9.5.3

EE24BTECH11055 - Sai Akhila Reddy Turpu

Question:

$$\frac{dy}{dx} + \frac{y}{x} = x^2 \tag{0.1}$$

Using Finite Differences to solve the given Differential Equation:

The finite difference method is a numerical technique for solving differential equations by approximating derivatives with differences. The first forward difference approximation of the derivative of $f(x)$ at x is given by:

$$\frac{dy}{dx} = \frac{f(x+h) - f(x)}{h} \tag{0.2}$$

Let $f(x+h) = y_{n+1}$ and $f(x) = y_n$

$$\implies y_{n+1} = h \times \frac{dy}{dx} + y_n \tag{0.3}$$

We already know that our derivative is given by:

$$\frac{dy}{dx} = x^2 - \frac{y}{x}$$

Let's assume the intial conditions to be: $x_0 = 1$ and $y_0 = 0.25$ The derivative at the initial conditions can be caluculated by substituting x_0 and y_0 in equation (1).

$$\frac{dy}{dx}|_{x=x_0} = 1 - 0.25 = 0.75$$

Let $h = 10^{-3}$ and $f_1(x) = f(x+h)$

We get $f_1(x) = f(x) + h \times \frac{dy}{dx}|_{x=x_0}$

$f_1(x) = 0.25 + 10^{-3} \times 0.75 = 0.25075$
Thus, $x_1 = x_0 + h = 1 + 10^{-3} = 1.001$,
$y_1 = y_0 + h \times \frac{dy}{dx}|_{x=x_0} = 0.25075$

What we've essentially done is, obtaining a point which is very close to the initial point along the direction of the derivative at that point.
To obtain the entire curve we repeat the process consecutively.
For finding $f_2(x)$, find $\frac{dy}{dx}|_{x=x_1}$ using equation (1) and substitute it in equation (2) Find $f_2(x), f_3(x)$ and so on and plot the points to get the curve.

JEE Method:

$\frac{dy}{dx} + \frac{y}{x} = x^2$
Finding the Integrating factor: $e^{\int \frac{1}{x}\,dx}$ $=e^{\ln x}$=x
Thus $IF = x$
$y(x) = \int x^2 \times x\,dx$
$y.x = \frac{x^4}{4} + C$ Let's assume $C = 0$, we get:

$$y = \frac{x^3}{4}$$

We can now verify that the given graph is $y = \frac{x^3}{4}$