Homework 2

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Question 1: Totally Unimodular Matrices:

$$A_1 = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

The above matrix is **not totally unimodular**:

1. All the elements are 1,0. (property 1 satisfied)
2. There are at most 2 non-zero elements in each column. (property 2 satisfied)
3. Is not a TU (Totally Unimodular matrix) because the rows of A1 cannot be partitioned into two disjoint sets such that if a column has two entries of the same sign, their rows are in different sets.

$$A_2 = \begin{pmatrix} -1 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{pmatrix}$$

The above matrix is **totally unimodular** because:

1. All the elements are 1,0,-1. (property 1 satisfied)
2. There are at most 2 non-zero elements in each column. (property 2 satisfied)
3. The rows of A2 can be partitioned into two disjoint sets such that if a column has two entries of the same sign, their rows are in different sets & if a column has two entries of different signs, their rows are in the same set.

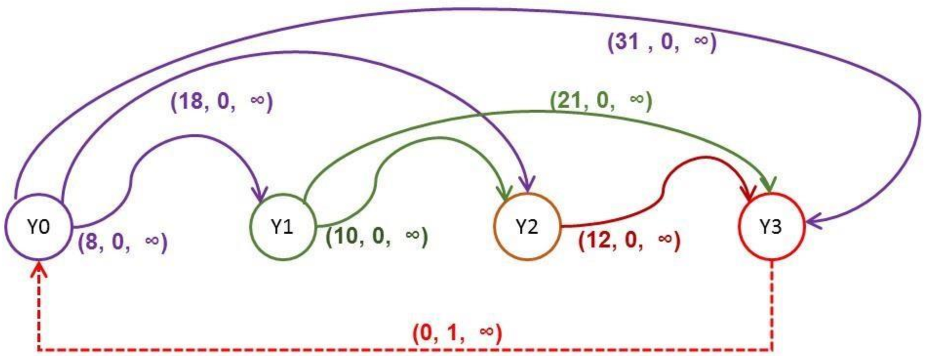
   Set 1:

   $$\begin{matrix} -1 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{matrix}$$

   Set 2:

$$\begin{matrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{matrix}$$

Question 2: Airport Tractor



Answer:
0-1, 1-3
Total Cost: $29,000

Question 3: Christmas Party and Summer Camp
Q3. A

a. Assumptions:

1. The Administrative Assistant (Jane)'s list of compatible pairs includes more than 15 pairs and includes all employees (i.e. each person is listed at least once).
2. Each person's compatible pairs can be organized in order of preference.

b. Solution:

1. Create a 30 x 30 matrix (Each row presents a unique employee)
2. List each's person's compatible pairs by order of preference (1 being the first choice, 2 being second and so forth).
3. Select the lowest number from each row such that there is no other number in that row and the column (where the lowest number was found). If there is a lower number in that column, select the lower number and drop the earlier one.
4. This will result in a total of 15 entries in the 30 X 30 matrix and most compatible pairing.
5. Integer Program Formulation:

   Let $x_{i,j}$ binary variable where, i = 1, 2, . . ., 30 and j = 1, 2, . . ., 30 and matrix P[i, j] be the preference matrix where I = 1, 2, ….., 30 and j = 1, 2, ….., 30 (as described in step 2).
   Objective Function: minimize: $sum \sum^{30}{}_{i=1} \sum^{30}{}_{j=1} Pair[i,j] * x[i,j]$ Constraints:

   $\sum^{30}{}_{j=1} x[i,j] \leq 1$

   $\sum^{30}{}_{i=1} x[i,j] \leq 1$

6. This will result in x[i, j] matrix with only 15 of the total 900 variables to have a value of 1.

Q. 3 b: **Given:**

1. 2 groups of 10 children each.
2. Each child has ranked children in other group from 1 to 10 (10 being most preferred and 1 being least preferred).
3. The task is to pair these 2 groups for an outing such that to maximize sum of weights

Solution:

This is classical 'Stable Marriage' problem except for the stability testing part. We present a solution that also includes 'Stability' testing.

