Question 1: Mr. X

Assume a continuous decision variable 0 <=x <= 1000. Write constraint(s) to meet the following requirements. You can add any auxiliary variables as you like, but you must define them. Make sure your constraints are as “tight" as possible.

(a) x = 0 or x >=10

Variable:

Y is a binary variable. {Y = 0 for x = 0, Y = 1 for x >= 10}

Constraints:

x <= 1000*Y

x >= 10*Y

(b) 0 <= x <= 15 or 30 <= x <= 100

Variables:

Y1, Y2 are binary variables.

Constraints:

Y1 + Y2 =1

x <= 15*Y1 + 100*Y2

x >= 30*Y2

(c) x belongs to {12, 12.3, 87, 99.1}

Set I = {1,2,3,4}

x belongs to K[i], for i in set I; (K[i] are the keys)

Variables:

Y[i] is a binary variable, for i in set I

Constraints:

{sum of i in I} Y[i] =1

{sum of i in I} Y[i]* K[i] =x

Question 2: Valid inequalities

Identify two valid inequalities for the following mathematical program. Explain how each is a valid inequality.

min 14x1 + 2x2 + 11x3 + 9x4 + x5

s.t. 3x1 - 4x2 + 2x3 - 3x4 + x5 <= -2

xi belongs to {0,1}, for i = 1,2,3,4,5

Two valid inequalities are:

1. {sum of xi} for i = 1,2,3,4,5 > 0

   Reasoning: if {sum of xi} for i = 1,2,3,4,5 = 0; then 0 <= -2; condition will not be satisfied.

2. {sum of xi} for i = 1,2,3,4,5 < 5

   Reasoning: if {sum of xi} for i = 1,2,3,4,5 = 5; then -1 <= -2; condition will not be satisfied.

Question 3: Gizmos and Gadgets:

Decision Variables:

X1 (Integer) = Number of widgets bought from WII
X2 (Integer) = Number of widgets bought from WRS
X3 (Integer) = Number of widgets bought from WU
X4 (Integer) = Number of widgets bought from WOW

d1 (Integer) = Number of widgets bought from WOW at 5.50$
d2 (Integer) = Number of widgets bought from WOW at 3.50$
d3 (Integer) = Number of widgets bought from WOW at 2.00$

y1 (binary) = Used to calculate d1, d2
y2 (binary) = Used to calculate d2, d3

B1 (binary) = B1=0 if X2 = 0 & B1=1 if X2 >= 7500
B2 (binary) = B2=0 if X3 = 0 & B2=1 if X3 > 0

Objective Function:

Minimize Cost: 4.25*X1 + 3.15*X2 + (1.90*X3+15000) *B2 + 5.50*d1 + 3.50*d2 + 2*d3

Constraints:

```
subject to WII: X1 <= 10000;
subject to WRS: X2 <= 15000;
subject to WU: X3 <= 9000;
subject to WOW: X4 <= 25000;

subject to demand: X1+X2+X3+X4 = 17000; #17k,18k,19k,28k,32k

subject to B1_1: X2 <= 15000*B1;
subject to B1_2: X2 >= 7500*B1;

subject to B2_1: X3 <= 9000*B2;
subject to B2_2: X3 >= B2;

subject to d1_delta1: 5000*y1 <= d1;
subject to d1_delta2: d1 <= 5000;

subject to d2_delta1: 7500*y2 <= d2;
subject to d2_delta2: d2 <= 7500*y1;

subject to d3_delta1: 0 <= d3;
subject to d3_delta2: d3 <= 12500*y2;
subject to WOW_SUM: X4 = d1+d2+d3;
```

RESULTS:

Demand	WII	WRS	WU	WOW	COST
17000	2000	15000	0	0	$ 55750
18000	3000	15000	0	0	$ 60000
19000	0	10000	9000	0	$ 63600
28000	3000	0	0	25000	$ 91500
32000	0	7500	0	24500	$ 101375

