ODE45 uses varying time steps, creating discrete jumps in the solution.
2. The voltage is approximated at each time step based on the prior step values.
3. Slight discrepancies compound over the thousands of steps.
4. Rounding of current and voltage values to a set precision.

However, these minor differences of <1.5V between the analytical and simulated voltages are insignificant in the context of the overall model. The MATLAB simulation closely replicates the analytical exponential decay and divergence characteristics. They are expected numerical artifacts from the discrete step and iterative approach used in the ODE45 algorithm.

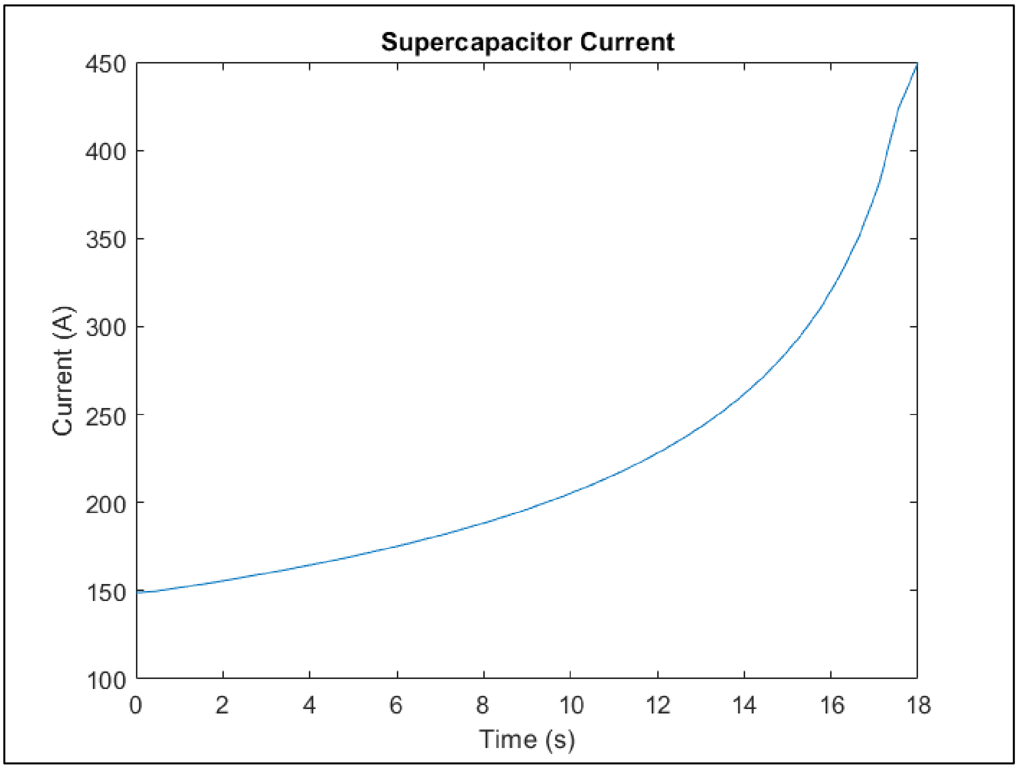

Figure 4-8 MATLAB result for current

The small differences between the analytical and simulated voltages have a negligible effect on the discharge current profile I(t).
The discharge current is determined by equation 3-2-6.

$$i(t) = -C\,\frac{dE(t)}{dt}$$

where, $\frac{dE(t)}{dt}$ can be calculated from equation 3-2-1.
The minor 0.11V lower terminal voltage from the MATLAB simulation causes the discharge current to be slightly higher compared to the analytical model.

However, the current discrepancy is very minor:
At 18s, with V(t) = 51.75V (analytical) , the current is I = 25000W / 51.75V = 483.09A.
With the lower MATLAB V(t) = 51.64V, the current is I = 25000W / 53.08V = 470.98A.
The 12.11A (2.50%) higher current in the analytical simulation is insignificant.

The key reasons the small voltage difference has minimal current impact:
The voltage gap between the models is less than 1.5V out of ~170V.
Current is inversely proportional to voltage. A small change in V causes an even smaller ∆I.

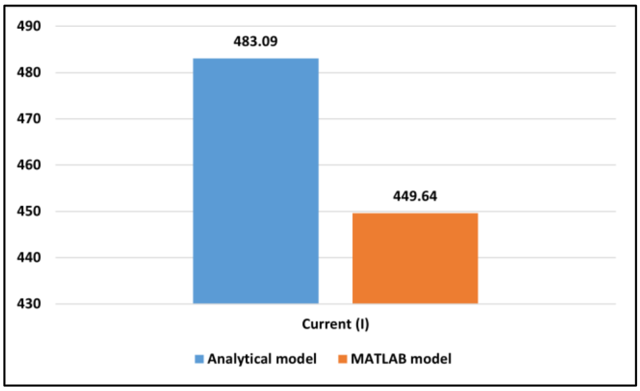

Figure 4-9 Current at 18th second

The exponential I(t) behaviour is maintained in the MATLAB simulation.

The MATLAB terminal voltage to 53.08V at 18s reduces the discharge current proportionally, increases divergence from internal voltage, but does not affect power.

The model still shows an exponential I(t) rise and validates the analytical approach.
This demonstrates how the model responds to changes in the voltage specification.

