Bode Plot Analysis of RC Circuits

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1 Objective

- Analyze the frequency response of different RC circuits:
  - Single RC circuit
  - Cascaded RC circuit
  - Twice-cascaded RC circuit
- Derive the transfer function $H(s)$ for each case.
- Apply logarithmic transformation and simplify the expressions.
- Generate magnitude and phase Bode plots to understand system behavior.

2 Theory

2.1 Transfer Function $H(s)$

The transfer function $H(s)$ is determined for each RC circuit using circuit analysis techniques. Then, the logarithmic transformation is applied to simplify the expressions for magnitude and phase calculations.

2.2 Bode Plot Analysis

Bode plots represent the logarithmic magnitude $\log|H(s)|$ vs. $\log\omega$ and the phase response. These plots are useful for understanding the frequency-dependent behavior of the circuits.

3 Experimental Setup

3.1 Materials Required

- Resistors and capacitors for each circuit configuration.($R = 100\Omega$ and $C = 100\mu F$)
- A signal generator to provide input voltage.
- An oscilloscope to measure the output voltage.

3.2 Procedure

- Assemble the circuit configurations as per the given schematics.
- Apply an input signal and measure the output at different frequencies.
- Record the voltage gain and phase shift at each frequency.
- Compute the transfer function $H(s)$ for each configuration.
- Apply logarithmic transformation to obtain magnitude and phase equations.
- Plot the magnitude and phase Bode plots.

4 Results and Discussion

4.1 Case 1: Single RC Circuit

4.1.1 Circuit Diagram

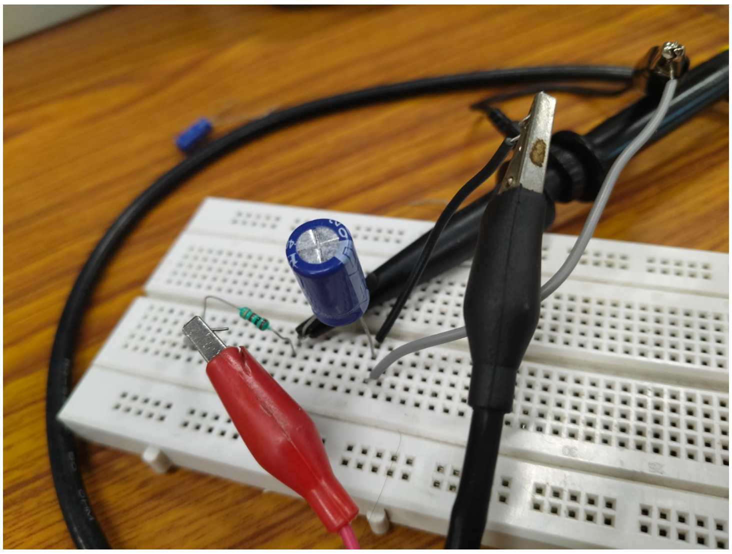

Figure 1: RC Circuit Diagram

4.1.2 Transfer Function

Derived as:

$$H(s) = \frac{1}{1 + sRC} H(j\omega) = \frac{1}{1 + \omega RCj} \tag{1}$$

Applying logarithm:

$$\log |H(j\omega)| = -\frac{1}{2} \log(1 + R^2C^2\omega^2) \tag{2}$$

$$\log |H(j\omega)| = -\log(RC) - \frac{1}{2} log(\frac{1}{R^2C^2} + \omega^2) \tag{3}$$

$$20 \log |H(j\omega)| = -20 log(RC) - 10 \log(\frac{1}{R^2C^2} + \omega^2) \tag{4}$$

For nth order circuit, we get

$$\phi = -n \tan^{-1}(RC\omega) \tag{5}$$

We get phase to be :

