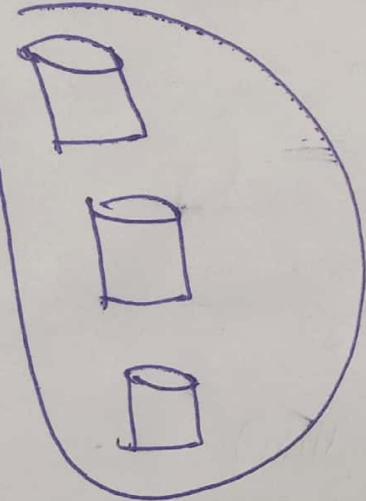
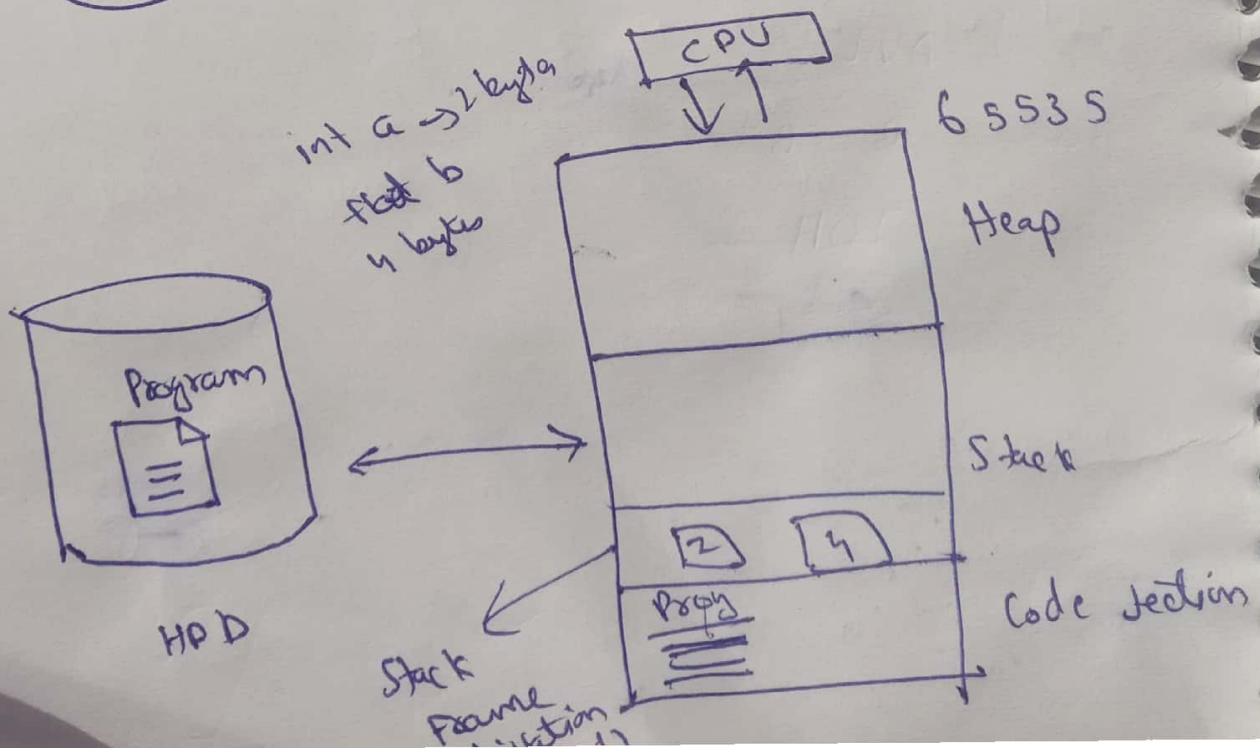


Big Data



Algorithm written for data warehouse  
Big are called data mining



void fun2 (int i)

2 int a;

3

void fun1 ()

2 int x;  
fun2(x);

3

void main ()

2 int a;  
float b;  
=

fun1();

Programs cannot access heap memory directly.  
They can access only using pointers.

void main ()

2 int \*p;  
depends on size of integer usually

p = new int[5];  
3 C p = (int \*)malloc(245);  
allocates

p = NULL;