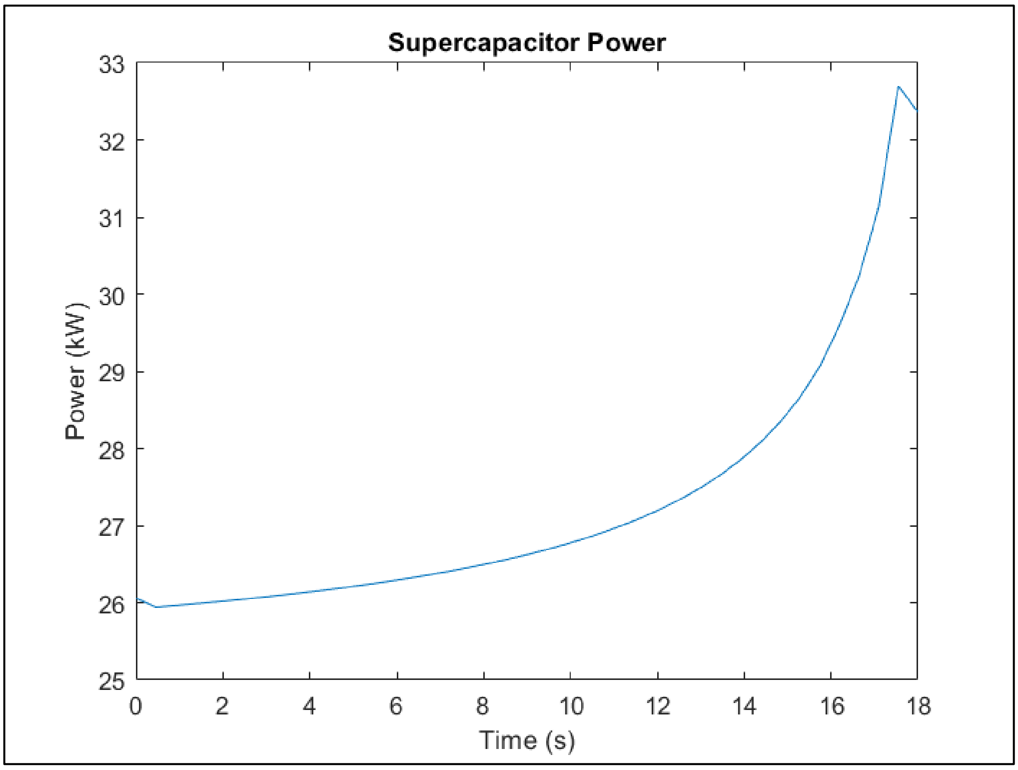

Figure 4-10 MATLAB result for power

The minor differences between the analytical and simulated internal and terminal voltages will have a negligible effect on the power demand for the application:

The 0.08V lower E(t) from MATLAB only causes a 0.08V * 470.98A = 37.6W difference in power at 18s.

The 1.33V lower V(t) from MATLAB only causes a 1.33V * 470.98A = 624.40W difference in power at 18s.

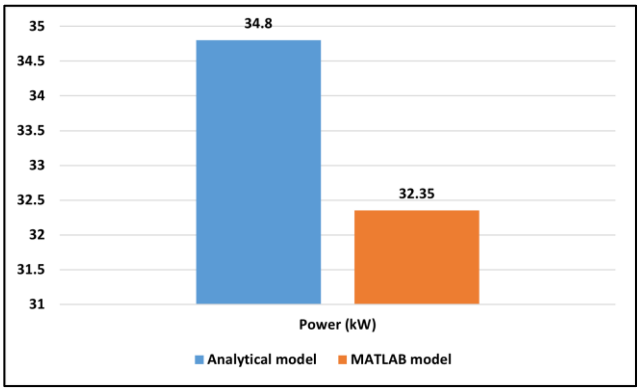

Figure 4-11 Power at 18th second

Given the overall power demand is 25,000W, these discrepancies in the power values are insignificant.

The peak power demand at 18s is still well above 25kW in the MATLAB simulation, meeting the application requirement.

4.4 Validation of non-ideal model with experimental data

Experimental data is taken from[17] as shown in figure 4-12, experiment has been carried out only for one cell. In the same way MATLAB script code is updated as per experimental data.

```
% Parameters

R = 0.7*10^-3; % Ohms - Resistance for one cell
C_cell = 2098; % Farads - Capacitance of the cell
n = 1; % Number of cells
C = C_cell/n; % Capacitance of an individual cell
p = 200; % Watts - Power
T = 36; % Seconds - Total simulation time
R_ESR = n * R; % Ohms - Equivalent Series Resistance
```

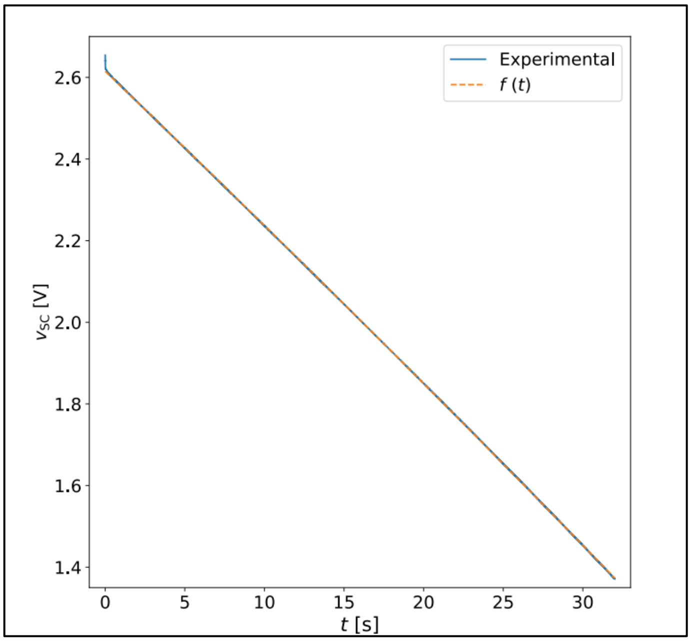

Figure 4-12 experimental graph for validation[[17]]

