Software Tools-Dec Support with Python

ISE-5123

Project Report

On

Facility Location using P-median Optimization

by

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Facility Location using P-median Optimization

Abstract: *Locating and choosing the optimal site for hospitals is important as they are vital for any populated area. The main objective of this project is to find the best possible set of locations for a network of hospitals. We selected three types of hospitals i.e. small, medium, large hospitals with varying budgets for each of them. After looking at various methods to do this we selected the P-median optimization method. By the end of the project, we select various locations based on this method at optimal locations while balancing the budget, distance and various other factors. We came up with two small, one medium and one large hospital.*

Introduction and Importance of the problem.

When we are planning to provide services to a community, we need to take many things into consideration. The location of these said services plays an important role in defining their effectiveness at serving these communities. The aim of any kind of service, whether profit oriented or service oriented is to reach the maximum number of people. This problem aims to find five different locations among seven different locations available for building hospitals surrounding Oklahoma City. People in general try to find the locations that are available at low cost and easy availability. But is that the right way to select the location? There are many factors to be considered while setting up a facility based on the purpose it is meant. To list a few

- The population of each area
- The noise levels in that area
- The connectivity of that area to the roads
- The climate conditions
- The nearness to the industrial areas
- The economic status of the people the facility is meant to serve most
- Risk factors like, natural disasters in that area.
- The capital available.
- The nearness to the supporting facilities
- Logistics and support

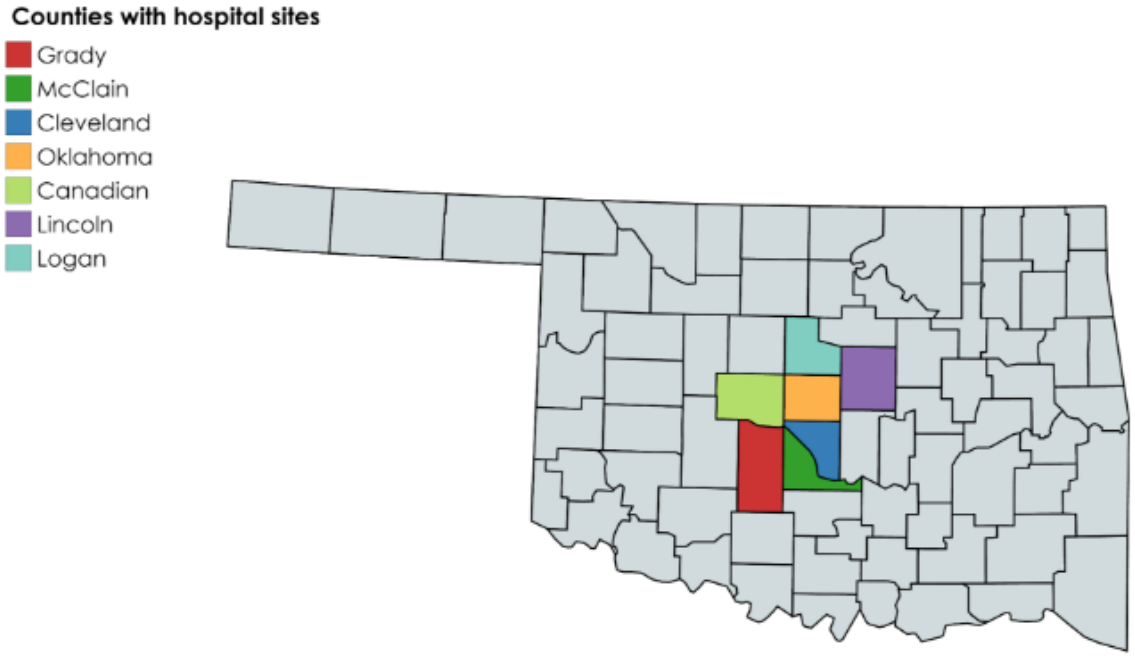
This problem can be extended to Shopping centers, schools, fire stations, warehouses, Grocery stores, restaurants and similar businesses. Various methods have been proposed to build these models. We consider a model called discrete location model. We consider a specific kind of discrete location model called P-median problem. This problem in literature is popularly known as facility location problem.

In this problem, we aim to decrease the sum of distance between the demand location and the supply location(facility). This problem in general is very important because the hospitals are meant to serve the population and that too as early as possible. Hospitals come under emergency services. The time it takes to reach the hospital location is very important. It is matter or life and

death. So, out of all the above listed factors we considered for the facility location the (demand-weighted) average distance between the facility and demand location is of prime importance. we formulate the above problem as integer linear problem.