$$\phi = -\tan^{-1}(RC\omega) \tag{6}$$

4.2 Oscilloscope- Practically measured VPP

4.2.1 w = 10

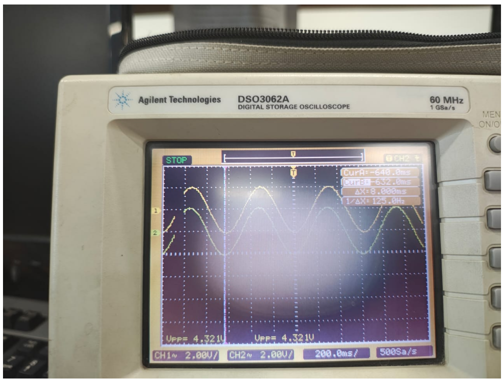

Figure 2: Oscilloscope Reading for w = 10

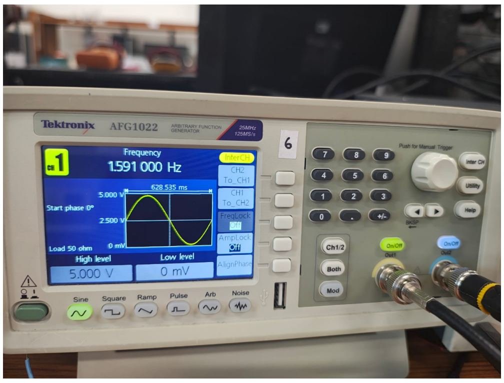

Figure 3: Function Generator Output for w = 10

4.2.2 w = 100

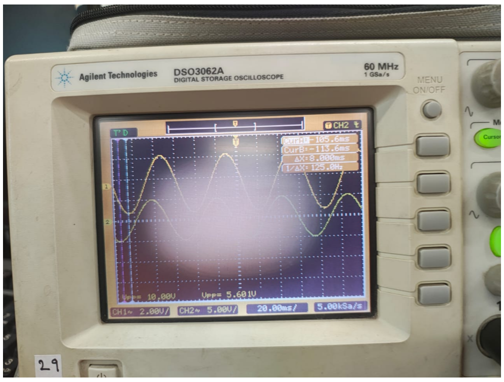

Figure 4: Oscilloscope Reading for w = 100

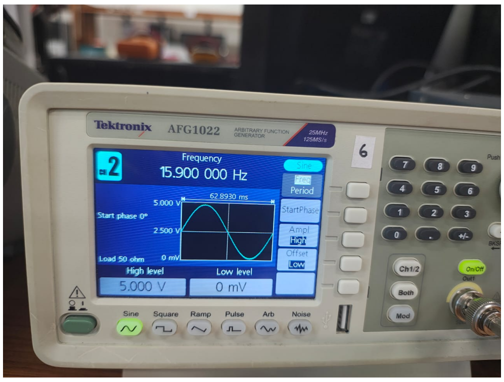

Figure 5: Function Generator Output for w = 100

4.2.3 w = 1000

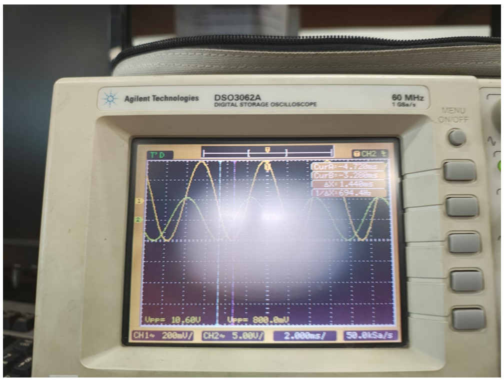

Figure 6: Oscilloscope Reading for w = 1000

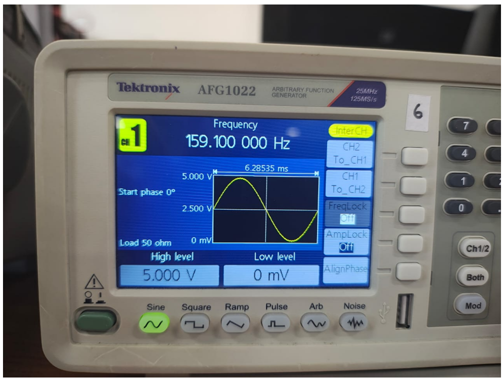

Figure 7: Function Generator Output for w = 1000

4.3 Theoretical values

$$
\begin{aligned}
\phi &= -0.572^\circ, & \omega &= 10 \text{ rad/s} && (7)\\
\phi &= -5.7^\circ, & \omega &= 10 \text{ rad/s} && (8)\\
\phi &= -45^\circ, & \omega &= 100 \text{ rad/s} && (9)\\
\phi &= -84.29^\circ, & \omega &= 1000 \text{ rad/s} && (10)
\end{aligned}
$$