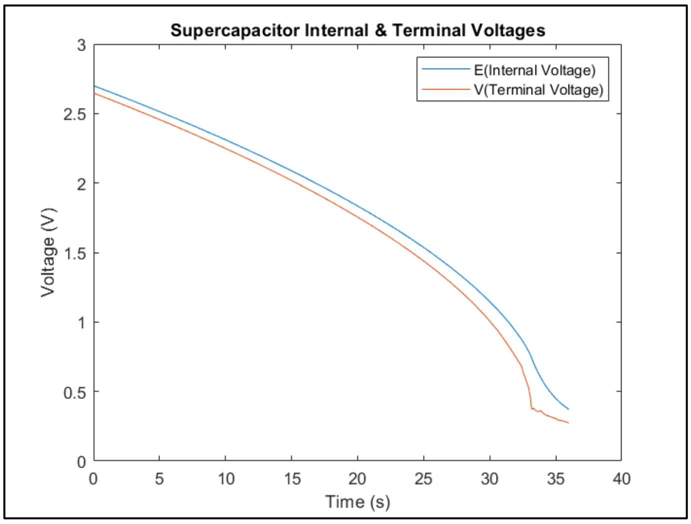

Figure 4-13 non-ideal voltages for one cell

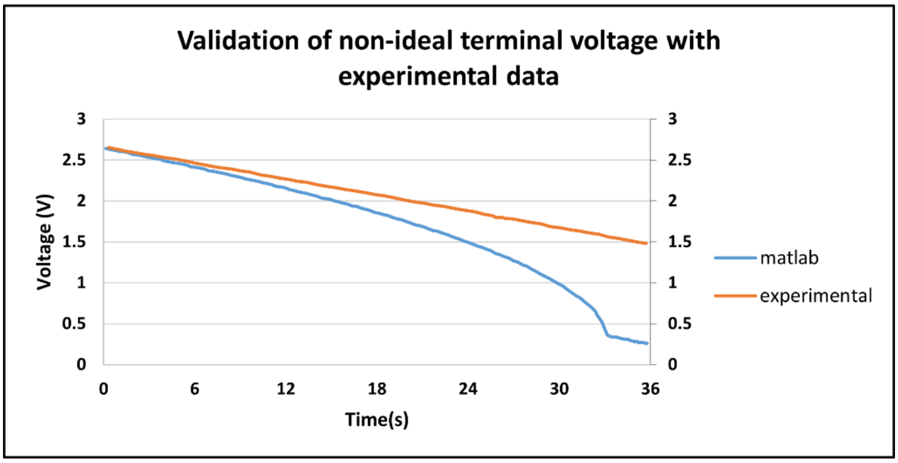

Figure 4-14 Validation of non-ideal with experimental model

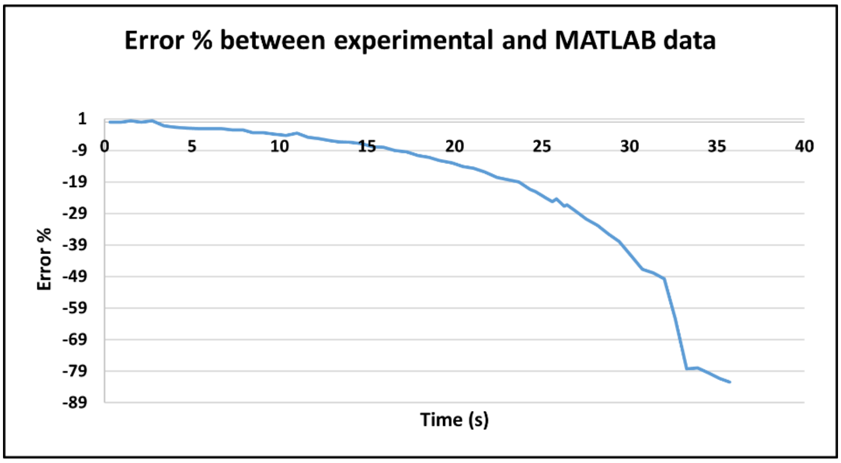

Figure 4-15 Error% of Non-ideal model

From the figure 4-14 & 4-15, the modelled time constant differs significantly from the actual transient response. The single exponential decay behaviour leads to increasing divergence from the experimental profile over time. After only 10 seconds, the absolute error reaches 1.5V (37%) and continues dropping to a maximum of 0.25V (83.3%) by the end of the 36s discharge.

This large deviation stems from improper modelling of the key electrical parameters(such as capacitance, ESR, voltage rating, self-discharge rate, temperature dependence, cycling stability & specific energy and specific power) that dictate supercapacitor performance. The oversimplified RC branch is only valid for approximate analysis rather than high-fidelity simulation. This underscores the requirement for more physics-based modelling approaches. Hence two branch model is used as alternative to validate with experimental data in next subsection.

4.5 Results of two branch model from MATLAB script

```
% Parameters
R_cell = 0.7 * 10^-3; % Resistance for one cell
n = 1; % Number of cells
R_ESR = n * R_cell; % Ohms - Equivalent Series Resistance
C0 = 2098; % Farads - Capacitance of the cell
K1 = 248; % Constant F/V
E_cells = 2.7; % Voltage of one cell
C1 = K1 * E_cells; % Capacitance of additional capacitor
C_cell = C0 + C1; % Total capacitance
C = C_cell / n; % Capacitance of an individual cell
p = 200; % Watts - Power
T = 36; % Seconds - Total simulation time

% Time points
t = 0:0.01:T; % Time vector with a step of 0.01 seconds

% Initial conditions
E_cells = E_cells * n; % Initial voltage across the cell

% Solve the differential equation for cell voltage
dE_cellsdt = @(t, E_cells) ((-p/E_cells) * (1/((C0 + ((2 * K1) * (E_cells -
((p * R_ESR) / E_cells)))) * (1/(1 + (p * R_ESR) / E_cells^2)))));
[t, Y] = ode45(dE_cellsdt, [0, T], E_cells);

% Extracting results
E_cells = Y(:, 1);
i = -C * gradient(E_cells, t);

% Calculate power in kilowatts
power_w = (E_cells.* i) ;

% Calculate terminal voltage
V = E_cells - i * R_ESR;

% Calculate power in kilowatts
%power_w = (V.* i) ;

% Plots
figure(1)
plot(t, E_cells, t, V)
title('Supercapacitor Internal & Terminal Voltages')
legend('E(Internal Voltage)', 'V(Terminal Voltage)')
xlabel('Time (s)'); ylabel('Voltage (V)')

figure(2)
plot(t, i)
title('Supercapacitor Current')
xlabel('Time (s)'); ylabel('Current (A)')

figure(3)
plot(t, power_w)
title('Supercapacitor Power')
xlabel('Time (s)'); ylabel('Power (W)')
```