Problem definition

In our problem, we have seven different counties. They are Canadian county with a population of 126123, Cleveland county with a population of 269340, Grady county with a population of 53685, Lincoln county with a population of 34351, Logan county with a population of 44422, McClain county with a population of 36511, Oklahoma county with a population of 755245.



We need to build five different hospitals in these counties. The hospitals are categorized into three distinct categories.

- **Small Hospital**

A small hospital requires a minimum of 4.91 acres out of which construction area cannot exceed 70%. The construction cost is calculated from the construction area. The bed cost is calculated by multiplying the cost of each bed by number of beds i.e. 50. In the case of small hospital. The total cost is calculated by adding the construction cost, land cost and bed cost.

- **Medium Hospital**

A medium hospital requires a minimum of 16.26 acres out of which construction area cannot exceed 60%. The bed cost is calculated by multiplying the cost of each bed by number of beds i.e. 250 in the case of small hospital.

- **Large Hospital**

A medium hospital requires a minimum of 36.13 acres out of which construction area cannot exceed 45%. The bed cost is calculated by multiplying the cost of each bed by number of beds i.e. 375 in the case of small hospital.

	Small	**Medium**	**Large**
Population Served	Less than 100,000	100,000 - 500,000	> 500,000
No of Beds	50	250	375
Total Land (in Acres)	4.916043557	16.2610958	36.13576844
Total Land Cost	491604.3557	1626109.58	3613576.844
Construction Area Cannot Exceed (%)	70	60	45
Construction Area (Acres)	3.44123049	9.75665748	14.2561983
Construction Area (Sq. ft.)	149,900	425,000	621,000
Total Construction Cost	40173200	113900000	166428000
Total bed cost	35000000	175000000	262500000
Total Cost	75664804.36	290526109.6	432541576.8

Figure 1: Various types of hospitals

We have taken the land cost per acre in the above counties as 100000$ and construction cost is 268$ per sq. ft. The variable cost is 700000$ per bed which includes all the equipment attached to the bed. The details are given in the Table below.

County	**Population**	**Land Cost Per Acre**	**Construction Cost Per foot**	**Cost Per bed**
Canadian	126,123	100000	268	700000
Cleveland	269,340	100000	268	700000
Grady	53,685	100000	268	700000
Lincoln	34,351	100000	268	700000
Logan	44,422	100000	268	700000
McClain	36,511	100000	268	700000
Oklahoma County	755,245	100000	268	700000

Figure 2: Various costs in different counties

We have chosen a location in each of the county and calculated the distance in miles between the center of the county to the selected location and as well as the distance between each county using google maps and is tabulated below

	(Hospital Canadian) 1206 Sunset Drive, El Reno	(Hospital_Cleeveland) 909 Alameda Street	(Hospital_Glady) Ok-9, Blanchard	(Hospital_Lincoln) 810 Hickory St	(Hospital_Logan)5308 S Sooner Road	(Hospital_McClain) 404 S Main Street	(Hospital_Oklahoma)100 N Lee Avenue
Canadian	6.4	57.6	57.2	70.7	62.4	45.9	34.4
Cleveland	51.7	6.8	27	46.1	48.6	24.4	24.1
Grady	40.3	44.8	12.6	78.6	71.2	23.9	43.2
Lincoln	82	62.2	80.4	20	56.9	69.1	53
Logan	53.7	62.4	64.2	55.5	18	52.9	44.9
McClain	60.9	19.8	18.4	65.2	57.8	21.2	33.3
Oklahoma	46.8	35.2	47.4	20.3	19.7	36.1	17.8

Figure 3: Distance matrix across various counties and hospital locations

PROBLEM FORMULATION

We formulate the above problem as integer linear program as we need integer decision variables to decide on which facility location to choose. The following are the available sets for the problem

- The set of available counties:
  I = {"canadian","cleveland","grady","lincoln","logan","McClain","Oklahoma"}
- The set of hospital sites in seven counties:
  J = {"Hospital_Canadian", "Hospital_Cleveland", "Hospital_Grady", "Hospital_Lincoln", "Hospital_Logan", "Hospital_McCalin", "Hospital_Oklahoma"}
- The set of hospital types:
  k = {'small','medium','large'}

We considered the following parameters while formulating the problem:

- **arc_dist (d) :** The distance which represents the distance between the counties and the hospital sites
- **pop_county :** The population of each county
- **budget:** The total available budget for hospitals
- **total_cost:** The cost of construction of each type of hospital

DECISION VARIABLES:

This problem has **190 decision variables**:

- $\boldsymbol{x_{ijk}}$**:** A Binary decision variable which represents whether a particular county i is being served by a hospital j of type k.
  Representation: $\boldsymbol{x_{ijk}}$: for i in county, j in hospital, k in hospital type.
  The problem has 7x7x3 = 147 such variables

- $\boldsymbol{y_{jk}}$**:** A binary variable that represents whether a particular site j has been chosen to build a hospital of type k.
  Representation: $\boldsymbol{y_{jk}}$**:** for j in hospital, k in hospital type
  This problem has 21 such variables.