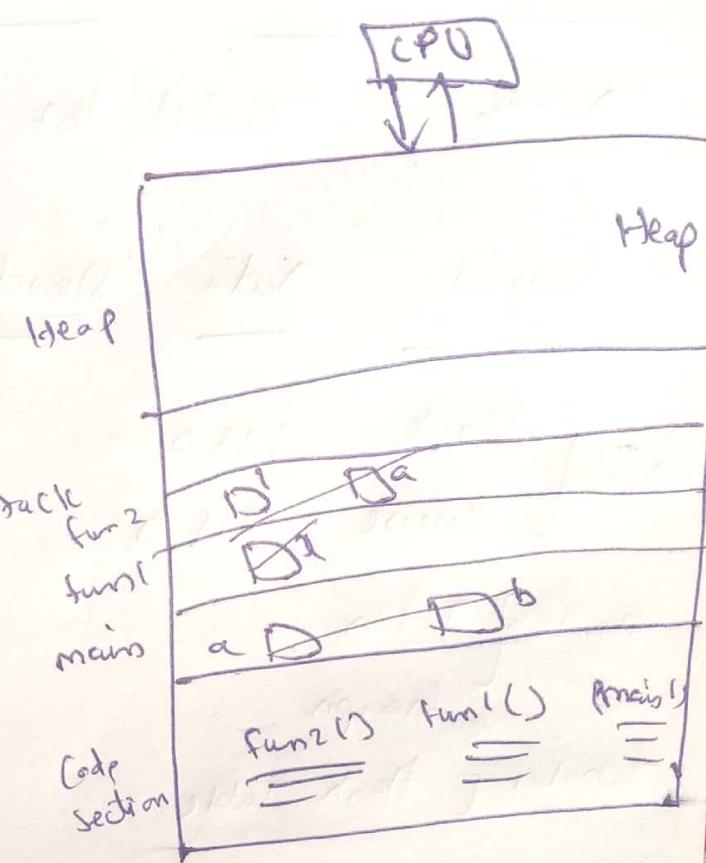
delete [] p;

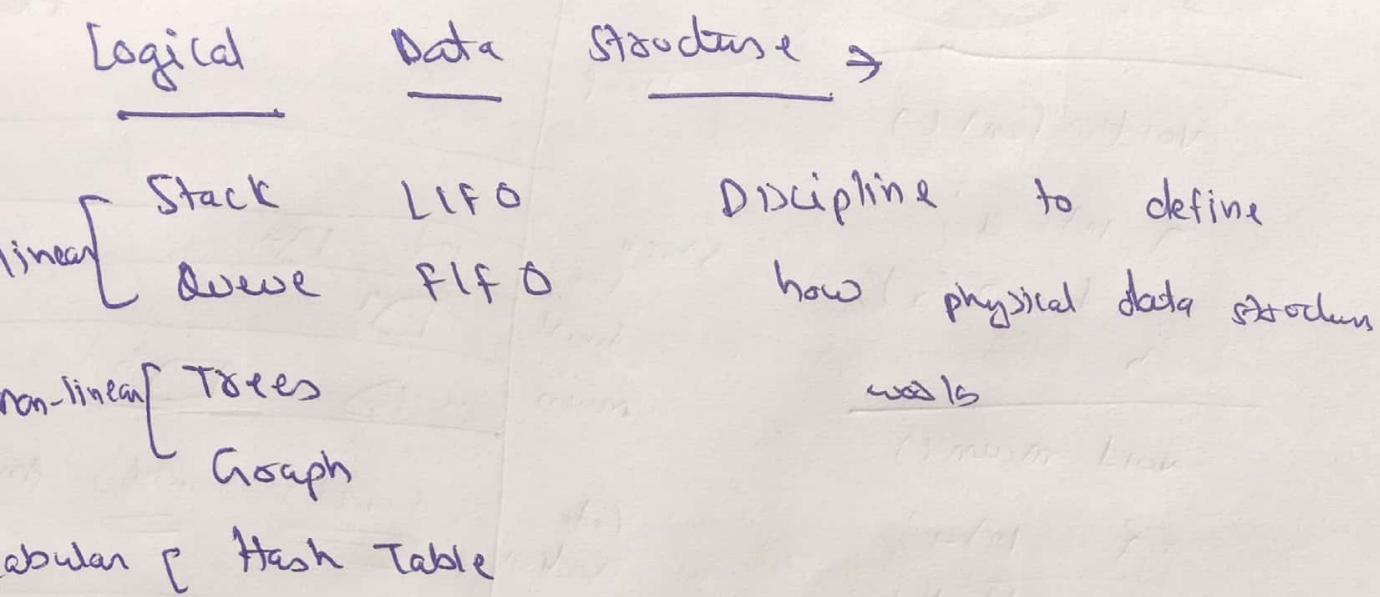
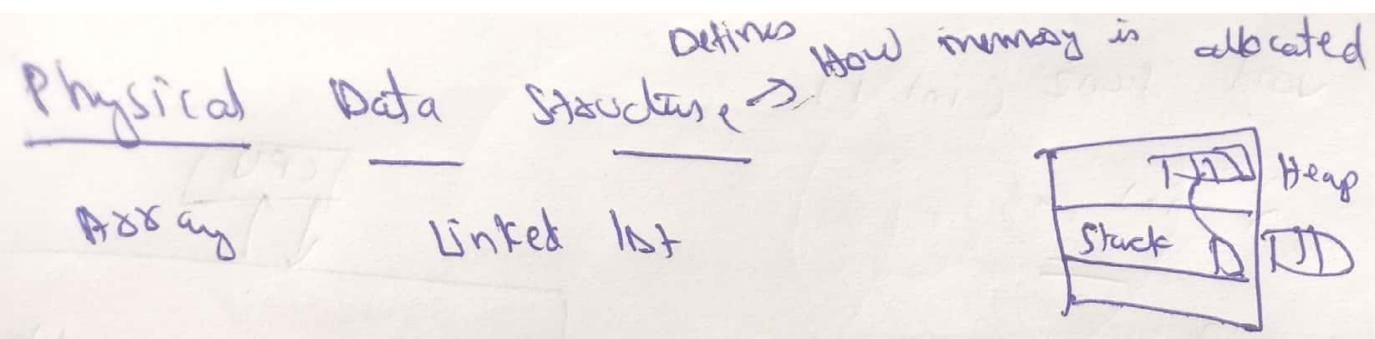
to first block.

