UNIT –I
MATRICES

1	a) Define rank of the matrix.	[L1][CO1]	**[2M]**
	b) Reduce the matrix A= $\begin{bmatrix}1&2&1\\-1&0&2\\2&1&-3\end{bmatrix}$ into Echelon form and find its rank?	[L3][CO1]	**[2M]**
	c) State Cauchy–Binet formulae.	[L1][CO1]	**[2M]**
	d) What is the Consistency and Inconsistency of system of linear equations?	[L1][CO1]	**[2M]**
	e) Solve by Gauss-Seidel method $x-2y=-3\,;\,2x+25y=15.$ [Only two iterations]	[L3][CO1]	**[2M]**
2	a) Reduce the matrix A= $\begin{bmatrix}\mathbf{2}&\mathbf{3}&\mathbf{5}&\mathbf{1}\\\mathbf{1}&\mathbf{3}&\mathbf{4}&\mathbf{5}\end{bmatrix}$ **1 2 3 2** into Echelon form and find its rank?	[L3][CO1]	**[5M]**
	b) Reduce the matrix A to normal form and hence find its rank A= $\begin{bmatrix}2&1&3&4\\0&3&4&1\\2&3&7&5\\2&5&11&6\end{bmatrix}$	[L3][CO1]	**[5M]**
3	**0** **2 3 1 1 6** a) If A= $\begin{bmatrix}\mathbf{1}&\mathbf{4}&\mathbf{2}\\\mathbf{0}&\mathbf{1}&\mathbf{1}\end{bmatrix}$ and $\begin{bmatrix}\mathbf{3}&\mathbf{2}&\mathbf{1}\\\mathbf{1}&\mathbf{2}&\mathbf{3}\end{bmatrix}$ B= Verify that $\lvert\boldsymbol{AB}\rvert=\lvert\boldsymbol{A}\rvert.\lvert\boldsymbol{B}\rvert$	[L2][CO1]	**[5M]**
	b) Find whether the following equations are consistent if so solve them $x+y+2z=4\,;\,2x-y+3z=9\,;\,3x-y-z=2.$	[L3][CO1]	**[5M]**
4	a) Reduce the matrix $\begin{bmatrix}\mathbf{1}&\mathbf{2}&\mathbf{3}&\mathbf{-1}\\\mathbf{1}&\mathbf{0}&\mathbf{1}&\mathbf{1}\\\mathbf{0}&\mathbf{1}&\mathbf{1}&\mathbf{-1}\end{bmatrix}$ $\mathbf{-2}\ \ \mathbf{-1}\ \ \mathbf{-3}\ \ \mathbf{-1}$ A= into Echelon form and find its rank?	[L3][CO1]	**[5M]**
	b) Solve completely the system of equations $4x+2y+z+3w=0\,;\,6x+3y+4z+7w=0\,;\,2x+y+w=0.$	[L3][CO1]	**[5M]**
5	Find the inverse of the matrix $\begin{bmatrix}-1&-3&3&-1\\1&1&-1&0\\2&-5&2&-3\\-1&1&0&1\end{bmatrix}$ A= using Gauss-Jordan method.	[L3][CO1]	**[10M]**
6	a) Solve completely the system of equations x+2y+3z=0, 3x+4y+4z=0, 7x+10y+12z=0.	[L3][CO1]	**[5M]**
	b) Show that the equations $x+y+z=4\,;\,2x+5y-2z=3\,;\,x+7y-7z=5$ are not consistent.	[L2][CO1]	**[5M]**
7	Show that the only real number λ for which the system $x+2y+3z=\lambda x\,;3x+y+2z=\lambda y\,;2x+3y+z=\lambda z$ has non-zero solution is 6. and solve them when λ=6.	[L2][CO1]	**[10M]**
8	Solve the equations $3x+y+2z=3\,;\,2x-3y-z=-3\,;\,x+2y+z=4$ Using Gauss elimination method.	[L3][CO1]	**[10M]**

9	Express the following system in matrix form and solve by Gauss elimination method. $\mathbf{2x_1 + x_2 + 2x_3 + x_4 = 6}$; $\mathbf{6x_1 - 6x_2 + 6x_3 + 12x_4 = 36}$; $\mathbf{4x_1 + 3x_2 + 3x_3 - 3x_4 = -1}$; $\mathbf{2x_1 + 2x_2 - x_3 + x_4 = 10}$.	[L2][CO1]	**[10M]**
10	Solve the following system of equations by Gauss-Jacobi Iteration method $27x + 6y - z = 85\,; x + y + 54z = 110\,; 6x + 15y + 2z = 72$.	[L3][CO1]	**[10M]**
11	Solve the following system of equations by Gauss-Siedel Iteration method $\mathbf{4x + 2y + z = 14\,; x + 5y - z = 10\,; x + y + 8z = 20}$.	[L3][CO1]	**[10M]**

