DSA 5113 : ASSIGNMENT # 1

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Question 1: SMULLYAN'S ISLAND REVISITED:

Inhabitants on Smullyan's Island are either always telling the truth or always lying. You meet three people, Gregor, Tywin, and Catelyn, who make the following statements, respectively:

Gregor: "Exactly one of us is telling the truth."

Tywin: "We are all lying."

Catelyn: "The other two are lying."

Truth Table:

	Assumptions			Verifying from statements			
S.No	Gregor	Tywin	Catelyn	Gregor	Tywin	Catelyn	Result
0	F	F	F	F	T	T	NO
1	F	F	T	T	F	T	NO
2	F	T	F	T	F	F	NO
3	F	T	T	F	F	F	NO
4	T	F	F	T	F	F	**YES**
5	T	F	T	F	F	F	NO
6	T	T	F	F	F	F	NO
7	T	T	T	F	F	F	NO

So, from the above truth table considering yes case we can conclude that,

Gregor is truth teller,

Tywin is liar &

Catelyn is also liar.

Question 2: Wildings and Humans:

Here is the investigation:

1. Jon (to Arya): Tell me about yourselves.

2. Arya: We are both insane.

3. Jon (to Sansa): Is that true?

4. Sansa: Of course not!

Sane Humans & Insane Wildings always say True statements.

Insane Humans & Sane Wildings always say False statements.

We have only 4 possibilities by following these 2 statements:

1. From the conversation of Arya & Sansa, we can understand that one of them is telling Truth & the other is telling False.
2. We know that one of them is Human & the other is wilding, so we can neglect cases where both of them are humans & both of them are wilding.

Assumptions		Possibilities				Verify from Statements		
A	**S**	**A**	**A**	**S**	**S**	**A**	**S**	**Result**
T	F	S	H	S	W	F	T	No
T	F	I	W	I	H	T	F	**Yes**
F	T	I	H	I	W	T	F	No
F	T	S	W	S	H	F	T	**Yes**

A – Arya S – Sansa T – True F – False
H – Human W – Wilding S – Sane I – Insane

From the result, considering Yes cases, we can conclude that Arya was Wilding & Sansa was Human.

Question 3: Working Capital Management:

a. Decision variables:

Here, we assumed that interest at the end of 6 th month needs to be maximum. So, decision variables are amounts which are to be invested at the start of each month in each of the 3 investment plans. But we cannot invest in the 3 month investment plan for the 5th and the 6 th months. Similarly, we cannot invest in the 6 month investment plan other than in the 1st month. The decision variable is represented by **i {M,P} >=0,** which is the investment in month M in the plan P**.** Here, M and P represent the following:
P = 1,3,6 (Investment plans)
M = 1,2,3,4,5,6 (number of months)
Hence, we get 11 decision variables: i {1,1}, i {1,3}, i {1,6}, i {2,1}, i {2,3}, i {3,1}, i {3,3}, i{4,1},i{4,3}, i {5,1}, i {6,1}.

b. Clearly state the objective and objective function:

Objective: We need to maximize the interest income (Return at the end of 6th month) by investing money in different plans at the start of each month while keeping in view of monthly expenditure.

Objective function: maximize final_return: 1.005*i[6,1]+1.021*i[4,3]+1.035*i[1,6];
It is equal to sum of (return of 6 month investment invested in 1st month, return of 3 month investment invested in 4th month, return of 1 month investment invested in 6th month).

c. **Clearly define and determine a functional form for each constraint:**
We equated the sum of investments made in month M to the money present for investment after adding the returns from previous investments and subtracting or adding the monthly expenditures.
subject to month1_constraints: sum{p in P} i[1,p] =300000+e[1];
subject to month2_constraints: sum{p in P} i[2,p] =e[2]+ir[1]*i[1,1];
subject to month3_constraints: sum{p in P} i[3,p] =e[3]+ir[1]*i[2,1];
subject to month4_constraints: sum{p in P} i[4,p] =e[4]+ir[1]*i[3,1]+ir[3]*i[1,3];
subject to month5_constraints: sum{p in P} i[5,p] =e[5]+ir[1]*i[4,1]+ir[3]*i[2,3];
subject to month6_constraints: sum{p in P} i[6,p] =e[6]+ir[1]*i[5,1]+ir[3]*i[3,3];
where e[i] represents expenditure for I th month &
ir{P} represents interest rates for plan P.

(d) Solve the problem using AMPL:

Investment	**Month 1**	**Month 2**	**Month 3**	**Month 4**	**Month 5**	**Month 6**
1 month investment	**50788.3**	**22885.6**	**0**	**0**	**0**	**13000**
3 month investment	**199212**	**40156.7**	**0**	**223395**	**NA**	**NA**
6 month investment	**0**	**NA**	**NA**	**NA**	**NA**	**NA**

<u>Question 4 : Seasonal Demand – Sunny Sandals:</u>

1. **Assumptions:**
We modeled assuming demand patters are stable year to year.
2. **Decision Variables:**
P{I} #Sandals Produced in Season i
C{I} #Inverntory stored at end of season i

3. **Objective & Objective function:**
<u>Objective :</u> Our objective is to minimize inventory costs, so as to meet demand in each season.

<u>Objective function</u>:
minimize Inventory_Costs: sum{i in I} 0.15 * C[i];
where C[i] refers to inventory stored at the end of season i.

