04-10-2023-shift-2(16-30)

EE24BTECH11055 - Sai Akhila Reddy Turpu

16) Let a dice be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to

a) 60 b) 30 c) 90 d) 15

17) Let a circle of radius 4 be concentric to the ellipse $15x^2 + 19y^2 = 285$. Then the common tangents are inclined to the minor axis of the ellipse at the angle

a) $\frac{\pi}{6}$ b) $\frac{\pi}{12}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$

18) Let $\overrightarrow{a} = 2\hat{i} + 7\hat{j} - \hat{k}$, $\overrightarrow{b} = 3\hat{i} + 5\hat{k}$ and $\overrightarrow{c} = \hat{i} + \hat{j} + 2\hat{k}$. Let $\overrightarrow{d}$ be a vector which is perpendicular to both $\overrightarrow{a}$ and $\overrightarrow{b}$, and $\overrightarrow{c} \cdot \overrightarrow{d} = 12$. Then $\left(-\hat{i} + \hat{j} - \hat{k}\right) \cdot \left(\overrightarrow{c} \times \overrightarrow{d}\right)$is equal to

a) 24 b) 42 c) 48 d) 44

19) Let $S = \left\{z = x + iy : \frac{2z-3i}{4z+2i} \text{is a real number}\right\}$. Then which of the following is NOT correct?

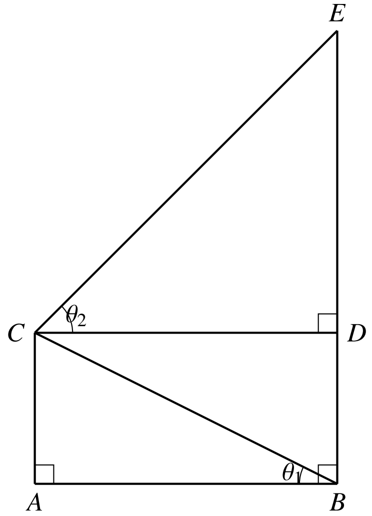
a) $y \in \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$
b) $(x, y) = \left(0, -\frac{1}{2}\right)$
c) $x = 0$
d) $y + x^2 + y^2 \neq -\frac{1}{4}$

20) Let the line $\frac{x}{1} = \frac{6-y}{2} = \frac{z+8}{5}$ intersect the lines $\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1}$ and $\frac{x+3}{6} = \frac{3-y}{3} = \frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x - 2y + z = 14$ is:

a) 3 b) $\frac{10}{3}$ c) 4 d) $\frac{11}{3}$

21) The sum of all four-digit numbers that can be formed using all the digits $2, 1, 2, 3$ is equal to ______.

22) In the figure, $\theta_1 + \theta_2 = \frac{\pi}{2}$ and $\sqrt{3}(BE) = 4(AB)$. If the area of $\triangle CAB$ is $2\sqrt{3} - 3\text{unit}^2$, when $\frac{\theta_2}{\theta_1}$ is the largest, then the perimeter of $\triangle CED$ is equal to ________.



23) Let the tangent at any point P on a curve passing through the points $(1, 1)$ and $\left(\frac{1}{10}, 100\right)$, intersect positive x-axis and y-axis at the points A and B respectively. If $PA : PB = 1 : k$ and $y = y(x)$ is the solution of the differential equation $e^{\frac{dy}{dx}} = kx + \frac{k}{2}, y(0) = k$, then $4y(1) - \log e^3$ is equal to ________.

24) Suppose $a_1, a_2, 2, a_3, a_4$ be in an arithmetico-geometric progression. If the common ratio of the corresponding geometric progression is 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to ________.

25) If the area of the region $\left\{(x, y) : |x^2 - 2| \leq x\right\}$ is A, then $6A + 16\sqrt{2}$ is equal to ________.

26) Let the foot of perpendicular from the point $A(4, 3, 1)$ on the plane $P : x-y+2z+3 = 0$ be N. If $B(5, \alpha, \beta), \alpha, \beta \in \mathbb{Z}$ is a point on plane P such that the area of triangle ABN is $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to ________.

27) Let S be the set of values of λ, for which the system of equations

$$6\lambda x - 3y + 3z = 4\lambda^2, \tag{27.1}$$

$$2x + 6\lambda y + 4z = 1, \tag{27.2}$$

$$3x + 2y + 3\lambda z = \lambda \tag{27.3}$$

has no solution. Then $12\sum_{\lambda\in S}|\lambda|$ is equal to ________.

28) If the domain of the function $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$ is $[\alpha, \beta) \cup (\gamma, \delta]$, then $|3\alpha + 10(\beta + \gamma) + 21\delta|$ is equal to ________.

29) Let the quadratic curve passing through the point $(-1, 0)$ and touching the line $y = x$ at $(1, 1)$ be $y = f(x)$. Then the x-intercept of the normal to the curve at the point $(\alpha, \alpha + 1)$ in the first quadrant is ________.

30) Let the equations of two adjacent sides of a parallelogram $ABCD$ be $2x - 3y = -23$ and $5x + 4y = 23$. If the equation of one of its diagonal AC is $3x + 7y = 23$ and the

distance of A from the other diagonal is d, then $50d^2$ is equal to ________.