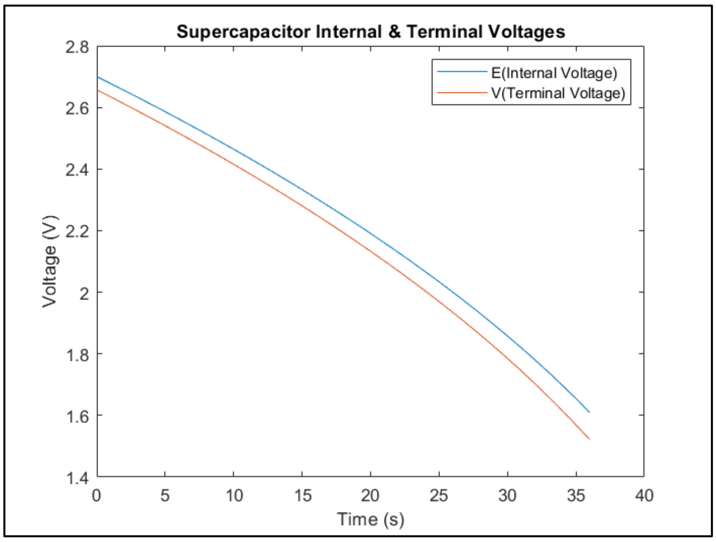

Figure 4-16 MATLAB results of Internal & terminal voltages

Figures 4-16, 4-17 & 4-18 are result of examination of a supercapacitor system through the development and analysis of a MATLAB script code. The code incorporates fundamental electrical parameters, including resistance (R_cell), capacitance (C), and voltage (E_cells), with a focus on capturing the dynamic response under a constant power input of 200W for a 36-second duration. The temporal evolution of internal and terminal voltages is elucidated in the figure 4-16, revealing an initial transient state and subsequent stabilization. Specifically, at t = 0 seconds, the internal voltage marginally exceeds the terminal voltage, indicative of an initial charge imbalance. Conversely, at t = 36 seconds, both voltages decline, signifying a discharge process.

Initial Internal Voltage Deviation: The internal voltage starts at 2.7V and decreases to 1.50V by the end of the simulation, reflecting a deviation of approximately 44.4%.

Terminal Voltage Deviation: The terminal voltage experiences a reduction from 2.65V to 1.52V, constituting a deviation of approximately 42.6%.

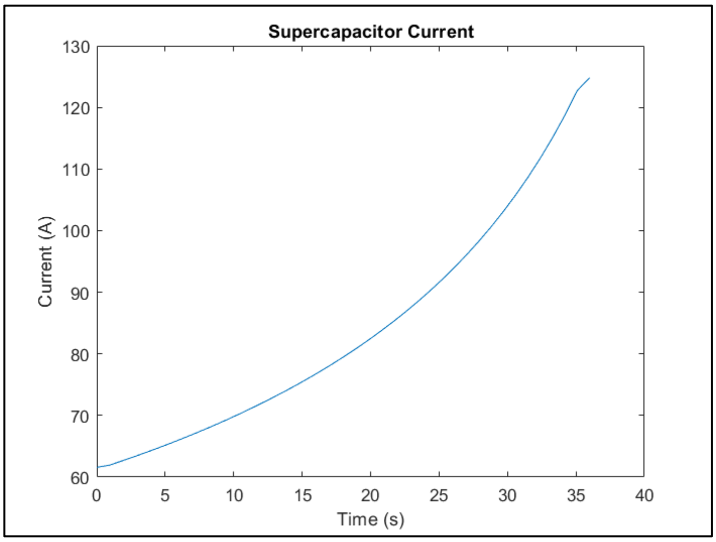

Figure 4-17 MATLAB result of current

Figure 4-17 details the current profile, exhibiting a notable increase from an initial 61.5A to 124.8A by the simulation's conclusion. This surge in current contributes to a corresponding rise in power dissipation.

Current Increase: The initial current of 61.5A escalates to 124.8A, representing a substantial increase of approximately 103.4%.

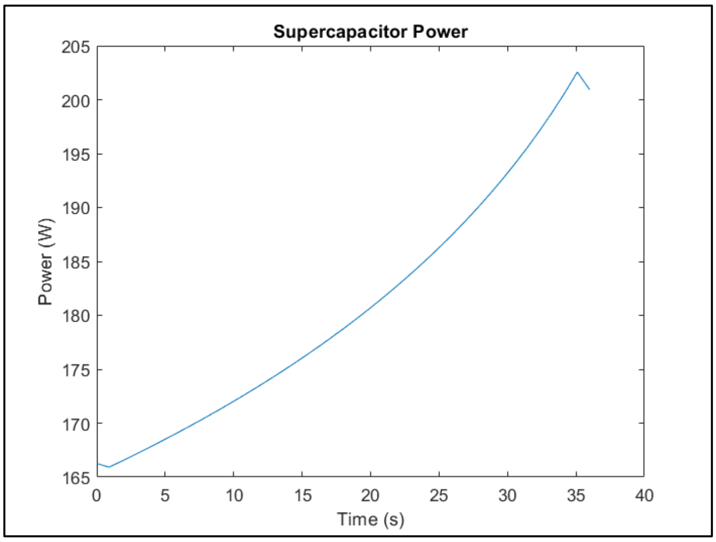

Figure 4-18 MATLAB result of power

Figure 4-18, escalating from 166.2W to 200.92W over the simulation interval. These outcomes provide intricate insights into the transient behaviour and energy dissipation characteristics of the supercapacitor system under specified power conditions, underscoring the significance of precise, physics-based modelling for predictive analysis and practical application optimization.

These findings contribute to a nuanced understanding of the supercapacitor system's behaviour and its practical implications for applications demanding transient energy storage and release.

Initial Power Dissipation: The power dissipation begins at 166.2W and rises to 200.92W, indicating an increase of approximately 20.8%.

This percentage-based analysis provides a quantitative perspective on the dynamic changes in key parameters throughout the simulation. The substantial increase in current and power dissipation aligns with the expected behaviour of a supercapacitor undergoing discharge. The percentage deviations in voltage parameters underscore the significance of transient effects and the system's response to continuous power input.