Stability: in simplest form this means that without stability consideration, a pair with scores of 9 and 1 will rank the same as pair with score of 5 and 5. However, with Stability constraint, a pair with scores of 9 and 1 will rank much lower than a pair with scores 5 and 5.

Procedure:

- Create 2 matrices size 10 x 10 reflecting rankings for each group.
- Let x[I, j] be a binary variable where i = 1 to 10 and j = 1 to 10.
- Objective function: maximize (F[i,j] * x [i, j] + A[i, j] * x[i, j])

Reference: Ref. DOI: 10.1007/978-3-643-03261-6_1, Bistarelli, Stefano, Santini, Francesco, July 2009

AMPL Code:

Ref. DOI: 10.1007/978-3-643-03261-6_1, Bistarelli, Stefano, Santini, Francesco, July 2009

```
set FRENCH;
set AMERICAN;

param F {i in FRENCH, j in AMERICAN};
param A {k in FRENCH, z in AMERICAN};

# Variables
var Pair {i in FRENCH, j in AMERICAN} binary;

# Objective
maximize weights: sum {i in FRENCH, j in AMERICAN}
                                        ((Pair[i,j] * F[i,j]) + (Pair[i,j] * A[i,j]));

# Constraints
subject to F_Pairs {i in FRENCH}: sum {j in AMERICAN} Pair[i,j] = 1; subject
to A_Pairs {j in AMERICAN}: sum {i in FRENCH} Pair[i,j] = 1;

subject to Stability {i in FRENCH, k in FRENCH, j in AMERICAN, z in AMERICAN:  (F[i,z]
        < F[i,j]) and (A[i,z] < A[k,z])}:Pair[i,j] + Pair[k,z] <= 1;

# Data
data hw2q3b.dat;
```

Solution **solve**;
display Pair;
(Even though we created a dummy data file, the model will not run due to limitations of the student version)

Q.3 c

a. As described in procedure above in part A, the constraint matrix (containing x[i, j] will have a total of 15 values that are non-zero (i.e. 1, one for each optimal pair, total 15 pairs). Each row will have at most one non-zero value and each column will have at most one non zero values.

$\begin{pmatrix} 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 0 \end{pmatrix}$: 30 x 30 matrix: Sum of each row ≤ 1, sum of each column ≤ 1 (15 rows and 15 columns will not have any non-zero values).

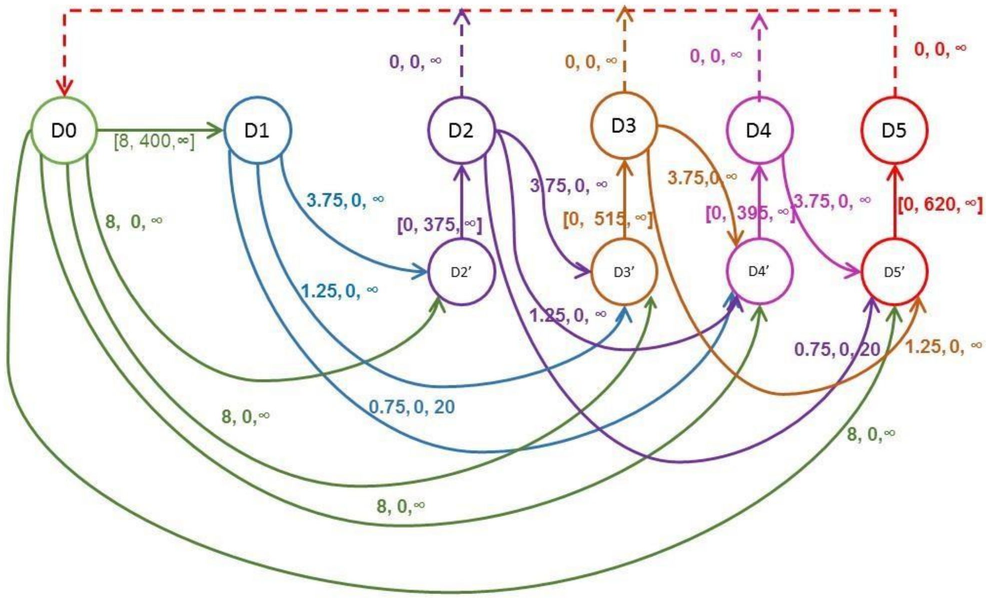
TU theorem Test:
Condition 1: All entries are 0, 1, or -1 (TRUE)
Condition 2: At most 2 non-zero entries appear in any column (TRUE)
Condition 3 A/B: Column has two entries of the same/different sign………: (TRUE)
Each column has at most one non-zero entry (condition does not apply)
All conditions are met. **The constraint matrix is TU**.