& doesn't

initially allocated memory is 0

free p





### ADT →

1. Representation of data
  2. Operations on data
- It is a datatype where only behavior is defined but not implementation. opposite of ADT is Concrete DT (CDT) where it contains implementation.

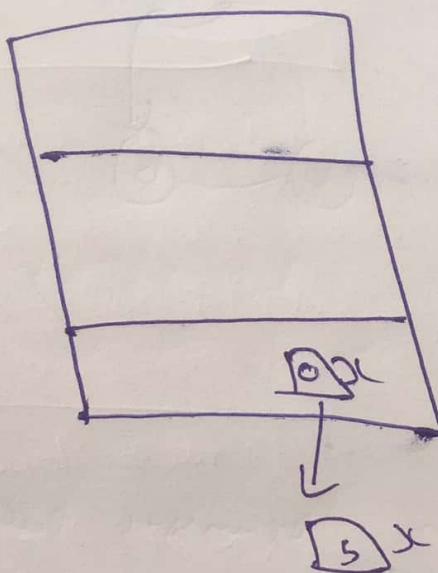
List      8      3      9      4      6      10      12

Data required for Storing list :

- ① Space for storing element
  - ② Capacity
  - ③  $\infty$
- Operations →

→ ArrayList, Map, Queue, Set, Stack, Table, Tree, etc are ADTs.

Static int i=0;



int func (int n)

{ static int x=0;

if (n > 0)

{ x++;

return func(n-1);

}

return 0; }

main ()

{ int a=5;

printf ("%.d", func(a));

}

### Types of Recursion

- ① Tail → Recursive call is in last step.  $\begin{cases} f(n) \\ f(n-1) \end{cases}$
- ② Head → first iteration
- ③ Tree ~~tree~~
- ④ Indirect
- ⑤ Nested

### Linear Recursion

fn calls itself only

one time

### Tree Recursion

fn(n-1)

fn(n-2)

3.