4.6 Validation of two branch model with experimental data

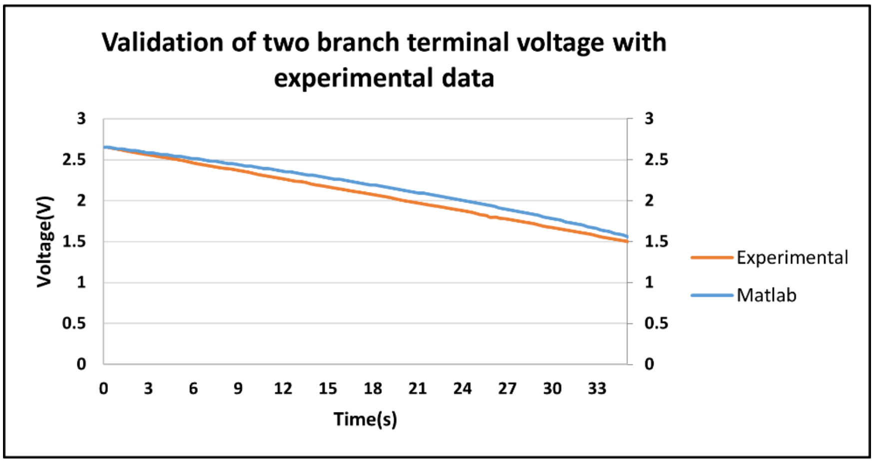

Figure 4-19 Validation of two branch with experimental model

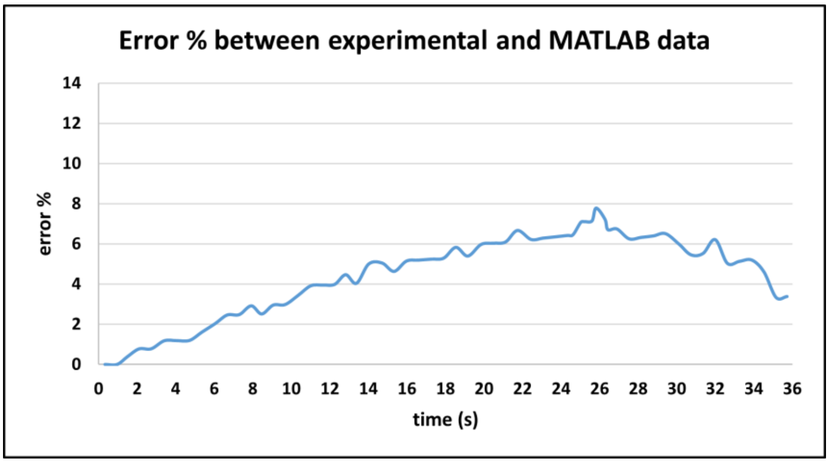

Figure 4-20 error% of two branch model

Figures 4-19 & 4-20 demonstrates validation of two branch model with experimental data. For a 36 second, 200W constant power discharge of a 2.7V cell, this model achieves a maximum absolute error of 7.77% and mean error of 4.54% relative to experimental results. The two-branch model emulates the better behaviour of the double-layer complex impedance and mitigates divergence over time. However, the model still exhibits steadily increasing deviation, reaching 3.37% by the end of discharge.

By comparing with non-ideal model, enhanced model (two branch) implicates experiment graphs. However, further improvement in parameter identification and optimization methods is required to drive higher accuracy over the full operating duration.

4.7 *Experimental vs non-ideal vs two branch models*

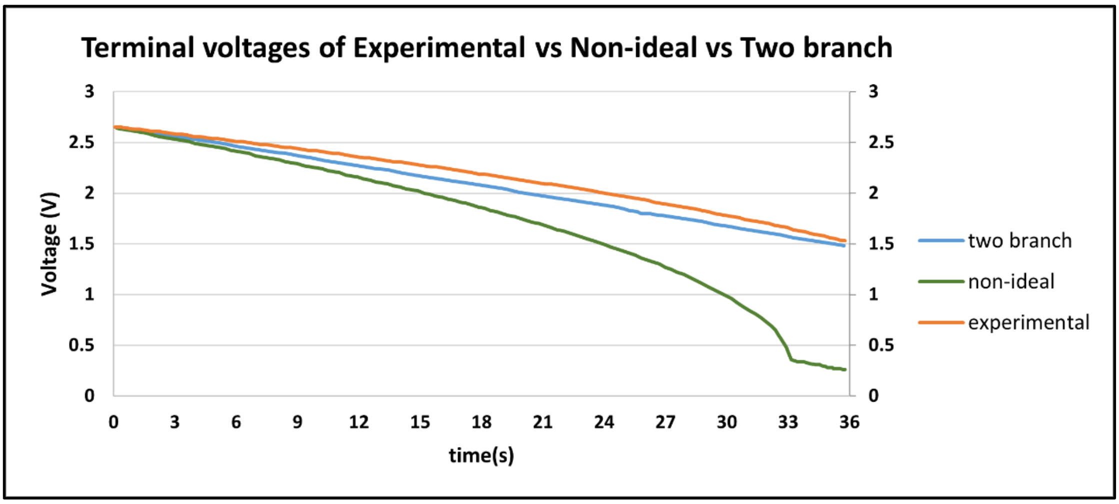

Figure 4-21 Experimental vs Non-ideal vs Two branch

The analysis of two distinct models provides valuable insights into the modelling and validation of supercapacitor performance. Non-ideal model illustrates the limitations of a single exponential decay model, highlighting significant deviations from experimental results over the 36-second discharge period. The observed discrepancies, reaching a maximum absolute error of 0.25V (83.3%), stems from the oversimplification of key electrical parameters influencing supercapacitor behaviour. The inadequacy of the RC branch model underscores the necessity for more sophisticated, physics-based modelling approaches.

A two-branch model as an alternative, demonstrating improved validation with experimental data. The model achieves a maximum absolute error of 7.77% and a mean error of 4.54%, reflecting better emulation of double-layer complex impedance behaviour and reduced divergence over time.

However, the model still exhibits increasing deviation, indicating the need for further refinement in parameter identification and optimization methods to enhance accuracy over the entire operating duration.

The utilization of the two-branch model represents progress in aligning simulation outcomes with experimental findings, emphasizing the ongoing pursuit of precision in supercapacitor modelling for real-world applications.