4.4 Magnitude Bode Plot

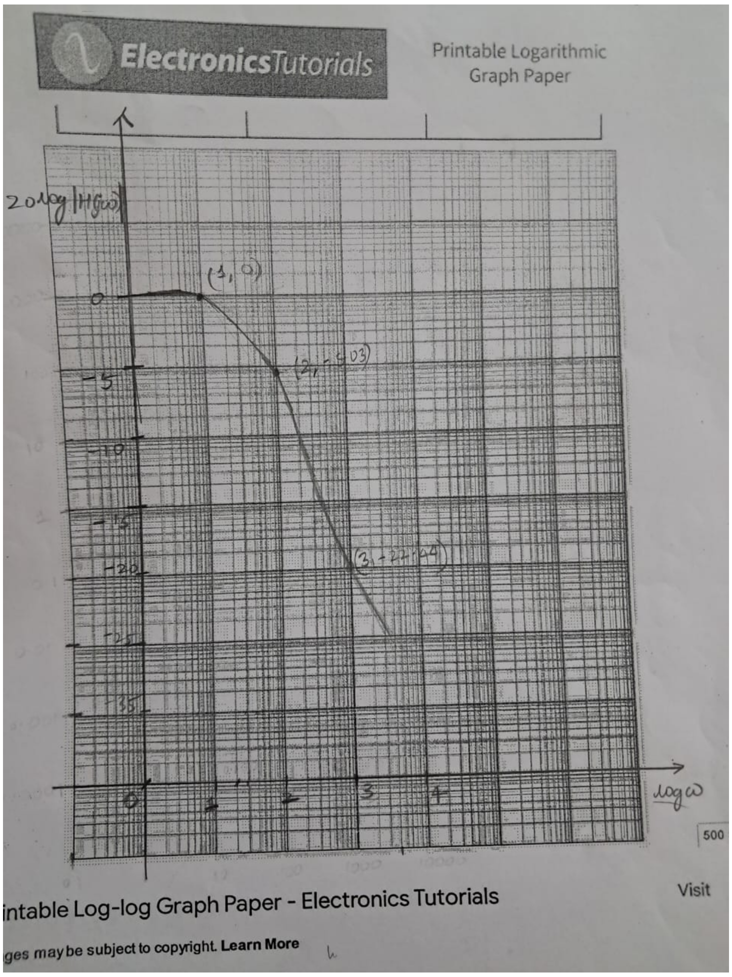

Figure 8: Magnitude Bode Plot for RC Circuit

Magnitude values(measured) are given by:

$$20 \log |H(j\omega)| = 20 \log V_{pp1}/V_{pp2}$$

$$20 \log |H(j\omega)| = 0.00 \text{ dB}, \qquad \omega = 10 \text{ rad/s} \tag{11}$$
$$20 \log |H(j\omega)| = -5.0362 \text{ dB}, \qquad \omega = 100 \text{ rad/s} \tag{12}$$
$$20 \log |H(j\omega)| = -22.4443 \text{ dB}, \qquad \omega = 1000 \text{ rad/s} \tag{13}$$

4.5 Phase Bode Plot

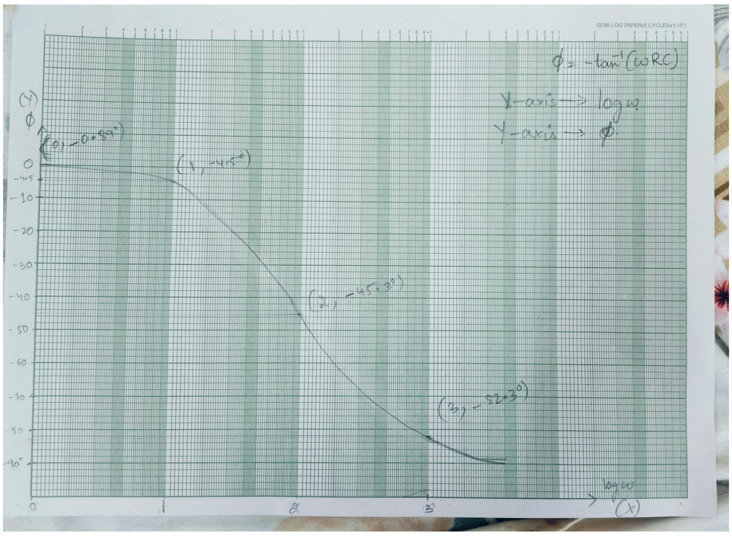

Figure 9: Phase Bode Plot for RC Circuit

Phase difference is given by:

$$\phi = \omega \times \delta t$$

$$\phi = -0.89^\circ, \qquad \omega = 1 \text{ rad/s} \tag{14}$$
$$\phi = -4.5^\circ, \qquad \omega = 10 \text{ rad/s} \tag{15}$$
$$\phi = -45.3^\circ, \qquad \omega = 100 \text{ rad/s} \tag{16}$$
$$\phi = -82.3^\circ, \qquad \omega = 1000 \text{ rad/s} \tag{17}$$

