Digital Logic Design Assignment 9 - EC2015-38

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1 Question:

A 3-input majority gate is defined by the logic function M(a, b,c) = ab + bc + ca . Which one of the following gates is represented by the function $M(\overline{M(a,b,c)}, M(a,b,\overline{c}), c)$?

(A) 3-input NAND gate

(B) 3-input XOR gate

(C) 3-input NOR gate

(D) 3-input XNOR gate

2 Solution :

It is requried to find the equivalent logic gate to function $M(\overline{M(a,b,c)}M(a,b,\overline{c}), c)$

Logic gates are 3-input gates with function M=ab+bc+ca .

This type of gates are also known as "Majority gates".

The meaning of majority gate is if majority of vaues is 1's it gives 1.If majority of values is 0's it gives 0.
Function of majority gate is :

$$M(a,b,c) = ab + bc + ca \tag{1}$$

Value of X from figure.1(Logic Circuit)

$$X = \overline{M(a,b,c)} = \overline{ab + bc + ca} \tag{2}$$

Value of Y from figure.1(Logic circuit)

$$Y = M(a,b,\overline{c}) = ab + b\overline{c} + \overline{c}a \tag{3}$$

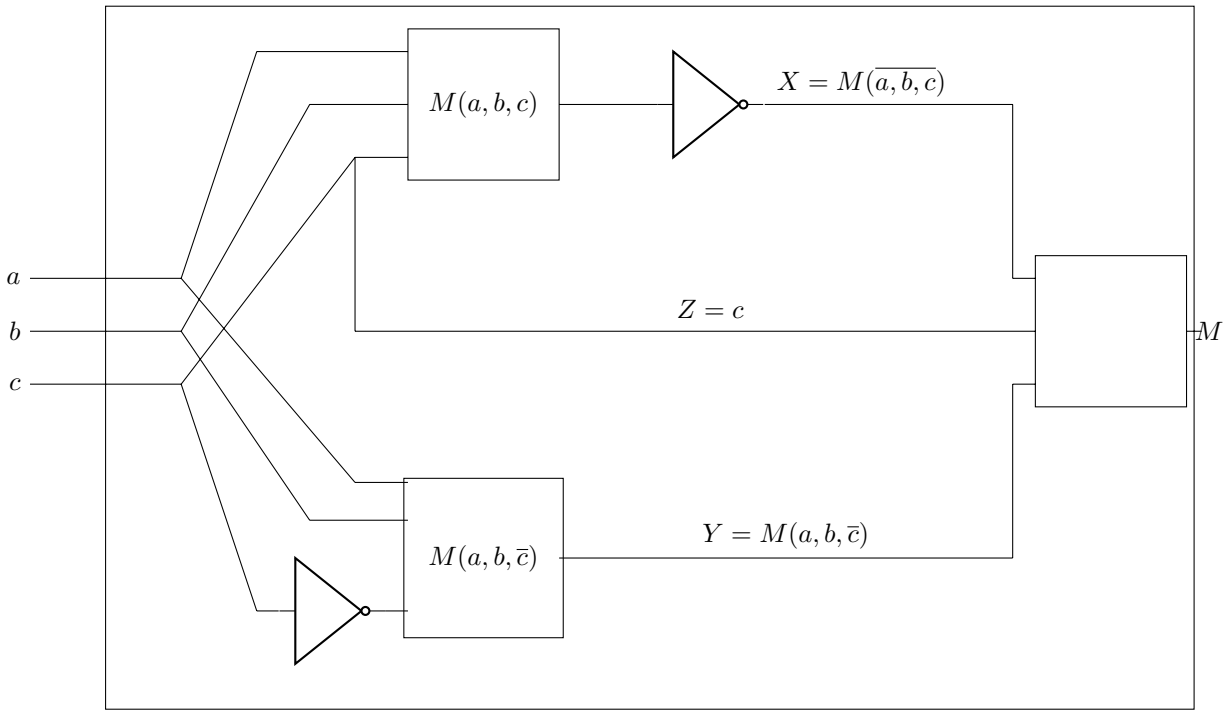

Figure 1: *Logic circuit equivalent of* $M(\overline{M(a,b,c)}M(a,b,\overline{c}),c)$ *used to solve this problem*