5. CONCLUSION AND FURTHER DISCUSSION

Bringing the analysis to a close, the analysis underscores the critical importance of optimal sizing in supercapacitor banks for high-power acceleration demands. The non-ideal model, employing analytical methods and validated through MATLAB simulations, provided crucial insights. Notably, the model emphasized the need for careful sizing to meet specific power requirements while minimizing mass. The study delved into voltage and current characteristics, revealing an exponential decay of internal capacitor voltage (E(t)) and the ensuing divergence between internal and terminal voltage (V(t)) due to Equivalent Series Resistance (ESR). This divergence reached 27.96%, showcasing the impact of ESR on system performance. The discharge current profile exhibited an exponential increase, reaching a peak of 483.09 A, well below the maximum rating of individual supercapacitor cells. The instantaneous power waveform showcased a peak power of 34.80 kW, exceeding the 25kW load power due to resistive power losses in the ESR. The MATLAB simulation closely replicated analytical results, with minor discrepancies deemed insignificant. Two branch model, discharge process over 36 seconds, evidenced by a 44.4% drop in internal voltage and a 42.6% decrease in terminal voltage. Concurrently, current surged by 103.4%, leading to a 20.8% increase in power dissipation. These findings emphasize the supercapacitor's efficiency in rapid energy discharge, particularly relevant for applications like start-stop system in Hybrid Electric Vehicles. While non-ideal model underscores the limitations of an oversimplified model, two branch model introduces a more sophisticated model that demonstrates improved validation. However, challenges persist in achieving higher accuracy throughout the supercapacitor's operational duration, indicating the ongoing need for refinement and optimization in modelling approaches.

Moving forward, potential research avenues encompass the refinement of numerical methods, experimental validation, exploration of dynamic power profiles, consideration of temperature effects, and integration with vehicle control systems. The synergistic use of analytical and numerical approaches stands as a robust strategy for advancing the comprehension and optimization of supercapacitor bank in HEV applications.

6. PROJECT MANAGEMENT REVIEW

In hindsight, the time management for my project has yielded valuable insights into navigating challenges and ensuring efficiency. Commencing on October 05, 2023, the project's initial plan focused solely on developing an analytical model. However, a pivotal suggestion from my professor prompted the expansion of the project scope to encompass both analytical and Simulink models, addressing both non-ideal and two-branch scenarios. A fortuitous discovery of a relevant paper streamlined the formulation of the non-ideal analytical model within just one month. This efficiency enabled the timely submission of an interim report. However, the journey toward the enhanced two-branch model proved to be more intricate and challenging. After three weeks and three collaborative meetings, a pertinent research paper was identified, setting the stage for the development of the enhanced model. Remarkably, both models were swiftly translated into MATLAB script code. Crucially, the culmination of this endeavour occurred just a week before the project deadline when we successfully validated both models using experimental data from a pertinent research paper. This time management strategy, characterized by adaptability, strategic collaboration, and swift problem-solving, played a critical role in ensuring the project's success. Despite the challenges, the project's completion on schedule underscored the significance of proactive effort, resourcefulness, and effective planning. This experience highlights the importance of adaptability and efficient time management in steering projects to successful outcomes within constrained timelines.

MSc Project

University of Hertfordshire

Sai Gopal Addagarala (21060080)

Project start: **05-10-2023**

Display week: **1.5**

Optimal sizing of supercapacitors for Hybrid electric vehicles

TASK	START	END	DAYS
Literature review			
Review existing studies and supercapacitor tech	05/10/23	08/10/23	3
Identify relevant publications and gather data	09/10/23	14/10/23	5
Survey Supercapacitor Models	15/10/23	17/10/23	2
Compile a list of available supercapacitor models	18/10/23	20/10/23	2
Analyze key characteristics and performance parameters	21/10/23	24/10/23	3
Non-ideal Circuit Model			
Research and select appropriate non-ideal model approaches	25/10/23	28/10/23	3
Implement chosen non-ideal circuit models	29/10/23	30/10/23	1
Numerical Resolution apply numerical iterative method- Euler's iterative method	31/10/23	03/11/23	3
Optimal Cell Count Calculation	04/11/23	06/11/23	2
Interim report	07/11/23	13/11/23	6
Model Enhancement (two branch model)			
Include term branch for leakage effect study	14/11/23	17/11/23	3
Leakage Effect Incorporation	18/11/23	21/11/23	3
Term branch model to account for leakage effects	22/11/23	26/11/23	4
Investigate literature for leakage effect modeling	27/11/23	01/12/23	4
Compute optimum number of cells for application in term branch model	02/12/23	05/12/23	3
MATLAB/Simulink and Documentation			
Implement Matlab code for Non-ideal and two branch models	06/12/23	18/12/23	12
Validating numerical models with Matlab code for non-ideal and two branch mo	19/12/23	22/12/23	3
Observing results, applications and future scope	23/12/23	24/12/23	1
Validating with experimental data and documentation	25/12/23	04/01/24	10
Changes in final report & Powerpoint presentation	05/01/24	08/01/24	3