4.6 Case 2: Cascaded RC Circuit

4.6.1 Transfer Function

Derived as:

$$H(j\omega) = \frac{\frac{1}{2RC}}{\frac{1-\omega^2R^2C^2}{2RC} + j\omega} \tag{18}$$

Applying logarithm:

$$\log(|H(j\omega)|) = -\log(2RC) - \frac{1}{2}(\log\frac{1-\omega^2R^2C^2}{2RC})^2 + \omega^2) \tag{19}$$

On simplification, we get:

$$20\log|H(j\omega)| = -40\log(RC) - 20\log(\omega^2 + \frac{1}{R^2C^2}) \tag{20}$$

For nth order circuit, we get

$$\phi = -n\tan^{-1}(RC\omega) \tag{21}$$

We get phase to be :

$$\phi = -2\tan^{-1}(RC\omega) \tag{22}$$

4.7 Oscilloscope- Practically measured VPP

4.7.1 w = 10

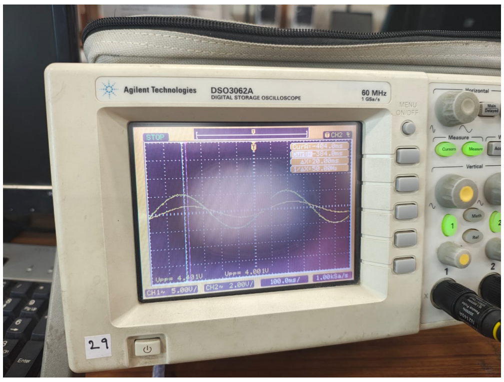

Figure 10: Oscilloscope Reading for w = 10

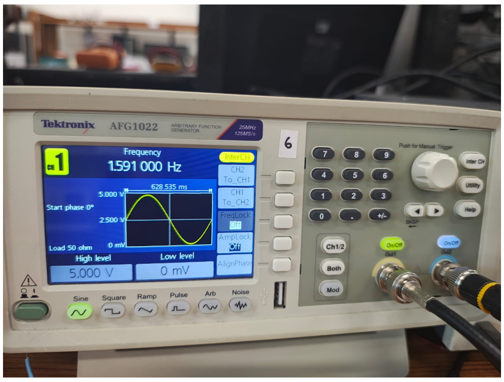

Figure 11: Function Generator Output for w = 10

4.7.2 w = 100

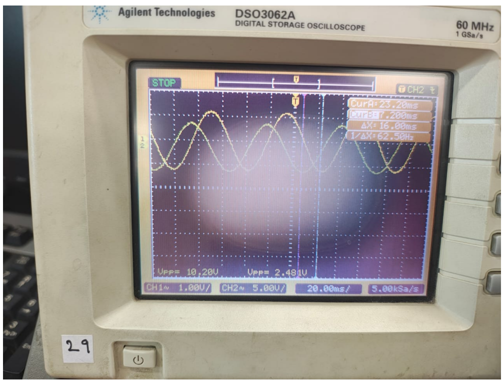

Figure 12: Oscilloscope Reading for w = 100

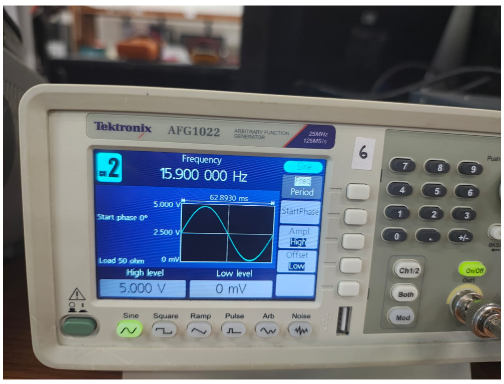