b. As described in procedure above in part B, the constraint matrix (containing x[i, j] will have a total of 10 values that are non-zero (i.e. 1, one for each optimal pair of American and French child, total 10 pairs). Each row will have exactly one non-zero value and each column will have at most one non zero values.

$\begin{pmatrix} 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 0 \end{pmatrix}$ 10 x 10 Matrix: Sum of each row = 1, sum of each column = 1

TU theorem Test:
Condition 1: All entries are 0, 1, or -1 (TRUE, All entries are 0 or 1)
Condition 2: At most 2 non-zero entries appear in any column (TRUE, Only one non-zero entry appear in each column)
Condition 3 A/B: Column has two entries of the same/different sign………: (TRUE)
Each column has exactly one non-zero entry (condition does not apply)
All conditions are met. **The constraint matrix is TU.**

Question 4: INFROMS Annual Meeting and Demand for Clean Table Cloths

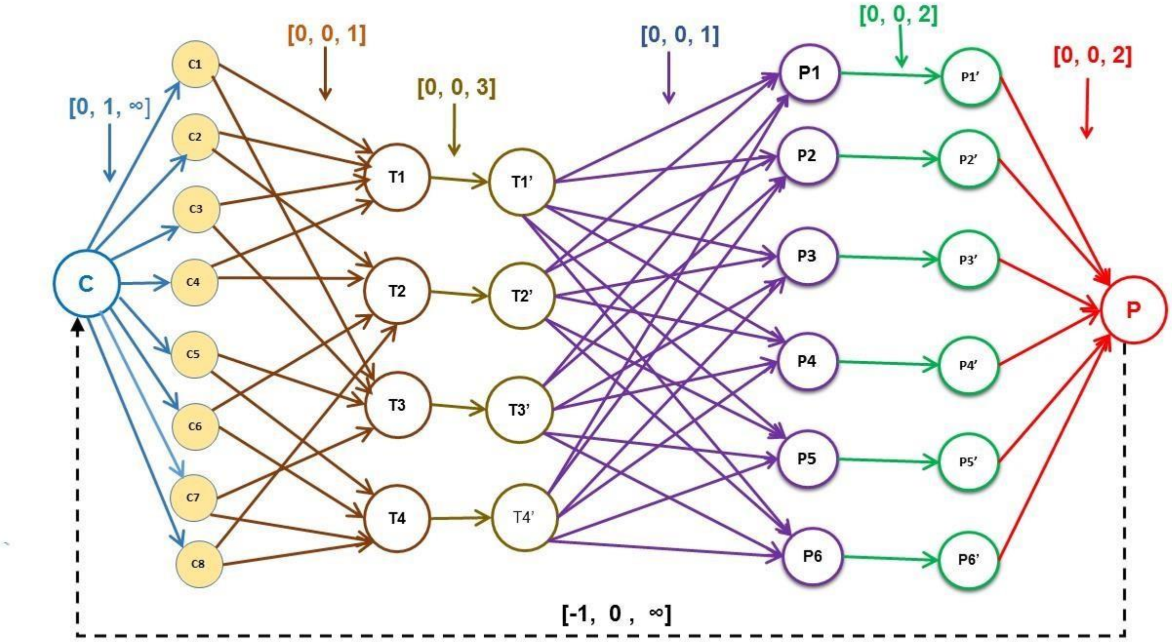


Answer: Total Purchase: 890 napkins

Total Cost: $ 9251.25

```
x [*,*] (flow of napkins between nodes)
:      D0    D1    D2    D2P   D3    D3P   D4    D4P   D5    D5P    := D0
.     400    .    375    .    115    .     0     .     0
D1     .     .     .     0     .    400    .     0     .     .
D2     0     .     .     .     .     0     .    375    .     0
D2P    .     .    375    .     .     .     .     .     .     .
D3     0     .     .     .     .     .     .     20    .    495
D3P    .     .     .     .    515    .     .     .     .     .
D4    270    .     .     .     .     .     .     .     .    125
D4P    .     .     .     .     .     .    395    .     .     .
D5    620    .     .     .     .     .     .     .     .     .
D5P    .     .     .     .     .     .     .     .    620    .
```

Question 5: Assigning Teachers and Classes

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C: Class (1 – 8)
T: Teacher (1 – 4)
P: Period (1 – 6)
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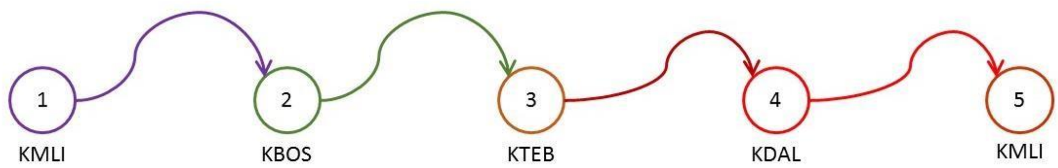
Answer:

		CLASS							
		C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
PERIOD	**P1**	T1					T2		
	P2								
	P3		T1						
	P4			T3	T1				
	P5					T4			
	P6							T3	T2

Question 6: Boomer Global Air Services



var: X[t]: Fuel purchased at node t (lbs)