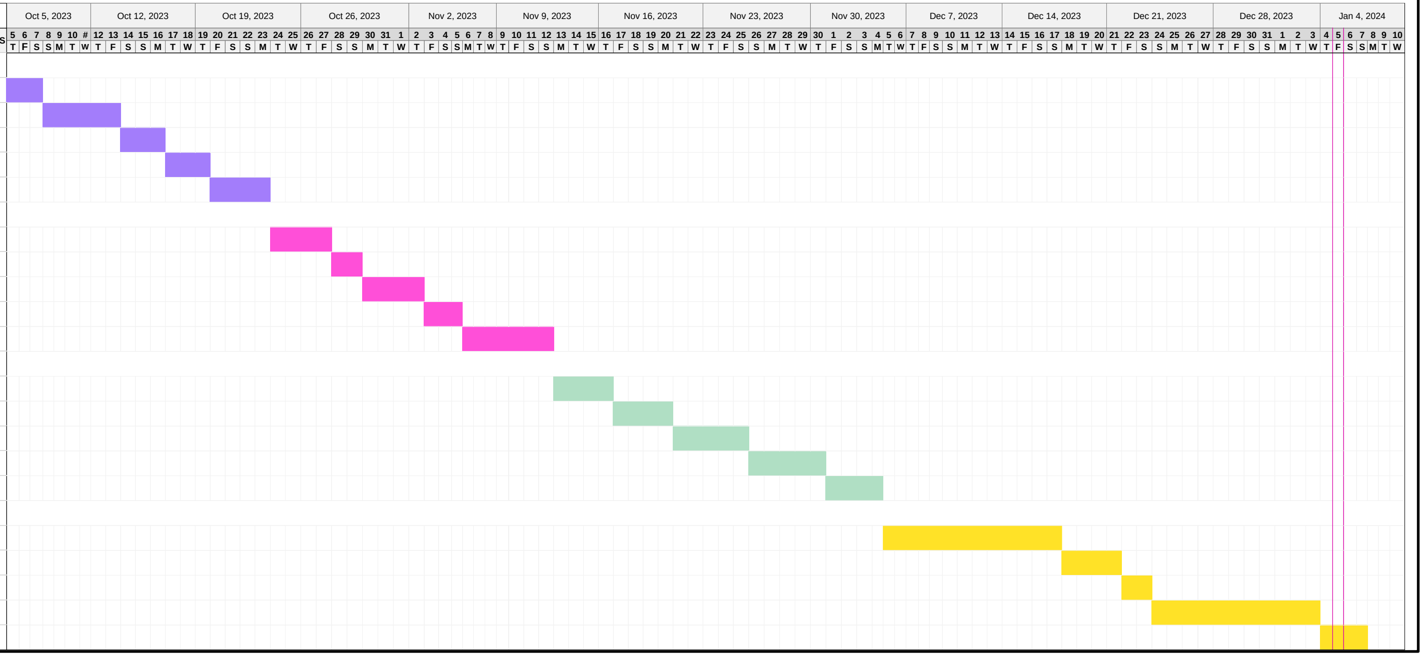

Figure 6-1 Project management-Gantt chart

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PROJECT LOGBOOK: Week 1

Name:	Sai Gopal Addagarala
Project title:	Optimal sizing of supercapacitors for hybrid electric vehicles
Supervisor:	**Christos Kalyvas**

Student notes & date:	**05/10/2023**
Planned tasks and Work Undertaken:	Understanding, aim and objective of the project. Roadmap to achieve required number of ultracapacitors required.
Problems encountered/ questions for supervisor:	

Project Meeting date & staff(s):	
Meeting notes:	Discussed about, Sizing Ultracapacitors for Hybrid Electric Vehicles by H. Douglas and P Pillay from IEEE Journal
Action point(s):	By the following week, need to achieve how the author approached to find number of capacitors required in ideal state
Checked by:	
Date:	**05/10/23**

PROJECT LOGBOOK: Week 2

Name:	Sai Gopal Addagarala
Project title:	Optimal sizing of supercapacitors for hybrid electric vehicles
Supervisor:	**Christos Kalyvas**

Student notes & date:	**13/10/2023**
Planned tasks and Work Under-taken:	To find number of ultracapacitors required for a constant power application.
Problems en-countered/ ques-tions for supervi-sor:	Whether to consider resistance or not.

Project Meeting date & staff(s):	
Meeting notes:	Discussed about, Sizing Ultracapacitors for Hybrid Electric Vehicles by H. Douglas and P Pillay from IEEE Journal
Action point(s):	Need to solve differential quadratic equation by using nu-merical method
Checked by:	
Date:	**20/10/23**

PROJECT LOGBOOK: Week 3

Name:	Sai Gopal Addagarala
Project title:	Optimal sizing of supercapacitors for hybrid electric vehicles
Supervisor:	**Christos Kalyvas**

Student notes & date:	**20/10/2023**
Planned tasks and Work Undertaken:	Considered constant power application, including equivalent series resistance. Solved numerical iterative approach by using Euler's iterative method in excel spread sheet.
Problems encountered/ questions for supervisor:	To merge number oof ultracapacitors achieved in which model? battery-ultracapacitors? Fuel cell- ultracapacitors?

Project Meeting date & staff(s):	
Meeting notes:	Discussed about, Sizing Ultracapacitors for Hybrid Electric Vehicles by H. Douglas and P Pillay from IEEE Journal And, to use achieved results in further steps of project.
Action point(s):	To complete "Effect of Supercapacitor Modelling and Unit Cell Capacitance Selection towards Economic Sizing of Energy Storage System in Electric Vehicle" by Sakshi Bansal, Praveen Nambisan, Pankaj Saha, and Munmun Khanra. And try to incorporate this in achieved model.
Checked by:	
Date:	**27/10/23**

PROJECT LOGBOOK: Week 4

Name:	Sai Gopal Addagarala
Project title:	Optimal sizing of supercapacitors for hybrid electric vehicles
Supervisor:	**Christos Kalyvas**

Student notes & date:	**27/10/2023**
Planned tasks and Work Under-taken:	Prepared Gantt chart and methodology.
Problems en-countered/ ques-tions for supervi-sor:	To consider series or parallel or series-parallel?

Project Meeting date & staff(s):	
Meeting notes:	Discussed about model 2 in Effect of Supercapacitor Mod-elling and Unit Cell Capacitance Selection towards Economic Sizing of Energy Storage System in Electric Vehicle by Sakshi Bansal, Praveen Nambisan, Pankaj Saha, Munmun Khanra.
Action point(s):	Need to work on model 2(paraller RC circuit)
Checked by:	
Date:	**03/11/23**

PROJECT LOGBOOK: Week 5

Name:	Sai Gopal Addagarala
Project title:	Optimal sizing of supercapacitors for hybrid electric vehicles
Supervisor:	**Christos Kalyvas**

Student notes & date:	**03/11/2023**
Planned tasks and Work Under-taken:	Interim report preparation, Literature review? Aims and Objectives?
Problems en-countered/ ques-tions for supervi-sor:	To consider series or parallel or series-parallel?

