Classical Mechanics

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Chapter 1

Introduction

1.1 Notation

$$\left(\vec{A} \times \vec{B}\right)_i = \epsilon_{ijk} A_j B_k$$

$$\vec{A}.\vec{B} = \sum_{ij} \delta_{ij} \vec{A}_i \vec{B}_j$$

$$\begin{aligned} \delta_{ij} &= 1 \text{ if } i = j \\ &= 0 \text{ otherwise} \end{aligned}$$

$$\begin{aligned} \epsilon_{ijk} &= 0 \text{ if any 2 of indices are equal} \\ &= 1 \text{ if indices are cyclic perm of (123)} \\ &= -1 \text{ if indices are anti-cyclic perm of (123)} \end{aligned}$$

1.2 Examples

Angular Momentum $\vec{L} = \vec{\rho} \times \vec{p}$

$$L_i = \epsilon_{ijk} \vec{\rho}_j \vec{p}_k$$

1.3 Gradient, Divergence and Curl

1.3.1 Gradient

$$f(\vec{\rho}) = f(x, y, z)$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x} \hat{e}_x$$

1.3.2 divergence

$$v_x(\vec{\rho}) = \frac{\partial v_x}{\partial x} e_x + \frac{\partial v_y}{\partial y} \hat{e}_y + \frac{\partial v_z}{\partial z} \hat{e}_z$$

$$\vec{\nabla}.\vec{v}(\vec{\rho}) = \left[\frac{d}{dx} e_x + \frac{d}{dz} e_y + \frac{d}{dz} e_z\right] . \left[v_x \hat{e}_x + v_y \hat{e}_y + v_z \hat{e}_z\right]$$

$$= \frac{dv_x}{dx} + \frac{dv_y}{dy} + \frac{dv_z}{dz}$$

1.3.3 curl

$$
\begin{aligned}
\left[\nabla \vec{\times} \vec{v}\right]_i &= \epsilon_{ijk}\partial_j v_k \\
\left[\nabla \vec{\times} \vec{v}\right]_1 &= \epsilon_{1jk}\partial_j v_k \\
&= \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}
\end{aligned}
$$

Chapter 2

Newtonian Mechanics

2.1 Action - Reaction pair

Action reaction forces always act on different bodies. Equal in magnitude, opposite but not necessarily along same line.

2.2 Second Law

$$\vec{F} = m\vec{a}$$

2.3 Mechanics of a single particle

$$\begin{aligned}\vec{v} &= \frac{\partial \vec{p}}{\partial t}\\ \vec{p} &= m\vec{v}\\ \vec{f} &= \frac{\partial \vec{p}}{\partial t} = \vec{p}\end{aligned}$$

If $m(t) \equiv m$,

$$\vec{F} = m\frac{d\vec{v}}{dt} = ma = m\frac{d^2\rho}{dt^2}$$

$$\begin{aligned}\vec{L} &= \vec{\rho} \times \vec{p}\\ \vec{\tau} &= \vec{\rho} \times \vec{F}\end{aligned}$$

By newtons law

$$\begin{aligned}\vec{\tau} &= \vec{\rho} \times \frac{d\vec{p}}{dt}\\ &= \vec{\rho} \times \frac{d}{dt}(m\vec{v})\end{aligned}$$

2.3.1 Work

$$\begin{aligned}W &= \int_i^f \vec{F} d\vec{s}\\ &= m\int_i^f \frac{d\vec{v}}{dt}.\vec{v}\ dt\\ &= \frac{m}{2}\int_i^f dt\frac{d}{dt}\left(\vec{v}.\vec{v}\right)\\ &= \frac{m}{2}\left(v_f^2 - v_i^2\right)\end{aligned}$$

Let $T = \frac{1}{2}mv^2$ then,

$$W = \frac{m}{2}\left(v_f^2 - v_i^2\right) = T_f - T_f$$

Conservative Forces

$\vec{F}$ is a conserved force if

$$\oint_{\mathcal{C}} \vec{F}.\vec{ds} = 0$$

then we can define the potential V as

$$\vec{F} = -\vec{\nabla}V.$$

$$\oint_{\mathcal{C}} \vec{F}.\vec{ds} = -\oint_{\mathcal{C}} \vec{\nabla}V.\vec{ds} = 0$$

Conservation Law

- If $\vec{F} = 0, \frac{\vec{dp}}{dt} = 0$ (liner momentum is conserved)
- If $\vec{\tau} = 0, \frac{\vec{L}}{dt} = 0$ (angular momentum is conserved)
- If $W = 0, \frac{\vec{T}}{dt} = 0$ (T is conserved)

Chapter 3

Hamiltonian Mechanics

3.1 Hamilton's Equations

3.2 Liouville's Theorem