Question 4: Facility Location:

a. Create a valid AMPL model file for this problem (include the model in your submission document as well as a separate attachment.)

```
Facility Location Problem (Minimizing Worst Case distance)

sets
set I;          #Number of districts
set J;          #Number of Fire places

parameters
param p{I};       #population in each district
param d{I,J};     #distance in km from district to site
param B;          #Budget
param f{J};       #fixed cost of site
param c{J};       #variable cost associated with site

decision variables

var X{I,J} binary;    # 1 if District I served by Site J, else 0
var Y{J} binary;      # 1 if fire station in site J, else 0
var S{J} integer;     # Total population served by each site
var Z binary;         # To select site 1 & 2 or site 3 & 4
var D;                # Maximum worst case Distance

Objective function:

minimize distances: D;

constraints

subject to exactly_1_firehouse {i in I}: sum{j in J} X[i,j] = 1;
subject to no_district_assigned_unused_site {j in J}: sum{i in I} X[i,j] <= Y[j] *45; #No district is assigned to a site where there is no fire station
```

subject to selection1: Y[1] + Y[2] >= 2*Z;
subject to selection2: Y[3] + Y[4] >= 2*(1-Z);
subject to population_each_site {j **in** J}: **sum**{i **in** I} p[i]*X[i,j] = S[j];
subject to Budget: **sum**{j **in** J} (c[j]*S[j] + f[j]*Y[j]) <= B;
subject to max_distance {i **in** I}: **sum**{j **in** J} d[i,j]* X[i,j] <= D;

b. Use the NEOS server:

i. Solve the provided instance of the facility location problem:

1.Determine optimal solution and objective value

Objective Value = 31.3 (Worst case Distance)

Objective Solution:

```
Y [*] :=
 1 1    4 1    7 0   10 0   13 0   16 0   19 1   22 1   25 0
 2 0    5 1    8 0   11 0   14 0   17 1   20 0   23 0
 3 1    6 0    9 0   12 0   15 0   18 0   21 0   24 0
```

```
x [*,*]
:      1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19 :=
1      1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
2      0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
3      0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
4      0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
5      0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
6      0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0
7      0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0
8      0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
9      0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
10     0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
11     0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
12     0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0
13     1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
14     0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
15     0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
16     1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
17     0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
18     0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
19     0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
20     1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
21     1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
22     0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
23     0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
24     0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
25     1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
26     1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
27     1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
28     0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
29     0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
30     0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
31     1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
32     0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
33     0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
```

34	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
37	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
42	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

:	20	21	22	23	24	25	:=
1	0	0	0	0	0	0	
2	0	0	1	0	0	0	
3	0	0	0	0	0	0	
4	0	0	0	0	0	0	
5	0	0	0	0	0	0	
6	0	0	0	0	0	0	
7	0	0	0	0	0	0	
8	0	0	0	0	0	0	
9	0	0	0	0	0	0	
10	0	0	0	0	0	0	
11	0	0	0	0	0	0	
12	0	0	0	0	0	0	
13	0	0	0	0	0	0	
14	0	0	0	0	0	0	
15	0	0	0	0	0	0	
16	0	0	0	0	0	0	
17	0	0	0	0	0	0	
18	0	0	1	0	0	0	
19	0	0	0	0	0	0	
20	0	0	0	0	0	0	
21	0	0	0	0	0	0	
22	0	0	0	0	0	0	
23	0	0	0	0	0	0	
24	0	0	1	0	0	0	
25	0	0	0	0	0	0	
26	0	0	0	0	0	0	
27	0	0	0	0	0	0	
28	0	0	0	0	0	0	
29	0	0	0	0	0	0	
30	0	0	0	0	0	0	
31	0	0	0	0	0	0	
32	0	0	0	0	0	0	
33	0	0	0	0	0	0	
34	0	0	0	0	0	0	
35	0	0	0	0	0	0	
36	0	0	0	0	0	0	
37	0	0	0	0	0	0	
38	0	0	0	0	0	0	
39	0	0	1	0	0	0	
40	0	0	0	0	0	0	
41	0	0	0	0	0	0	
42	0	0	0	0	0	0	
43	0	0	0	0	0	0	
44	0	0	0	0	0	0	
45	0	0	0	0	0	0	

