

9.5.3

EE24BTECH11055 - Sai Akhila Reddy Turpu

QUESTION:

$$\frac{dy}{dx} + \frac{y}{x} = x^2 \quad (0.1)$$

USING FINITE DIFFERENCES TO SOLVE THE GIVEN DIFFERENTIAL EQUATION:

The finite difference method is a numerical technique for solving differential equations by approximating derivatives with differences. The first forward difference approximation of the derivative of $f(x)$ at x is given by:

$$\frac{dy}{dx} = \frac{f(x+h) - f(x)}{h} \quad (0.2)$$

Let $f(x+h) = y_{n+1}$ and $f(x) = y_n$

$$\implies y_{n+1} = h \times \frac{dy}{dx} + y_n \implies y_{n+1} = h \left(x_n^2 - \frac{y_n}{x_n} \right) + y_n \quad (0.3)$$

We already know that our derivative is given by:

$$\frac{dy}{dx} = x^2 - \frac{y}{x}$$

Let's assume the initial conditions to be: $x_0 = 1$ and $y_0 = 0.25$ The derivative at the initial conditions can be calculated by substituting x_0 and y_0 in equation (1).

$$\frac{dy}{dx} \Big|_{x=x_0} = 1 - 0.25 = 0.75$$

Let $h = 10^{-3}$ and $f_1(x) = f(x+h)$

We get $f_1(x) = f(x) + h \times \frac{dy}{dx} \Big|_{x=x_0}$

$$f_1(x) = 0.25 + 10^{-3} \times 0.75 = 0.25075$$

Thus, $x_1 = x_0 + h = 1 + 10^{-3} = 1.001$,

$$y_1 = y_0 + h \times \frac{dy}{dx} \Big|_{x=x_0} = 0.25075$$

What we've essentially done is, obtaining a point which is very close to the initial point along the direction of the derivative at that point.

To obtain the entire curve we repeat the process consecutively.

For finding $f_2(x)$, find $\frac{dy}{dx} \Big|_{x=x_1}$ using equation (1) and substitute it in equation (2) Find $f_2(x), f_3(x)$ and so on and plot the points to get the curve.

JEE METHOD:

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

Finding the Integrating factor: $e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

Thus $IF = x$

$$y(x) = \int x^2 \times x dx$$

$y \cdot x = \frac{x^3}{4} + C$ Let's assume $C = 0$, we get:

$$y = \frac{x^3}{4}$$

We can now verify that the given graph is $y = \frac{x^3}{4}$

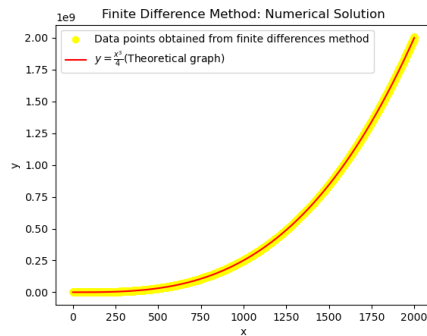


Fig. 0.1: $y = \frac{x^3}{4}$