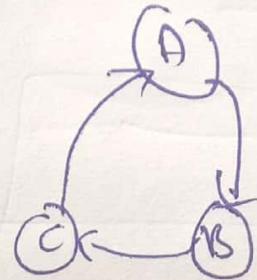
when  
one or  
more fn  
calls each other

AB

B(C)

B()

{ A(C) }



Nested →

Recursive fn passes recursive call as argument

```
int fn(int n)
{
    if (n > 100)
        return n - 10;
    else
        fn(fn(n + 1));
}
```

fn(qs);

m<sup>n</sup> →

```
int pow(int m, int n)
{
    if (n == 0)
        return 1;
    return pow(m, n - 1) * m;
}
```

```

int pow(int m, int n)
{
    if (n == 0)
        return 1;
    if ((n - 1) % 2 == 0)
        return pow(m * m, n / 2);
    else
        return m * pow(m * m, (n - 1) / 2);
}

```

Q Taylor using Horner's rule

$$1 + \left\{ \frac{a}{1} \left[ 1 + \frac{a}{2} \left[ 1 + \frac{a}{3} \left[ \dots \right] \right] \right] \right\}$$

```

int e(int x, int n)
{
    static int s = 1;
    if (n == 0)
        return s;
    s = 1 + x * n / s;
    return e(x, n - 1);
}

```

Q.  $n^{\alpha} \rightarrow$

```

int C(int n, int d)
{
    if (d == 0 || n == d)
        return 1;
    else
        return (n - d) * C(n - 1, d - 1) + C(n - 1, d);
}

```

int \*pA;

pA = new int[n][3]; (3) 24)

A[0] = new int[4];

Adds A[j] = lo + i \* w

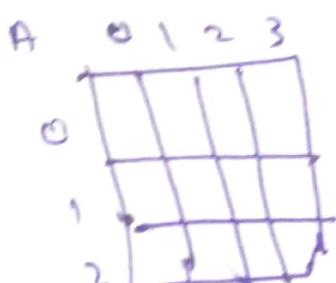
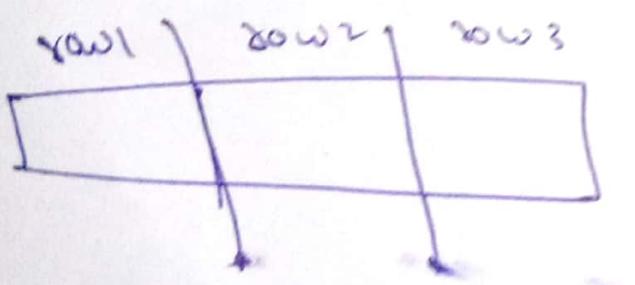
base index size of data type  
Add.

int A[1..5]

$$\Rightarrow \text{Add}[3] = 200 + 2 \times 2 \\ = 204$$

Row Major  $\rightarrow$

int A[m][n]



Adds [A[i][j]] = lo + [i \* n + j] \* w

int A[1..3][1..4]

Adds [A[i][j]] = lo + [(i-1)n + (j-1)] \* w

Column Major →

a <sub>00</sub>	a <sub>10</sub>	a <sub>20</sub>	a <sub>01</sub>	a <sub>11</sub>	a <sub>21</sub>	
col 0			col 1		col 2	col 3



$$\text{addr}[A[i][j]] = lo + [jxm + i] * w$$

ND

$$A[d_1][d_2][d_3][d_4]$$

Row Major

$$\text{addr}[A[i][j][k][l]]$$

$$= lo + i_1 * d_1 + i_2 * d_2 + i_3 * d_3$$

$$+ i_4 * d_4 + i_5 * d_5 \\ - i_6 * d_6$$

Column Major

$$\text{addr} = lo + [i_4 * d_4 * d_2 * d_3 + i_3 * d_3 * d_1 \\ + i_2 * d_1 + i_1] * w$$

- For finding missing elements in sequence

A[i]-i ≠ diff

- Permutations of a string

void perm (char S[], int l, int h)

{ int i; if (l == h) { if (S) use S; } else { for (i = l; i < h; i++) { swap (S[l], S[i]); perm (S[l+1], S[h]); swap (S[l], S[i]); } } }