4. **Constraints:**
subject to Demand {i in I} : P[i]+C[prev(i)] = D[i]+C[i];
subject to Production {i in I} : P[i] <= 1200;

5. **Result:**

Season	Production	Production per season	Inventory
1	2800	1200	0
2	500	650	150
3	100	1200	1250
4	850	1200	1600

<u>Question 5: Golden Canning Co</u>

a. We know that :

- Number of grade A tomatoes are 600,000 with a quality rating of 9.
- Number of grade B tomatoes are 2,400,000 with a quality rating of 5.
- Whole tomatoes have a minimum rating of 8, so in order to maximize the whole tomatoes production you need to maintain a quality of 8.
- Let, B represents the B grade tomatoes added to 600,000 grade A tomatoes in order to make quality 8.
- So,
  9 *(600,000) + 5*(B) = 8*(600,000+B)
  3*B = 600,000
  B = 200,000
- So, Whole tomato production is limited to: 600,000 + B = 800,000

b. <u>Tomato costs:</u>

- Cost of grade A tomatoes (rating 9) per pound is 9.32 cents per pound.
- Cost for 1 rating is 9.32 cents/9 = 1.036 cents (Since cost proportional to quality).
- Cost of grade B tomatoes (rating 5) per pound is 5.18 cents per pound.
- Cost for 1 rating is 5.18 cents/5 = 1.036 cents (Since cost proportional to quality).

- Tomato cost of whole canned tomatoes:
  = 1.036 * quality rating * no of pounds per case
  = 1.036*8*18 = 149.18 cents
  = 1.49$.

- Tomato cost of Tomato Juice:
  = 1.036 * quality rating* no of pounds per case
  = 1.036*6*20 = 124.32 cents
  = 1.24$.

- Tomato cost of Tomato paste:
  = 1.036 * quality rating* no of pounds per case

= 1.036*5*25 = 129.5 cents
= 1.30$

<u>Tomato count:</u>

- From table 3 we can understand that Tomato paste yields higher profit & Whole tomato gives loss.
- So, she gave priority to manufacture 80,000 cases (equal to demand forecast) of tomato paste, equals to 80,000 * 25 pounds = 2,000,000 & all tomatoes used for paste are grade B, since quality of 5 is sufficient.
- So, the remaining tomatoes are 400,000 of grade B & 600,000 of grade B. All these are used to manufacture tomato juice since Whole tomatoes gives loss.

<u>Wrong with reasoning</u>:

- Her basis of of pricing the product is , price directly proportional to quality.
- So, price = 1.036 * quality rating. (She has normalized cost based on quality)
- Quality of tomato juice she obtained from 600,000 grade A & 400,000 grade B is:
  Rating = 9*(600,000) + 5*(400,000) = Qj * (1,000,000)
  Qj = 7.4 (Quality of Juice).
- But, her pricing is based, considering Quality of juice as 6.0, which is against her reasoning. So, her pricing could be greater than the price which she calculated at threshold rating.
- And the other one is she didn't calculate Less Allocated OHD which Thomas has calculated. So, one of them may be wrong in this aspect.

(c) Ignoring for the moment the chance to buy additional A-grade tomatoes. Formulate the production question as a LP problem, **solve with AMPL**, or an equivalent software package, and answer the following questions.

Here I have modelled this problem by considering less allocated OHD (removing from contribution to profit).

i. How much whole, juice, and paste should be made?

From ample results:

Whole Tomatoes	38888.9 cases
Tomato Juice	15000 cases
Tomato Paste	80000 cases

ii. What is the contribution to profit?

Profit obtained is 12916.66667$

iii. Are there any tomatoes left over? If so, of what grade?

`There are no tomatoes left over.`

iv. What is the average quality point count of whole, juice, and paste?

Whole Tomatoes	**8.0**
Tomato Juice	**6.0**
Tomato Paste	**5.0**

v. What would be the worth of one additional pound of A-grade tomatoes?

The worth of one additional pound of A-grade tomatoes is `0.07225`$ (Shadow Price).

vi. Should Golden Canning Co buy the extra 80,000 pounds of A-grade tomatoes at the offered price?

No, they need not buy because the additional profit produced by 1 pound of grade A is less than 8.5 cents per pound.

(d) For the following questions, assume that the 80,000 pounds of A-grade is not in the formulation.

i. What product mix would result if Thomas's contribution figures were used? Is Thomas's contribution more or less than the contribution to your (earlier) Part (c) solution? Why the difference?

Thomas contribution mix was same as what is done in part (c)

ii. Alternatively, what product mix would result if Bollman's profit figures were used? What would the profit be? Is this more or less than you found in Part (c)?

<u>Product mix</u> :

Whole Tomatoes	0 cases
Tomato Juice	50000 cases
Tomato Paste	80000 cases

- Profit is 48000 $.
- Profit obtained by Bollman is greater than in part(c).

iii. Suppose an unlimited supply of A-grade tomatoes were available at $0.085 per lb. How much should Golden Canning Co buy? What would be the product mix?

For this part, I considered profit for each case as mentioned by Thomas, in table 2.

- The worth of one additional pound of A-grade tomatoes is -0.01275, we need to buy 1 pound of grade A tomatoes at 0.085$. so, we incur loss if we buy extra grade A tomatoes. So, we will not buy any extra grade A tomatoes.

Product mix:

Whole tomatoes	38888.9 cases
Tomato juice	15000 cases
Tomato Paste	80000 cases