

1) The angle between the straight lines, whose direction cosines are given by the equations  $2\mathbf{l} + 2\mathbf{m} - \mathbf{n} = 0$  and  $\mathbf{mn} + \mathbf{nl} + \mathbf{lm} = 0$ , is :

- a)  $\frac{\pi}{2}$  c)  $\cos^{-1}\left(\frac{8}{9}\right)$   
 b)  $\pi - \cos^{-1}\left(\frac{4}{9}\right)$  d)  $\frac{\pi}{3}$

2) Let  $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$  where  $[t]$  denotes the greatest integer less than or equal to  $t$ . If  $\det(A) = 192$ , then the set of values of  $x$  is the interval:

- a)  $[68, 69)$  c)  $[65, 66)$   
 b)  $[62, 63)$  d)  $[60, 61)$

3) Let  $M$  and  $m$  respectively be the maximum and minimum values of the function  $f(x) = \tan^{-1}(\sin x + \cos x)$  in  $\left[0, \frac{\pi}{2}\right]$ , then the value of  $\tan(M - m)$  is equal to:

- a)  $2 + \sqrt{3}$  c)  $3 + 2\sqrt{2}$   
 b)  $2 - \sqrt{3}$  d)  $3 - 2\sqrt{2}$

4) Each of the persons  $A$  and  $B$  independently tosses three fair coins. The probability that both of them get the same number of heads is:

- a)  $\frac{1}{8}$  b)  $\frac{5}{8}$  c)  $\frac{5}{16}$  d) 1

5) A differential equation representing the family of parabolas with axis parallel to  $y$ -axis and whose length of latus rectum is the distance of the point  $(2, -3)$  from the line  $3x + 4y = 5$ , is given by:

- a)  $10 \frac{d^2y}{dx^2} = 11$  c)  $10 \frac{d^2x}{dy^2} = 11$   
 b)  $11 \frac{d^2x}{dy^2} = 10$  d)  $11 \frac{d^2y}{dx^2} = 10$

6) If two tangents drawn from a point  $\mathbf{P}$  to the parabola  $y^2 = 16(x - 3)$  are at right angles, then the locus of point  $\mathbf{P}$  is:

- a)  $x + 3 = 0$  c)  $x + 2 = 0$   
 b)  $x + 1 = 0$  d)  $x + 4 = 0$

7) The equation of the plane passing through the line of intersection of planes  $\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$  and  $\mathbf{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$  and parallel to the  $x$ -axis is:

a)  $\mathbf{r} \cdot (\hat{j} - 3\hat{k}) + 6 = 0$   
 b)  $\mathbf{r} \cdot (\hat{i} + 3\hat{k}) + 6 = 0$

c)  $\mathbf{r} \cdot (\hat{i} - 3\hat{k}) + 6 = 0$   
 d)  $\mathbf{r} \cdot (\hat{j} - 3\hat{k}) - 6 = 0$

8) If the solution curve of the differential equation  $(2x - 10y^3)dy + ydx = 0$ , passes through the points  $(0, 1)$  and  $(2, \beta)$ , then  $\beta$  is a root of the equation:

a)  $y^5 - 2y - 2 = 0$   
 b)  $2y^5 - 2y - 1 = 0$

c)  $2y^5 - y^2 - 2 = 0$   
 d)  $y^5 - y^2 - 1 = 0$

9) Let  $\mathbf{A}(a, 0)$ ,  $\mathbf{B}(b, 2b + 1)$  and  $\mathbf{C}(0, b)$ ,  $b \neq 0, |b| \neq 1$ , be points such that the area of the triangle  $\mathbf{ABC}$  is 1 sq. unit, then the sum of all possible values of  $a$  is:

a)  $\frac{-2b}{b+1}$   
 b)  $\frac{2b}{b+1}$

c)  $\frac{2b^2}{b+1}$   
 d)  $\frac{-2b^2}{b+1}$

10) Let  $[\lambda]$  be the greatest integer less than or equal to  $\lambda$ . The set of all values of  $\lambda$  for which the system of linear equations  $x + y + z = 4$ ,  $3x + 2y + 5z = 3$ ,  $9x + 4y + (28 + [\lambda])z = [\lambda]$  has a solution is:

a)  $\mathbb{R}$   
 b)  $(-\infty, -9) \cup (-9, \infty)$   
 c)  $[-9, -8)$   
 d)  $(-\infty, -9) \cup (-8, \infty)$

11) The set of all values of  $k > -1$ , for which the equation  $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$  has real roots, is:

a)  $\left(1, \frac{5}{2}\right]$   
 b)  $[2, 3)$

c)  $\left[-\frac{1}{2}, 1\right)$   
 d)  $\left(\frac{1}{2}, \frac{3}{2}\right] - \{1\}$

12) A box open from top is made from a rectangular sheet of dimension  $a \times b$  by cutting squares each of side  $x$  from each of the four corners and folding up the flaps. If the volume of the box is maximum, then  $x$  is equal to:

a)  $\frac{a+b-\sqrt{a^2+b^2-ab}}{12}$   
 b)  $\frac{a+b-\sqrt{a^2+b^2+ab}}{6}$   
 c)  $\frac{a+b-\sqrt{a^2+b^2-ab}}{6}$   
 d)  $\frac{a+b+\sqrt{a^2+b^2-ab}}{6}$

13) The Boolean expression  $(p \wedge q) \implies ((r \wedge q) \wedge p)$  is equivalent to:

a)  $(p \wedge q) \implies (r \wedge q)$   
 b)  $(q \wedge r) \implies (p \wedge q)$

c)  $(p \wedge q) \implies (r \vee q)$   
 d)  $(p \wedge r) \implies (p \wedge q)$

14) Let  $\mathbb{Z}$  be the set of all integers,

$$A = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x - 2)^2 + y^2 \leq 4\},$$

$$B = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \leq 4\},$$

$$C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x - 2)^2 + (y - 2)^2 \leq 4\},$$

If the total number of relations from  $A \cap B$  to  $A \cap C$  is  $2^p$ , then the value of  $p$  is:

a) 16

c) 49

b) 25

d) 9

15) The area of the region bounded by the parabola  $(y - 2)^2 = (x - 1)$ , the tangent to it at the point whose ordinate is 3 and the x-axis is:

a) 9

b) 10

c) 4

d) 6