## Special Matrices (n × n)

- ① Diagonal  $m[i,j] = 0, i \neq j$
- ② Lower D  $m[i,j] = 0, i > j$
- ③ Upper D  $m[i,j] = 0, i < j$
- ④ Symmetric  
(3 diagonals)  $m[i,j] = m[j,i]$
- ⑤ Tridiagonal  $m[i,j] = 0 \mid i - j \mid > 1$
- ⑥ Sparse Band (Several diagonals along with main other are zero diagonal)
- ⑦ Toeplitz  $m[i,j] = m[i-1, j-1]$
- ⑧ Sparse (more no. of 0's)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

9

8

row	column	element
8	9	8
1	8	3
2	3	8
2	6	10

## Special Matrices ( $n \times n$ )

- ① Diagonal  $m[i,j] = 0, i \neq j$
- ② Lower D  $m[i,j] = 0, i > j$
- ③ Upper D  $m[i,j] = 0, i < j$
- ④ Symmetric  
(3 diagonals)  
 $m[i,j] = m[j,i]$
- ⑤ Tridiagonal  
 $m[i,j] = 0 \mid i - j \mid > 1$
- ⑥ <sup>Some</sup> Band  
Band  
(Several diagonals along with main diagonal)  
other are zero
- ⑦ Toeplitz  $m[i,j] = m[i-1,j-1]$
- ⑧ Sparse  
(more no. of 0's)

$$\begin{bmatrix}
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 8 & 0 & 0 & 10 & 0 & 0 & 0 \\
 * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \hline
 \end{bmatrix}^9_8$$

row	column	element
8	9	8
1	8	3
2	3	8
2	6	10

A [ 3, 8, 10, ... ]

IA [ 0, 1, 3, 5, 4, ... ]

A [ 3, 8, ... ]  
JA [ 2, 2, 3, 6, ... ] column

Coordinate list

i	1	1	1	1	1
rows	5	1			
columns	2	6	2		
non-zero	5	6	2		
element	1	2	3		

j	1	2	1	1
1	3	2	1	
2	1	2	3	
3	5	3	1	

compute b  
place rows

$$\left[ \begin{array}{ccccc} 0 & 0 & 0 & 6 & 0 \\ 0 & 7 & 0 & 0 & 0 \\ 0 & 2 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Polynomial Representation

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
2	1	2	3	4	5	6	7	8	9
3	5	3	1	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0

$$3x^5 + 2x^4 + 5x^2 + 2x + 2$$

Coeff	3	2	5	2	1
exp	5	4	2	1	0

In class, everything by default is private,  
in structure everything by default is public.

struct Node {  
    int data;

    2 bytes

    2 bytes  
    char node \* next;

```

struct Node *p;
p = (struct Node *) malloc (sizeof (struct Node));
p = new Node;
p->data = 10;
p->next = 0;

```

```

int count (struct Node *p)
{
    if (p == 0)
        return 0;
    else
        return 1 + count(p->next);
}

```

Time  
 $O(n)$   
Space  
 $O(n)$

### Searching

- ① linear  $O(1) O(n)$
- ② binary X can't be performed

### Insertion

$O(1) O(n)$

Remove Duplicates in a linked list  $\rightarrow$

Node \*p = first;

Node \*q = first  $\rightarrow$  next;

```

while( q != NULL)
{
    if( p->data == q->data )
    {
        p = q;
        q = q->next; }

    else
    {
        p->next = q->next;
        delete q;
        q = p->next; }

}

```

Reverse a linked list →

```

p = first;
q = NULL;
r = NULL;