Project Meeting date & staff(s):	
Meeting notes:	Discussed about model 2 in Effect of Supercapacitor Mod-elling and Unit Cell Capacitance Selection towards Economic Sizing of Energy Storage System in Electric Vehicle by Sakshi Bansal, Praveen Nambisan, Pankaj Saha, Munmun Khanra.
Action point(s):	Need to work on model 2(paraller RC circuit) and on Par-ticle swarm optimization method.
Checked by:	
Date:	**10/11/23**

PROJECT LOGBOOK: Week 6

Name:	Sai Gopal Addagarala
Project title:	Optimal sizing of supercapacitors for hybrid electric vehicles
Supervisor:	**Christos Kalyvas**

Student notes & date:	**10/11/2023**
Planned tasks and Work Undertaken:	Simulink file?
Problems encountered/ questions for supervisor:	To make sure about voltage leakages in term branch.

Project Meeting date & staff(s):	
Meeting notes:	Discussed about model 2 in Effect of Supercapacitor Modelling and Unit Cell Capacitance Selection towards Economic Sizing of Energy Storage System in Electric Vehicle by Sakshi Bansal, Praveen Nambisan, Pankaj Saha, Munmun Khanra.
Action point(s):	By implementing term branch in non-ideal ECM and observe leakage effect on SC bank
Checked by:	
Date:	**10/11/23**

PROJECT LOGBOOK: Week 7

Name:	Sai Gopal Addagarala
Project title:	Optimal sizing of supercapacitors for hybrid electric vehicles
Supervisor:	**Christos Kalyvas**

Student notes & date:	**17/11/2023**
Planned tasks and Work Undertaken:	Parallel leakage in term branch and Simulink files.
Problems encountered/ questions for supervisor:	To make sure about voltage leakages in term branch.

Project Meeting date & staff(s):	
Meeting notes:	Comparing of mathematical model with Simulink.
Action point(s):	After building Simulink model, compare %of error with excel file.
Checked by:	
Date:	**17/11/23**

PROJECT LOGBOOK: Week 8

Name:	Sai Gopal Addagarala
Project title:	Optimal sizing of supercapacitors for hybrid electric vehicles
Supervisor:	**Christos Kalyvas**

Student notes & date:	**24/11/2023**
Planned tasks and Work Undertaken:	Simulink files for RC and parallel
Problems encountered/ questions for supervisor:	To include Belhachemi values to compare.

Project Meeting date & staff(s):	
Meeting notes:	Error while finding two branch capacitors in script
Action point(s):	To work on two branch script code.
Checked by:	
Date:	**24/11/23**

PROJECT LOGBOOK: Week 9

Name:	Sai Gopal Addagarala
Project title:	Optimal sizing of supercapacitors for hybrid electric vehicles
Supervisor:	**Christos Kalyvas**

Student notes & date:	**01/12/2023**
Planned tasks and Work Undertaken:	Simulink files for two banch
Problems encountered/ questions for supervisor:	To make sure about voltage leakages in term branch.

Project Meeting date & staff(s):	
Meeting notes:	dE(t)/dt in Sizing Ultracapacitors For Hybrid Electric Vehicles has some error for two branch error
Action point(s):	Need to find another paper which has application for two branch model
Checked by:	
Date:	**01/12/23**

PROJECT LOGBOOK: Week 10

Name:	Sai Gopal Addagarala
Project title:	Optimal sizing of supercapacitors for hybrid electric vehicles
Supervisor:	**Christos Kalyvas**

Student notes & date:	**08/12/2023**
Planned tasks and Work Undertaken:	Script code for non-ideal model is finalised, need to work on two branch script code
Problems encountered/ questions for supervisor:	To make sure term branch voltage and terminal voltage is same.

Project Meeting date & staff(s):	
Meeting notes:	From "Analytical modelling and sizing of supercapacitors for spacecraft hybrid energy storage systems" paper , solve equation 12 in MATLAB script to find voltages in term branch.
Action point(s):	The voltage is same for both term branch and terminal.
Checked by:	
Date:	**08/12/23**

PROJECT LOGBOOK: Week 11

Name:	Sai Gopal Addagarala
Project title:	Optimal sizing of supercapacitors for hybrid electric vehicles
Supervisor:	**Christos Kalyvas**

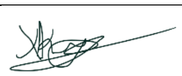
Student notes & date:	**15/12/2023**
Planned tasks and Work Under-taken:	To work on script code for two-branch , using equations 7-12 from "Analytical modelling and sizing of supercapacitors for spacecraft hybrid energy storage systems" paper.
Problems en-countered/ ques-tions for supervi-sor:	To make sure term branch voltage and terminal voltage is same.

Project Meeting date & staff(s):	
Meeting notes:	From "Analytical modelling and sizing of supercapacitors for spacecraft hybrid energy storage systems" paper , solve equation 12 in MATLAB script to find voltages in term branch, current and power for constant power application.
Action point(s):	Came to conclusion that voltages are same for both term branch and terminal.
Checked by:	
Date:	**15/12/23**

PROJECT LOGBOOK: Week 12

Name:	Sai Gopal Addagarala
Project title:	Optimal sizing of supercapacitors for hybrid electric vehicles
Supervisor:	**Christos Kalyvas**

Student notes & date:	**20/12/2023**
Planned tasks and Work Under-taken:	To work on script code for two-branch , using equations 7-12 from “Analytical modelling and sizing of supercapacitors for spacecraft hybrid energy storage systems” paper.
Problems en-countered/ ques-tions for supervi-sor:	Voltage can be found by using equation 10 from “Analytical modelling and sizing of supercapacitors for spacecraft hybrid energy storage systems” paper.

Project Meeting date & staff(s):	
Meeting notes:	As models are finalised in MATLAB script environment, need to start put all together in report.
Action point(s):	Need to start writing project report by 04/01/2024 for re-view before submitting on 08/01/2024.
Checked by:	
Date:	**20/12/23**

PROJECT LOGBOOK: Week 13

Name:	Sai Gopal Addagarala
Project title:	Optimal sizing of supercapacitors for hybrid electric vehicles
Supervisor:	**Christos Kalyvas**

Student notes & date:	**04/01/2024**
Planned tasks and Work Undertaken:	Validation with experimental data for both the models.
Problems encountered/ questions for supervisor:	Is it valid from “New Parameter Identification Method for Supercapacitor Model” from figure 8?

Project Meeting date & staff(s):	
Meeting notes:	Final dissertation and defence documentation preparation and validation of both models
Action point(s):	Validation and documentation
Checked by:	
Date:	**04/01/24**