Figure 13: Function Generator Output for w = 100

4.7.3 w = 1000

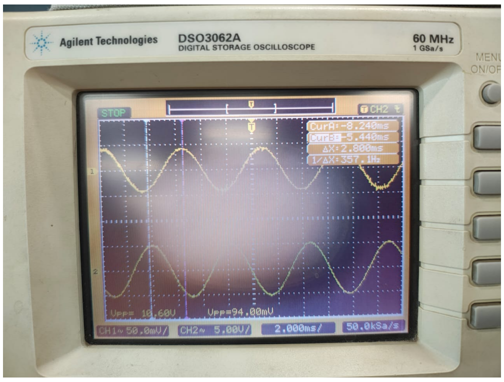

Figure 14: Oscilloscope Reading for w = 1000

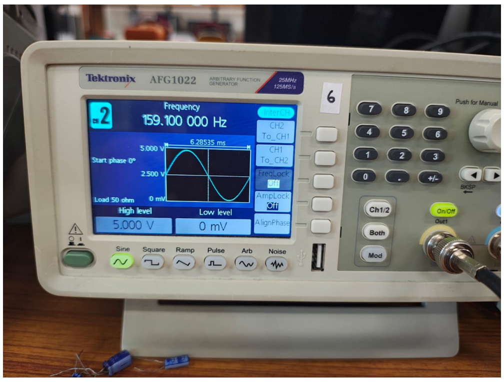

Figure 15: Function Generator Output for w = 1000

4.8 Theoretical values

$$\phi = -11.4^\circ, \qquad \omega = 10 \text{ rad/s} \tag{23}$$
$$\phi = -90^\circ, \qquad \omega = 100 \text{ rad/s} \tag{24}$$
$$\phi = -168.58^\circ, \qquad \omega = 1000 \text{ rad/s} \tag{25}$$

4.9 Magnitude Bode Plot

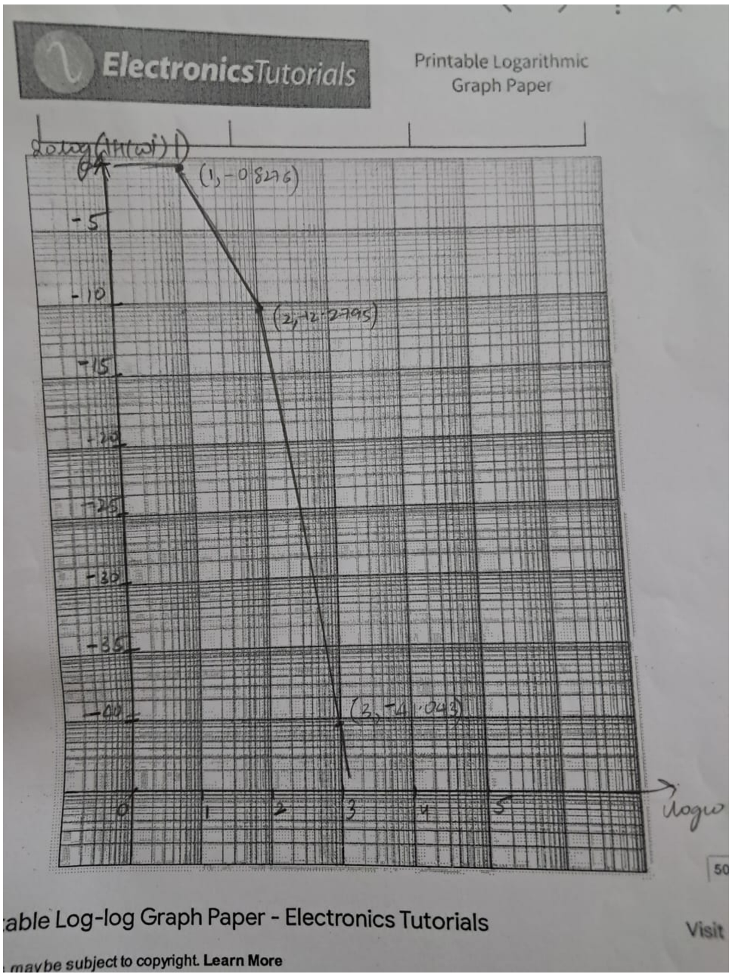

Figure 16: Magnitude Bode Plot for RC Circuit

Magnitude values(measured) are given by:

$$20 \log |H(j\omega)| = 20 \log V_{pp1}/V_{pp2}$$

$$20 \log |H(j\omega)| = -0.8276 \text{ dB}, \qquad \omega = 10 \text{ rad/s} \tag{26}$$
$$20 \log |H(j\omega)| = -12.2795 \text{ dB}, \qquad \omega = 100 \text{ rad/s} \tag{27}$$
$$20 \log |H(j\omega)| = -41.043 \text{ dB}, \qquad \omega = 1000 \text{ rad/s} \tag{28}$$