var: B[t]: Binary variable (used to calculate cost associated with ramp fee at node t
var: Q[t]: Binary variable (used to incorporate 100 gal min. fuel requirement)
param: P[t]: No. of passengers taking off from node t
param: F[t]: Fuel consumed for trip t (lbs) : $t \rightarrow t+1$
param: C[t]: Fuel cost ($/lb) at node t
param: Y[t]: Ramp Fee at node t ($)
param: W[t]: Threshold weight of fuel to avoid ramp fees at node t

Solution:

Selection of Airplane:

The problem statement does not provide complete data like it has for CE750 for the other 2 (Gulfstream VE and Cessna CE680). Chase should compare all 3 aircrafts and all other criteria before selecting the aircraft or prepare cost estimate for all the aircraft and present it to her supervisor for approval.

(a) Formulate the problem and solve for an optimum fuel plan for Chase for the upcoming trip.

<u>Optimum Fuel Plan:</u>

Location	Number of Pounds of fuel purchased
KMLI (Starting)	6,000
KBOS	0
KTEB	2,010
KDAL	2,590
KMLI (Ending)	4,600

COST = $ 11,718.50

1. Compare your results with a no tankering solution :

No tankering refers to having fuel that is just sufficient to go to next location.

Location	Number of Pounds of fuel purchased
KMLI (Starting)	200
KBOS	2,000
KTEB	5,300
KDAL	3,100

KMLI (Ending)	4,600

COST = $ 15,008.80

2. What are the most important limitations of the model? How might these be addressed?

- Here fuel consumed from one location to the other location is fixed, but actually fuel consumption depends on number of passengers. Since, weight directly proportional to fuel consumed. This can be addressed, since we know the number of passengers travelling from one location to the other.
- Another limitation is that only one type of aircraft was analyzed for fuel cost in detail.
  Solution: Prepare similar cost estimates for other 2 aircrafts.
- There is no mention in the description that the one of the CE750 is available on the proposed date.
  Solution: Ascertain aircraft availability

(b) Suppose the BGAS department manager wished to modify the model to require that "if you buy any gas, you must buy at least 100 gallons".

Location	Number of Pounds of fuel purchased
KMLI (Starting)	6,000
KBOS	0
KTEB	2,010
KDAL	2,590
KMLI (Ending)	4,600

COST = $ 11,718.50

There is no change in cost & solution compared to optimum solution.