```



```
while( p != NULL)
```

```

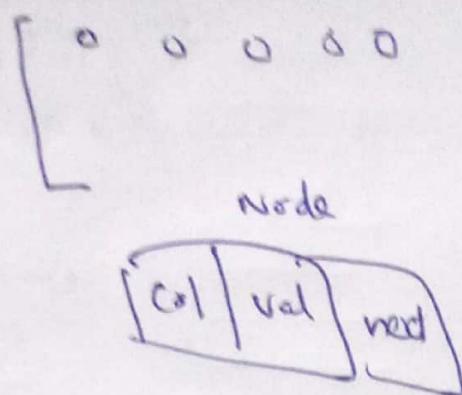
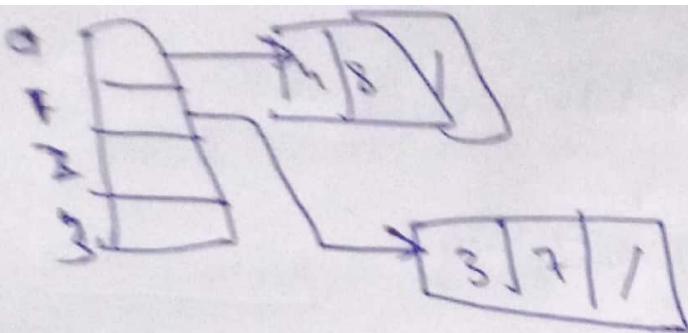
{
    r = q;
    q = p;
    p = p->next;
    q->next = r;
}

first = q;

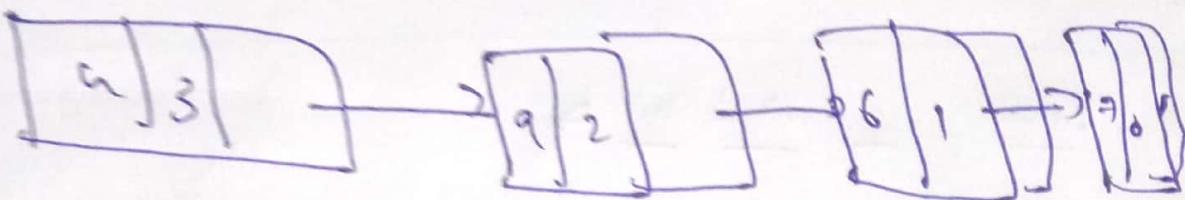
```

Sparse matrix Representation →

[	0	0	0	0	8	0	]	5x6
0	0	0	7	0	0			
5	0	0	0	9	0			
0	0	0	0	0	3			
6	0	0	4	0	0			



$$4x^3 + 9x^2 + 6x + 7$$



```
#include <stdio.h>
```

```
struct Node {
    int coeff; exp;
} Node *node;
3 *poly=NULL;

int main() {
    create(); printf("1/d", biolog[], eval(poly));
    return 0;
}
```

```
void Display (struct Node *p)
```

```
{ printf("%d%d%d", p->coeff, p->exp);
    p=p->next;
    printf("\n");
}
```

```
void create() {
    struct Node *t, *last;
    int num, i;
    printf ("Enter no. of terms ");
    scanf ("%d", &num);
    for (i=0; i<num; i++) {
        t = (struct Node *) malloc
            (sizeof (struct Node));
        scanf ("%d%d%d", &t->coeff,
               &t->exp);
        t->next=NULL;
        if (poly==NULL) {
            poly=last=t;
        } else {
            last->next=t;
            last=t;
        }
    }
}
```

long eval (struct node \*p, int z)

{ long val;

while (p)

{ val += p->coeff \* pow(z, p->exp);

p = p->next;

}

return val;

Application

3

Polynomial Addition

Stack →

LIFO

ADT Stack

Data

- Space for storing elements
- top pointer

Operations

1. push()
2. pop()
3. peek(index)
4. stackTop()
5. isEmpty()
6. isfull()

struct Stack

{ int size;

int top;

int \*S;

}

int main()

struct Stack st;

printf("Enter stack size: ");

scanf("%d", &size);

st.S = new int[size];

st.top = -1;

Empty

underflow

top = -1

full

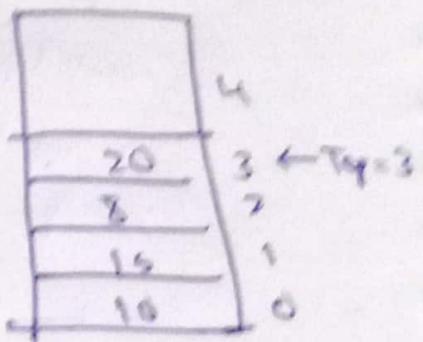
top = size - 1

O(1) →

pop push

Peek  $\rightarrow$

pos	Index = top - pos + 1
1	$3 = 3 - 1 + 1$
2	$2 = 3 - 2 + 1$
3	$1 = 3 - 3 + 1$



peek front  $\rightarrow$

int peek (stack st, int pos)