UNIT –II

EIGEN VALUES, EIGEN VECTORS AND ORTHOGONAL TRANSFORMATION

1	a) Define Eigen values and Eigen vectors of a matrix.	[L1][CO2]	**[2M]**
	b) Find the Eigne values of the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$	[L3][CO2]	**[2M]**
	c) State Cayley Hamilton theorem	[L1][CO2]	**[2M]**
	d) Convert the symmetric matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix}$ into the quadratic form.	[L2][CO2]	**[2M]**
	e) Find the symmetric matrix corresponding to the quadratic form $\boldsymbol{ax^2 + 2hxy + by^2}$.	[L3][CO2]	**[2M]**
2	a) For the matrix A= $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{-3} \\ \mathbf{0} & \mathbf{3} & \mathbf{2} \\ \mathbf{0} & \mathbf{0} & \mathbf{-2} \end{bmatrix}$ find the Eigen values of $\boldsymbol{3A^3 + 5A^2 - 6A + 2I}$.	[L3][CO2]	**[5M]**
	b) Determine the Eigen values of A^{-1} *where* $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$	[L3][CO2]	**[5M]**
3	Find the Eigen values and corresponding Eigen vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.	[L3][CO2]	**[10M]**
4	Find the Eigen values and corresponding Eigen vectors of the matrix A and also find the eigen values of A^{-1} where $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.	[L3][CO2]	**[10M]**
5	Determine the modal matrix P of $\boldsymbol{A} = \begin{bmatrix} \mathbf{-2} & \mathbf{2} & \mathbf{-3} \\ \mathbf{2} & \mathbf{1} & \mathbf{-6} \\ \mathbf{-1} & \mathbf{-2} & \mathbf{0} \end{bmatrix}$ .Verify that $\boldsymbol{P^{-1}AP}$ is a diagonal matrix.	[L2][CO2]	**[10M]**
6	a) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$.	[L2][CO2]	**[5M]**
	b) Show that the matrix $A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ satisfies its characteristic equation.	[L2][CO2]	**[5M]**
7	Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and find A^{-1} *and* A^4 using Cayley Hamilton theorem.	[L3][CO2]	**[10M]**

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8	Show that the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation. Hence find A^{-1} .	[L2][CO2]	**[10M]**
9	a) State the nature of the Quadratic form $2x_1x_2 + 2x_1x_3 + 2x_2x_3$.	[L1][CO2]	**[5M]**
	b) Identify the nature of the Quadratic form $-3x_1^2 - 3x_2^2 - 3x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$.	[L2][CO2]	**[5M]**
10	Reduce the Quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ into canonical form by Orthogonal transformation and Find the Rank, Index and Signature of the canonical form.	[L3][CO2]	**[10M]**
11	Reduce the Quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy - 2xz - 2yz$ into the canonical form by Orthogonal transformation and discuss its nature.	[L3][CO2]	**[10M]**