4.10 Phase Bode Plot

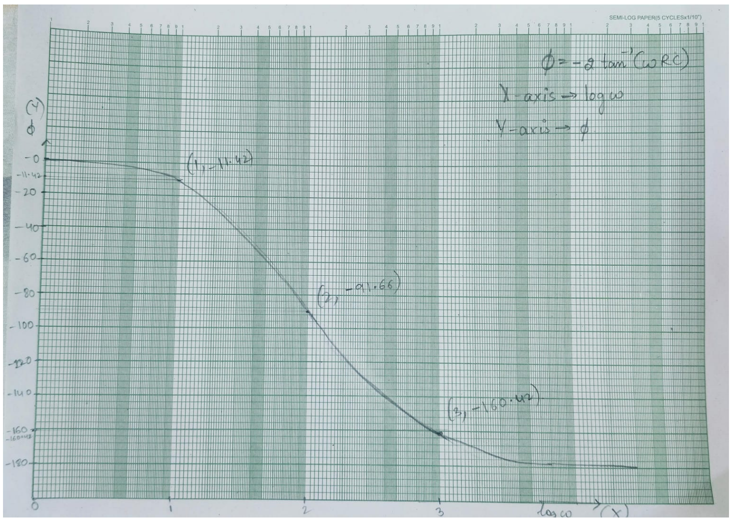

Figure 17: Phase Bode Plot for RC Circuit

Phase difference is given by:

$$\phi = \omega \times \delta t$$

$$\phi = -11.42°, \qquad \omega = 10 \text{ rad/s} \qquad (29)$$
$$\phi = --91.66°, \qquad \omega = 100 \text{ rad/s} \qquad (30)$$
$$\phi = -160.42°, \qquad \omega = 1000 \text{ rad/s} \qquad (31)$$

4.11 Case 3: Twice-Cascaded RC Circuit

4.11.1 Circuit Diagram

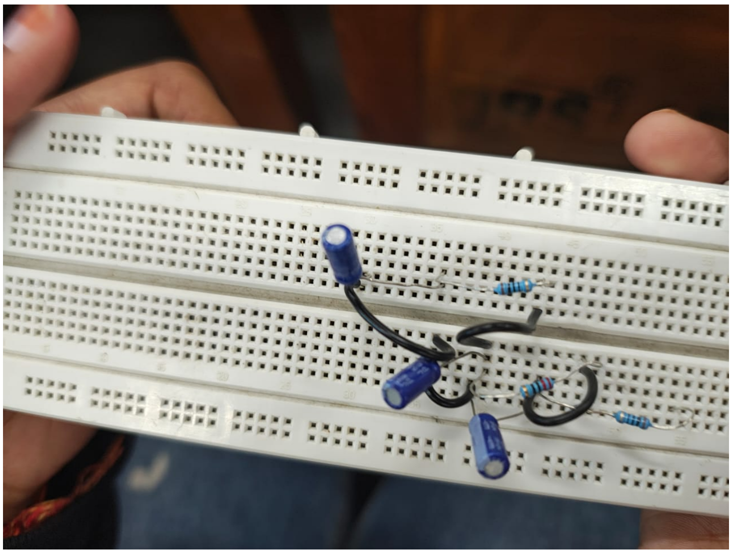

Figure 18: RC Circuit Diagram

4.11.2 Transfer Function

Derived as:

$$H(s) = \frac{\frac{1}{3RC+s^2R^3C^3}}{\left(\frac{1+3s^2R^2C^2}{3RC+s^2R^3C^3} + s\right)} \tag{32}$$

Since $s = j\omega$

$$H(j\omega) = \frac{\frac{1}{3RC-\omega^2R^3C^3}}{\frac{1-3\omega^2R^2C^2}{3RC-\omega^2R^3C^3} + j\omega} \tag{33}$$

Applying logarithm:

$$\log|H(s)| = -\log\left(3RC - \omega^2R^3C^3\right) - \frac{1}{2}\log\left(\left(\frac{1-3\omega^2R^2C^2}{3RC-\omega^2R^3C^3}\right)^2 + \omega^2\right) \tag{34}$$

On simplification, we get:

$$\log|H(s)| = -\frac{3}{2}\left(2\log(RC) + \log\left(\omega^2 + \frac{1}{R^2C^2}\right)\right) \tag{35}$$

$$20\log|H(s)| = -30\left(2\log(RC) + \log\left(\omega^2 + \frac{1}{R^2C^2}\right)\right) \tag{36}$$

For nth order circuit, we get

$$\phi = -n\tan^{-1}(RC\omega) \tag{37}$$

We get phase to be :