2 int z = -1;

if (st.top - pos + 1 < 0)

printf("Invalid position");

else

$z = st[s.top - pos + 1];$

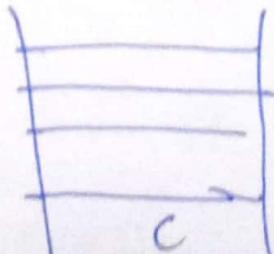
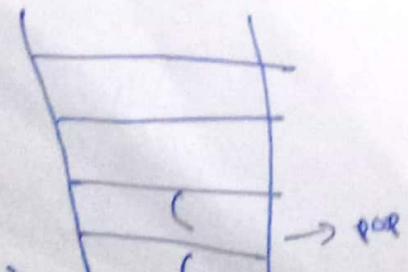
return z;

3

Application  $\rightarrow$

$$(a+b)+(c-d)$$

↑



(	(	a + b)	*	(	c - d)	)	)	)	)	b	
0	1	2	3	4	5	6	7	8	9	10	11

```

int isBalance (char *exp)
{
    stack stack st;
    for (i=0; exp[i] != '\0'; i++)
    {
        if (exp[i] == '(')
            push (&st, exp[i]);
        else if (exp[i] == ')')
        {
            if (!Empty (st))
                return false;
            pop (&st);
        }
    }
    return !Empty (st) ? false : true;
}

```

```

int main()
{
    char *exp = "(a+b)*(c-d))";
    printf ("%s", isBalanced (exp));
    return 0;
}

```

```

int isBalanced (char *exp) {
    int i;
    for (i=0; exp[i] != '\0'; i++)
    {
        if (exp[i] == '(') else if (exp[i] == ')')
        {
            push (exp[i]);
            if (top == NULL)
                return 0;
            pop();
        }
    }
}

```

```

if (top == NULL)
    return 1;
else
    return 0;
}

```

Conversions →

Infix → Postfix

$$8 + 3 * (9 - 0) / 2^2 + 6 / 2$$

Procedure

+	-	1
*	/	2
(	)	3

$$(a + b * c)$$

$$a + b c \quad \text{prefix}$$

$$abc * + \quad \text{postfix}$$

$$a + b + c + d$$

$$\rightarrow a + b + [c + d]$$

$$ab + cd +$$

$$ab + cd +$$

$$[ab+] + [cd+]$$

$$[ab + cd+] +$$

$$(a + b) * (c - d)$$

$$(ab) * (cd -)$$

$$\rightarrow ab + cd - *$$

## Queue ADT

### Data

- ① Space for storing elements
- ② front → deletion
- ③ Rear → insertion

### Operations

enqueue(x) dequeue()  
isEmpty() isFull()  
first() last()

Insert O(1)

Delete O(n)

Empty →

front == rear

initially

front = Rear = -1

enqueue → O(1)

dequeue → O(1)