UNIT –III
CALCULUS

1	a) State Rolle's theorem.	[L1][CO3]	**[2M]**
	b) Verify the Rolle's Theorem can be applied to the function f(x) = tan x in [0,□]	[L2][CO3]	**[2M]**
	c) State Lagrange's mean value theorem.	[L1][CO3]	**[2M]**
	d) State Cauchy's mean value theorem.	[L1][CO3]	**[2M]**
	e) Expand Taylor's series of the function f(x) in powers of (x-a).	[L2][CO4]	**[2M]**
2	a) Verify Rolle's Theorem for the function f(x) = $\frac{\sin x}{e^x}$ in $[0, \pi]$	[L2][CO3]	**[5M]**
	b) Verify Lagrange's mean value theorem for f(x) = $\log_e x$ in $[1, e]$.	[L2][CO3]	**[5M]**
3	a) Verify Rolle's Theorem for the function f(x) =log$\left[\frac{x^2+ab}{x(a+b)}\right]$ in $[a, b]$; a,b>0	[L2][CO3]	**[5M]**
	b) Test whether the Lagrange's Mean value theorem holds f(x)=$x^3 - x^2 - 5x + 3$ in $[0,4]$ and if so find approximate value of c.	[L4][CO3]	**[5M]**
4	a) Verify Rolle's theorem for the function f(x) = x(x+3)$e^{-\frac{x}{2}}$ in [-3,0]	[L2][CO3]	**[5M]**
	b) Verify Cauchy's mean value theorem for f(x) = e^x and g(x) = e^{-x} in [a, b].	[L2][CO3]	**[5M]**
5	a) Show that for any $x > 0, 1 + x < e^x < 1 + xe^x$ using Lagrange's mean value theorem.		
	b) Verify Cauchy's Mean value theorem for $f(x) = x^3$ and $g(x) = x^2$ in [1,2]	[L2][CO3]	**[5M]**
6	a) Prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}\left(\frac{3}{5}\right) > \frac{\pi}{3} - \frac{1}{8}$ using Lagrange's mean value theorem.	[L2][CO3]	**[5M]**
	b) Verify Cauchy's mean value theorem for $f(x) = sinx$; $g(x) = cosx$ in $\left[0, \frac{\pi}{2}\right]$.	[L2][CO3]	**[5M]**
7	a) Express the polynomial $2x^3 + 7x^2 + x$ -6 in power of $(x-2)$ by Taylor's series.	[L3][CO4]	**[5M]**
	b) Expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$ up to the term containing $\left(x- \frac{\pi}{2}\right)^4$ assigning Taylor's series.	[L2][CO4]	**[5M]**
8	a) Expand $\log_e x$ in powers of (x-1) and hence evaluate $\log 1.1$ correct to 4 decimal places using Taylor's theorem.	[L2][CO4]	**[5M]**
	b) Obtain the Maclaurin's series expression of the following functions: i) e^x ii) cos$x$ iii) sin x	[L2][CO4]	**[5M]**

9	Verify Taylor's theorem $f(x) = (1-x)^{\frac{5}{2}}$ for with Lagrange's form of remainder up to 2 terms in the interval [0,1].	[L2][CO4]	**[10M]**
10	a) Calculate the approximate value of $\sqrt{10}$ correct to 4 decimal places using Taylor's theorem.	[L3][CO4]	**[5M]**
	b) Show that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ by Maclaurin's theorem.	[L2][CO4]	**[5M]**
11	Using Maclaurin's series expand $\tan x$ up to the fifth power of x and hence find the series for log (sec x).	[L3][CO4]	**[10M]**

UNIT –IV

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PARTIAL DIFFERENTIATION AND APPLICATIONS
(MULTI VARIABLE CALCULUS)

1	a) Define Continuity of a function of two variables at a point.	[L1][CO5]	**[2M]**
	b) Evaluate $\lim\limits_{\substack{x\to 1\\ y\to 2}} \frac{2x^2y}{x^2+y^2+1}$.	[L5][CO5]	**[2M]**
	c) If $x = u(1-v);\ y = uv$ then prove that $J\left(\frac{x,y}{u,v}\right) = u$	[L2][CO5]	**[2M]**
	d) State Functional Dependence.	[L1][CO5]	**[2M]**
	e) Define Extreme value of a function of two variables.	[L1][CO5]	**[2M]**
2	a) If $U = log(x^3+y^3+z^3-3xyz)$, prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 U = \frac{-9}{(X+Y+Z)^2}$	[L5][CO5]	**[5M]**
	b) If $u = tan^{-1}\left[\frac{2xy}{x^2-y^2}\right]$ then Prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = 0$.	[L5][CO5]	**[5M]**
3	a) $u = sin^{-1}(x-y), where\ x = 3t, y = 4t^3$, then show that $\frac{du}{dt} = \frac{3}{\sqrt{1-t^2}}$ by total derivative.	[L2][CO5]	**[5M]**
	b) If $u = f(y-z, z-x, x-y)$ prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0$ by using Chain rule .	[L3][CO5]	**[5M]**
5	Expand $x^2y+3y-2$ in powers of $(x-2)$ and $(y+2)$ up to the term of 3rd degree.	[L2][CO5]	**[10M]**
6	a) Expand $e^x siny$ in powers of x and y by Maclaurin series.	[L2][CO5]	**[5M]**
	b) If $u = x^2-2y;\ v = x+y+z,\ w = x-2y+3z$,then find Jacobian $J\left(\frac{u,v,w}{x,y,z}\right)$.	[L1][CO5]	**[5M]**
7	a) If $\boldsymbol{u = \frac{x+y}{1-xy}}$ **and** $\boldsymbol{v = tan^{-1}x + tan^{-1}y}$ **, find** $\frac{\partial(u,v)}{\partial(x,y)}$?	[L1][CO5]	**[5M]**
	b) Verify if $\boldsymbol{u = 2x-y+3z, v = 2x-y-z,\ w = 2x-y+z}$ are functionally dependent and if so, find the relation between them.	[L5][CO5]	**[5M]**
8	Examine the maxima and minima, if any, of the function $f(x) = x^3y^2(1-x-y)$.	[L4][CO5]	**[10M]**
9	a) Examine the function for extreme value $\mathbf{f(x,y) = x^4+y^4-2x^2+4xy-2y^2}$; (x>0,y>0).	[L4][CO5]	**[5M]**
	b) Find the minimum value of $x^2+y^2+z^2$ given $x+y+z = 3a$.	[L1][CO5]	**[5M]**
10	a) Find the stationary points of $u(x,y) = sinx.siny.sin(x+y)$ where $0 < x < \pi, 0 < y < \pi$ and find the maximum of u.	[L1][CO5]	**[5M]**
	b) Find the shortest distance from origin to the surface $xyz^2 = 2$.	[L1][CO5]	**[5M]**
11	a) Find a point on the plane $\mathbf{3x+2y+z-12 = 0}$, which is nearest to the origin.	[L1][CO5]	**[5M]**
	b) Find the points on the sphere x^2 □y^2 □z^2 □4 that are closest and farthest from the point (3, 1,-1).	[L1][CO5]	**[5M]**