Z = 0

2. Determine how much of the budget was used

Budget used is: $ 14990700

3. Determine total solution time; determine the total number of branch-and-bound nodes used in the algorithm

Solution Time = 3.89241 seconds

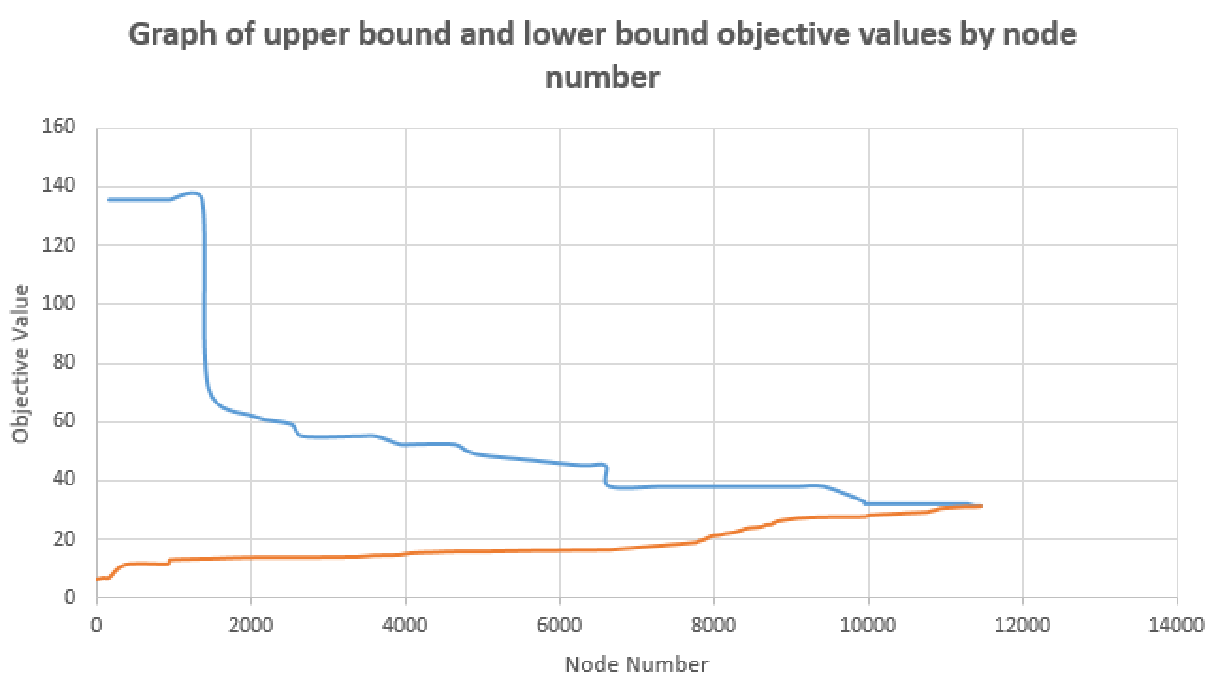
branch-and-bound nodes = 11,452

4. What was the root relaxation value? What and when (node number) was the the first incumbent value found?

Root Relaxation Value: 6.1359

Found first incumbent of value 135.800000 after 0.06 sec at Node 170.

ii. Create a graph of upper bound and lower bound objective values by node number (if there are too many nodes, you collect data and plot values for every k nodes, where k > 1 according to the size of the node tree.)



- From the above graph, we can understand that gap between upper and lower bound decreases and graph converges to an appropriate value as Node number increases.
- We can also infer that, there is sharp decrease in gap between upper and lower bounds at Incumbent nodes.

iii. Turn the cutting plane options on (mipcuts and splitcuts) and re-solve. Describe how this affeects the enumeration tree and solution time.

```
Solution Time = 4.48432 seconds
branch-and-bound nodes = 6,123
```

By turning the cutting plane options on, Solution time has increased from 3.89241 seconds to 4.48432, which is about 20% increase in time, besides this the size of the enumeration tree has decreased as its branch-and-bound nodes decreases from 11,452 to 6,123. We can also note that there is no change in the objective value.

C. Reformulate the problem to minimize the average distance (instead of minimizing the worst-case distance) and resolve. Note: the original problem is called the p-center problem, the new variation is called the p-median problem. How does your solution change?

Objective Value = 10.9867 (Average Distance)

Objective Solution:

```
Y [*] :=
 1 1    4 1    7 0   10 0   13 0   16 0   19 0   22 1   25 0
 2 0    5 1    8 0   11 0   14 0   17 1   20 0   23 0
 3 1    6 0    9 0   12 1   15 0   18 0   21 1   24 1
```

X [*,*]