- $\boldsymbol{p_{jk}}$**:** Integer variable that indicates the population served by a hospital of each category (small, medium, large).
  Representation: $\boldsymbol{p_{jk}}$**:** for j in hospital, k in hospital type
  This problem has 21 such variables
- **D:** A continuous variable which represents total sum of distances between each county and the hospital associated with that particular county.
  Representation: D
  This problem has one such variable.

Objective function:

- Minimize the Distance variable "D".

Constraints:

- This project has total of **80 constraints.**
- The below constraint states that there should be exactly one hospital allocated for each county. There are seven such constraints (There is a constraint for each county)

For i in county $\sum_1^j \sum_1^k X_{ijk}$ =1

- The below constraint says that the weight distance should be at most the value of "D". Here D is the sum of distance between each county and its allocated hospital.

$$\Sigma_1^i \sum_1^j \sum_1^k X_{ijk} * d_{ij} <= D$$

- The above constraint takes care that the total cost for construction for the hospitals should not cross the available budget.

$$\sum_1^j \sum_1^k y_{jk} * C_k <= \text{budget}$$

- The maximum of 5 hospitals should be constructed in 7 counties.

$$\sum_1^j \sum_1^k y_{jk} \leq 5$$

- No county should be assigned to a site which has not been chosen for a hospital to be built.

$$\sum_1^i X_{ijk} \leq y_{jk} * 7$$

There are total of 7 sites, each site can have any type of hospital. So, this puts to 21 constraints.

- Population served by each site.

$$\sum_1^i x_{ijk} * p_i = p_{jk}$$

P[j][k] has refers to population associated to hospital j of type k. So, this puts to 21 constraints.

- Not more than one hospital located at the same site. There

$$\sum_1^k y_{jk} \leq 1$$

We cannot assign, more than one type of hospital at the same location. We have 7 such constraints, one for each hospital.

- Type of hospital selected based on population served by each site.

$$0 \leq p_{j,small} \leq y_{j,small} * 100000$$

$$y_{j,medium} * 100000 \leq P_{j,medium} \leq y_{j,medium} * 500000$$

$$y_{j,large} * 500000 \leq P_{j,large}$$

If the population assigned to a site is less than 100,000 then Small hospital is being selected. If population assigned is between 100,000 to 500,000 then medium hospital is

being selected. If population assigned is more than 500,000 then large hospital is being built. We have 21 such constraints, 1 for each hospital and its type.

RESULTS:

- ·The five hospitals assigned to seven counties are:

S.No	Hospital	Hospital Type
1	Hospital Canadian	Medium
2	Hospital Cleveland	Large
3	Hospital Grady	Small
4	Hospital Lincoln	Small
5	Hospital Logan	Small

- Table below shows hospitals, hospital type assigned to each county along with the distance between each county and it's assigned hospital:

S.No	County	Associated Hospital	Hospital Type	Distance (Miles)
1	McClain	Hospital Grady	Small	18.4
2	Oklahoma	Hospital Cleveland	Large	35.2
3	Canadian	Hospital Canadian	Medium	6.4
4	Cleveland	Hospital Cleveland	Large	6.8
5	Grady	Hospital Grady	Small	12.6
6	Lincoln	Hospital Lincoln	Small	20.0
7	Logan	Hospital Logan	Small	18

- Minimum Distance is 117.4 miles (18.4+35.2+6.4+6.8+12.6+20+18). 117.4 miles is the sum of distance of each county to its associated hospital.
- Total Budget used is 432,541,576 $.

This formulation is very robust and can be implemented to all emergency services like fire station etc.

Sensitivity Analysis:

- For this problem, we mainly focused on minimizing the total distance rather than minimizing the budget. The budget used is nearly 432 million Dollars.
- we tried to minimize the budget below 400 million dollars but the solution became infeasible.
- We can also observe that the solution is more robust because our objective is not considering budget but to minimize distance.

CONCLUSION:

From the above results, we can notice that there are three small hospitals, one medium, one large hospital need to be built. Which indicates that more number of small hospitals are built within the same county. There by the hospital will be available close to the residents of the county. There by it decreases the total distance. We can also observe that a large hospital is being built in the Cleveland county, which is very close to Oklahoma County. In this facility location problem, Cleveland, Grady, Canadian, Lincoln, Logan have hospitals within the county. This instance can be observed even for fire stations. Fire stations are in general located very near to the densely-located places.