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UNIT –V
MULTIPLE INTEGRALS
(MULTI VARIABLE CALCULUS)

1	a) Evaluate $\int_0^2 \int_0^x y\, dy\, dx$	[L5][CO6]	**[2M]**
	b) Evaluate $\int_0^{\pi} \int_0^{a\sin\theta} r\, dr\, d\theta$	[L5][CO6]	**[2M]**
	c) Transform the integral into polar coordinates, $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)\, dy\, dx$.	[L2][CO6]	**[2M]**
	d) Find the area enclosed by the parabolas $x^2 = y \text{ and } y^2 = x$.	[L1][CO6]	**[2M]**
	e) Evaluate $I = \int_0^1 \int_1^2 \int_2^3 xyz\, dx\, dy\, dz$.	[L5][CO6]	**[2M]**
2	a) Evaluate $\int_0^5 \int_0^{x^2} x(x^2+y^2)dxdy$	[L5][CO6]	**[5M]**
	b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$	[L5][CO6]	**[5M]**
3	a) Evaluate $\iint (x^2+y^2)dxdy$ in the positive quadrant for which $x+y \le 1$.	[L5][CO6]	**[5M]**
	b) Evaluate $\int_0^{log2} \int_0^x \int_0^{x+y} e^{x+y+z}\, dzdydx$.	[L5][CO6]	**[5M]**
4	a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)dydx$	[L5][CO6]	**[5M]**
	b) Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)}dxdy$ by converting to polar coordinates.	[L5][CO6]	**[5M]**
5	a) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.	[L2][CO6]	**[5M]**
	b) Evaluate the integral by transforming into polar coordinates $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}\, dxdy$.	[L3][CO6]	**[5M]**
6	a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)dxdy$.	[L5][CO6]	**[5M]**
	b) Evaluate the integral by changing the order of integration $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dydx$.	[L5][CO6]	**[5M]**
7	Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} (xy)dydx$ and hence evaluate the same.	[L1][CO6]	**[10M]**
8	a) By changing order of integration, evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$.	[L3][CO6]	**[5M]**
	b) Evaluate $\int_{-1}^{1} \int_0^{z} \int_{x-z}^{x+z} (x+y+z)dxdydz$	[L5][CO6]	**[5M]**
9	a) Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	[L1][CO6]	**[5M]**
	b) Evaluate $\int_1^e \int_1^{logy} \int_1^{e^x} logz\, dzdxdy$.	[L5][CO6]	**[5M]**
10	a) Find the volume common to the cylinders $x^2+y^2=a^2 \text{ and } x^2+z^2=a^2$.	[L1][CO6]	**[5M]**
	b) Evaluate $\int\int\int (x^2+y^2+z^2)\, dx\, dy\, dz$ taken over the volume enclosed by the sphere $x^2+y^2+z^2=1$, by transforming into spherical polar coordinates.	[L5][CO6]	**[5M]**
11	a) Evaluate the triple integral $\iiint xy^2zdxdydz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$.	[L5][CO6]	**[5M]**
	b) Calculate the volume of the solid bounded by the planes $x=0, y=0, x+y+z=a \text{ and } z=0$	[L1][CO6]	**[5M]**