$$\phi = -3\tan^{-1}(RC\omega) \tag{38}$$

4.12 Oscilloscope - Practically Measured VPP's

4.12.1 w = 10

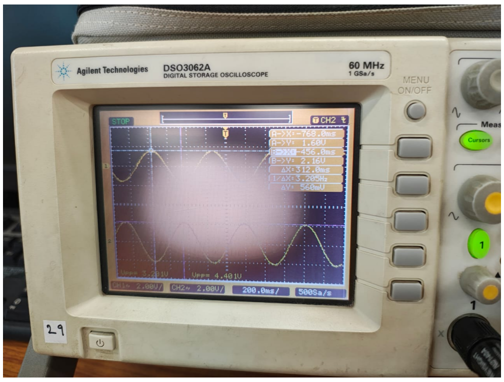

Figure 19: Oscilloscope Reading for w = 10

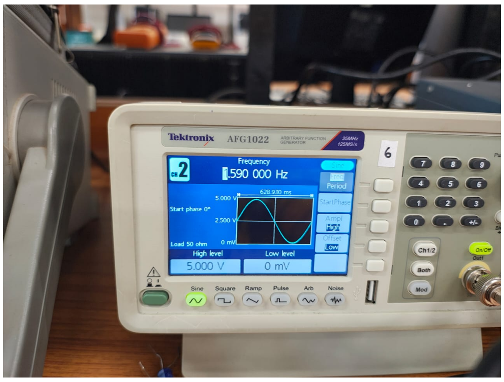

Figure 20: Function Generator Output for w = 10

4.12.2 w = 100

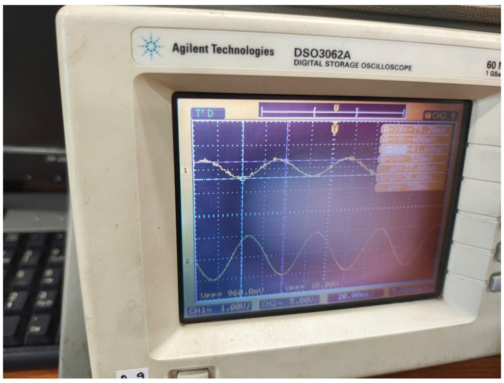

Figure 21: Oscilloscope Reading for w = 100

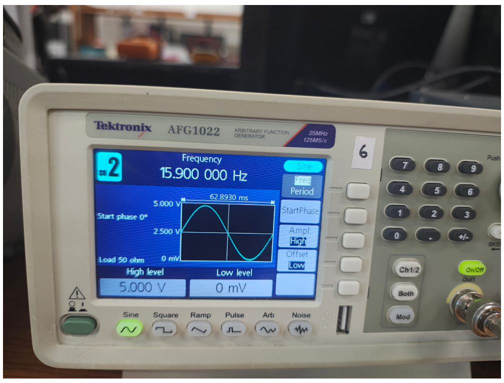

Figure 22: Function Generator Output for w = 100

4.12.3 w = 1000

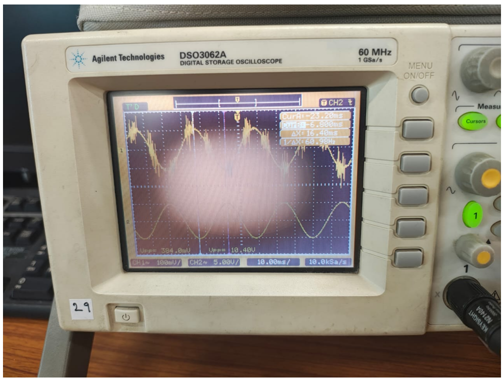

Figure 23: Oscilloscope Reading for w = 200

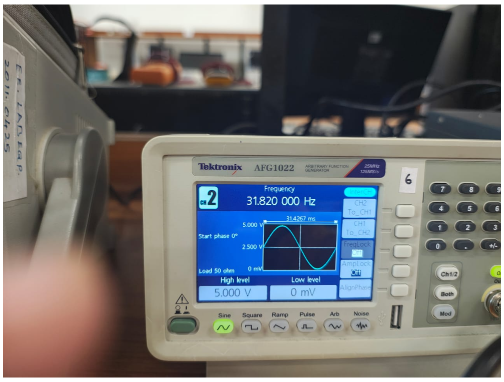

Figure 24: Function Generator Output for w = 200

4.13 Theoretical values

$$\phi = -17.1^\circ, \qquad \omega = 10 \text{ rad/s} \tag{39}$$
$$\phi = -135^\circ, \qquad \omega = 100 \text{ rad/s} \tag{40}$$
$$\phi = -252.87^\circ, \qquad \omega = 1000 \text{ rad/s} \tag{41}$$