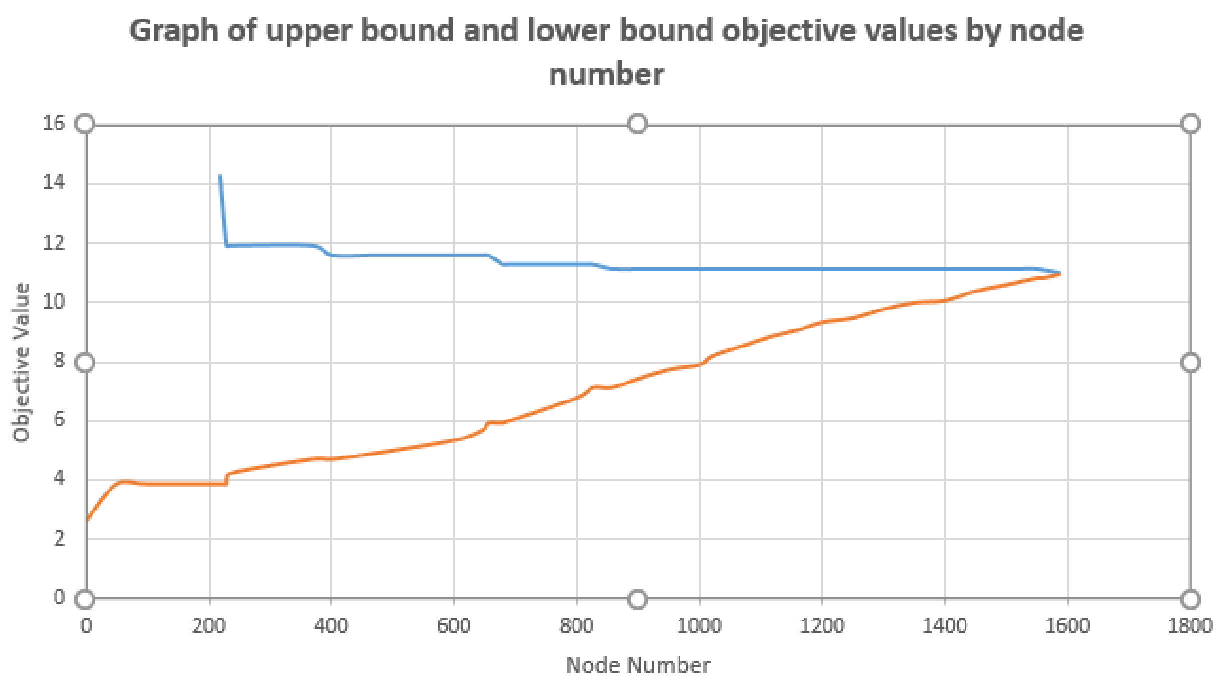
:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19 :=
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
6	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
9	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
16	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
20	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

29	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
31	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
34	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
37	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

:	20	21	22	23	24	25	:=
1	0	0	0	0	0	0	
2	0	0	0	0	1	0	
3	0	0	0	0	0	0	
4	0	0	0	0	0	0	
5	0	0	0	0	0	0	
6	0	0	0	0	0	0	
7	0	0	0	0	0	0	
8	0	0	0	0	0	0	
9	0	0	0	0	0	0	
10	0	1	0	0	0	0	
11	0	1	0	0	0	0	
12	0	0	0	0	0	0	
13	0	0	0	0	0	0	
14	0	0	0	0	0	0	
15	0	0	0	0	0	0	
16	0	0	0	0	0	0	
17	0	0	0	0	0	0	
18	0	1	0	0	0	0	
19	0	0	0	0	0	0	
20	0	0	0	0	0	0	
21	0	0	0	0	0	0	
22	0	0	0	0	0	0	
23	0	0	0	0	0	0	
24	0	0	1	0	0	0	
25	0	0	0	0	1	0	
26	0	0	0	0	1	0	
27	0	0	1	0	0	0	
28	0	0	0	0	0	0	
29	0	0	0	0	0	0	
30	0	0	0	0	0	0	
31	0	0	0	0	0	0	
32	0	0	0	0	0	0	
33	0	0	0	0	0	0	
34	0	0	0	0	0	0	
35	0	1	0	0	0	0	
36	0	0	0	0	0	0	
37	0	0	0	0	0	0	
38	0	0	0	0	0	0	
39	0	0	1	0	0	0	
40	0	0	0	0	0	0	
41	0	0	0	0	1	0	
42	0	0	0	0	0	0	
43	0	0	0	0	0	0	
44	0	0	0	0	0	0	

45 0 0 0 0 0 0

- Z = 0
- Budget = $ 14982300
- branch-and-bound nodes = 1,587
- Solution Time = 0.638903 seconds
- Root Relaxation Value: 2.6181
- Found first incumbent of value 14.308889 after 0.05 sec at Node 219 (This can be observed from the graph)



- For the p center problem, the objective value (distance) is 31.3, but for p-median problem the objective value (distance) is 10.9867, which results in change of X, Y values. We can notice that, there is significant change in the objective values for these two cases, but there is no considerable change in the budget.
- Solution time has decreased from 3.89241 seconds to 0.638903 seconds, since number of branch-and-bounds decreased from 11,452 to 1,587.