Value of Z from figure.1(Logic circuit)

$$Z = c \tag{4}$$

The logical representation of function $M(\overline{M(a,b,c)}, M(a,b,\overline{c}), c)$ in terms of X,Y,Z gives from equations (2),(3),(4)

$$M(\overline{M(a,b,c)}, M(a,b,\overline{c}), c) = M(X,Y,Z) \tag{5}$$

From equation(1) the functon of "M"

$$M(X,Y,Z) = XY + YZ + ZX \tag{6}$$

From equation (2),(3),(4) using in (6)

$$M(X,Y,Z) = (\overline{ab+bc+ca})(ab+b\overline{c}+\overline{c}a) + (ab+b\overline{c}+\overline{c}a)(c) + (\overline{ab+bc+ca})(c) \tag{7}$$

Simplifying Eq(3) further using de Morgan's law

$$\overline{ab}.\overline{bc}.\overline{ca} = (\overline{a}+\overline{b})(\overline{b}+\overline{c})(\overline{c}+\overline{a}) \tag{8}$$

By the function using equation(1)

$$M(a,b,\overline{c}) = ab + b\overline{c} + \overline{c}a \tag{9}$$

On making few more manipulations using of above equation

$$M(\overline{M(a,b,c)}, M(a,b,\overline{c})c) = (\overline{ab}.\overline{bc}.\overline{ca})(ab+b\overline{c}+\overline{c}a) + (ab+b\overline{c}+\overline{c}a)(c) + (\overline{ab+bc+ca})(c) \tag{10}$$

$$M = (\overline{a}+\overline{b})(\overline{b}+\overline{c})(\overline{c}+\overline{a}))(ab+b\overline{c}+\overline{c}a) + abc + (\overline{a}+\overline{b})(\overline{b}+\overline{c}) + (\overline{c}+\overline{a})c \tag{11}$$

$$M = (\overline{a}\overline{b} + \overline{b}\overline{c} + \overline{ca})(a+b+c) + abc \tag{12}$$

$$\overline{a}\overline{b}\overline{c} + b\overline{ca} + c\overline{a}\overline{b} + abc \tag{13}$$

$$M(\overline{M(a,b,c)}, M(a,b,\overline{c}), c) = a \oplus b \oplus c \tag{14}$$

From kmaps a,b,c are exclusive of each other so gate is "exclusive OR gate" or "XOR gate" with 3-inputs

Solution using truth table and karnaugh map

3 Truth Table

By Truth table M is max for minterms 1,2,4,7

Corresponding values of a,b,c for M=1 are :

for m_1 is $a = 0, b = 0, c = 1$
for m_2 is $a = 0, b = 1, c = 0$
for m_4 is $a = 1, b = 0, c = 0$
for m_7 is $a = 1, b = 1, c = 1$

a	b	c	X	Y	Z	M
0	0	0	1	0	0	0
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	0	0	1	0
1	0	0	1	1	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	0	1	1	1

Table 1: Truth Table for eq.(6)

4 K-map for the function $M(\overline{M(a,b,c)}, M(a,b,\overline{c}), c)$

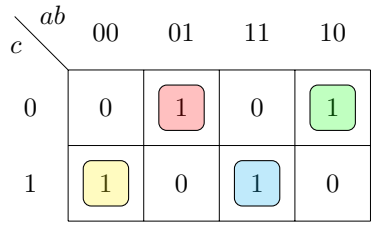

Figure 2: K-map for SOP expression

$$M = a \oplus b \oplus c \tag{15}$$

The expression obtains using the K-map is the same as that one obtains for SOP earlier . Alternatively, we can also make a K-map for obtaining the POS expression:

5 K-map for the function $M(\overline{M(a,b,c)}, M(a,b,\overline{c}), c)$

From the kmap a,b,c are independent.a,b,c are exclusive for function M by kmaps
From kmaps a,b,c are exclusive of each other so gate is "exclusive OR

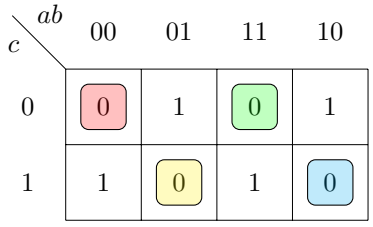

Figure 3: K-map for POS expression

gate" or XOR gate with 3-inputs $M(\overline{M(a,b,c)}, M(a,b,\overline{c}), c)$

$$M(\overline{M(a,b,c)}, M(a,b,\overline{c}), c) = a \oplus b \oplus c \tag{16}$$

Equation of obtained using Algerbraic method SOP and POS kmaps are same i.e 3-input XOR gate -from equation 14,15,16
Answer:3-input XOR gate