4.14 Magnitude Bode Plot

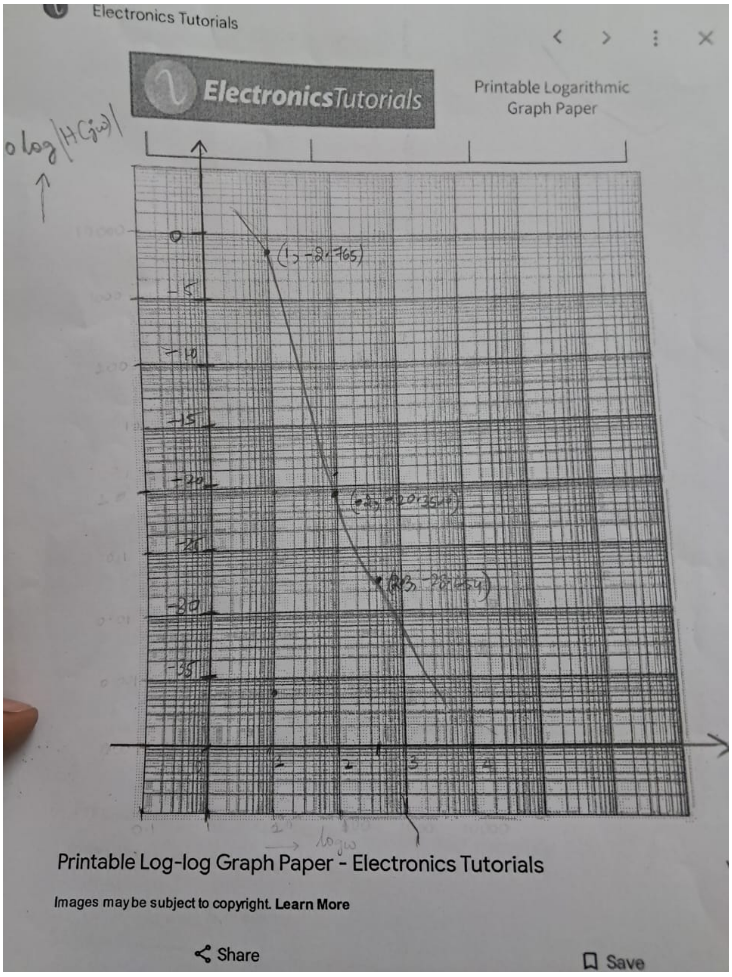

Figure 25: Magnitude Bode Plot for RC Circuit

Magnitude values(measured) are given by:

$$20 \log |H(j\omega)| = 20 \log V_{pp1}/V_{pp2}$$

$$20 \log |H(j\omega)| = -2.765 \text{ dB}, \qquad \omega = 10 \text{ rad/s} \tag{42}$$
$$20 \log |H(j\omega)| = -20.3546 \text{ dB}, \qquad \omega = 100 \text{ rad/s} \tag{43}$$
$$20 \log |H(j\omega)| = -28.654 \text{ dB}, \qquad \omega = 1000 \text{ rad/s} \tag{44}$$

4.15 Phase Bode Plot

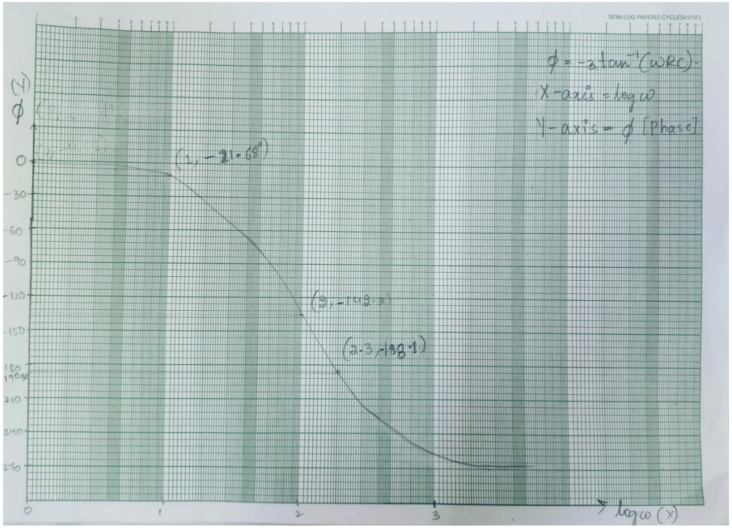

Figure 26: Phase Bode Plot for RC Circuit

Phase difference is given by:

$$\phi = \omega \times \delta t$$

$$\phi = -21.68^\circ, \qquad \omega = 10 \text{ rad/s} \tag{45}$$

$$\phi = -142.2^\circ, \qquad \omega = 100 \text{ rad/s} \tag{46}$$

$$\phi = --188.1^\circ, \qquad \omega = 200 \text{ rad/s} \tag{47}$$

5 Conclusion

The results demonstrate how cascading RC circuits affects frequency response. The magnitude plot reveals increasing attenuation with cascading, and phase plots show a shift in phase behavior. These findings are essential in filter design and signal processing applications.