full →

Rear == size - 1

disadvantage

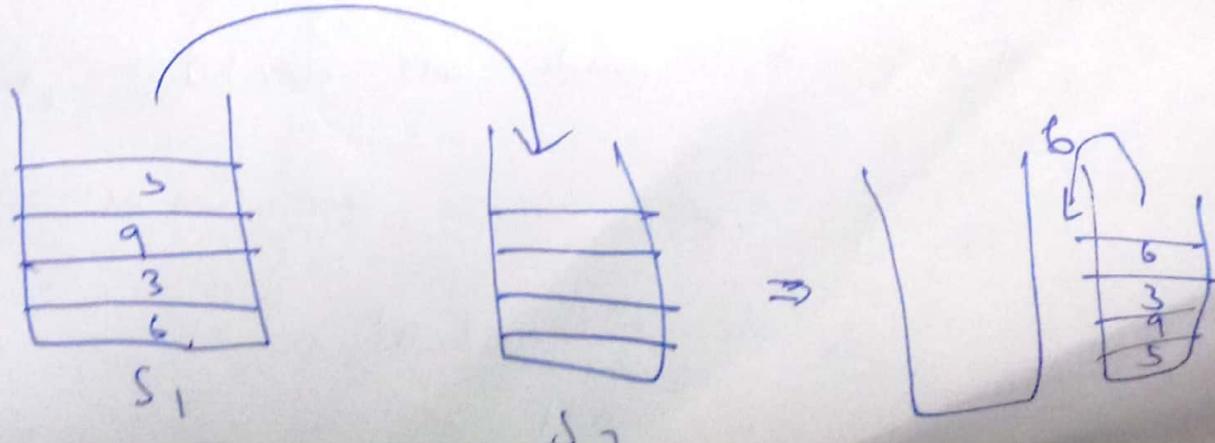


solutions

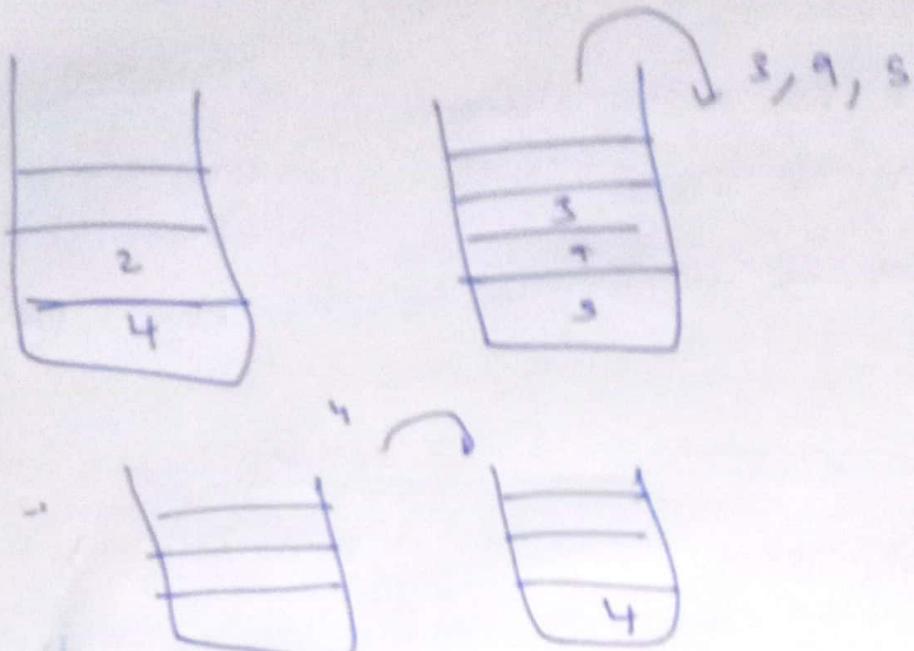
resetting pointers  
Circular Queue

Ques using 2 stacks →

elements → 6, 3, 9, 5, 4, 2, 8, 1, 10, ..



Exercise 4



Ans

Trees  $\rightarrow$

Degree  $\rightarrow$  no. of direct children

Degree of tree =  $\min_{\text{node}} (\text{max. degree of node})$

Forest = Collection of trees obtained from omitting the root

Binary tree  $\deg(T) = 2$ .

Catalan no. =  $\frac{2^n C_n}{n+1}$  (Shapes)

Binary trees possible with max height =  $2^{n-1}$

No. of ways binary tree can be

filled =  $\frac{2^n (n \times n!)}{n+1}$

## Height vs Nodes

$$\min \text{ nodes} = h+1$$

$$\max \text{ nodes} = 2^h - 1$$

for any tree

degree  $n$

$3 = \text{any tree}$   
 $\in \{1, 2, 3\}$

$$\boxed{\deg(o) = \deg(r) + 1}$$

Stack Binary tree

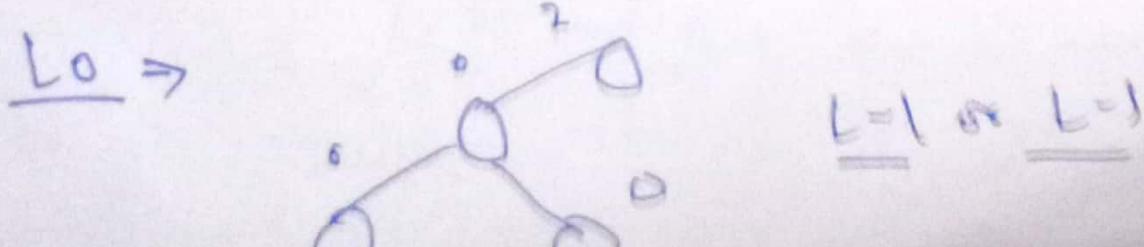
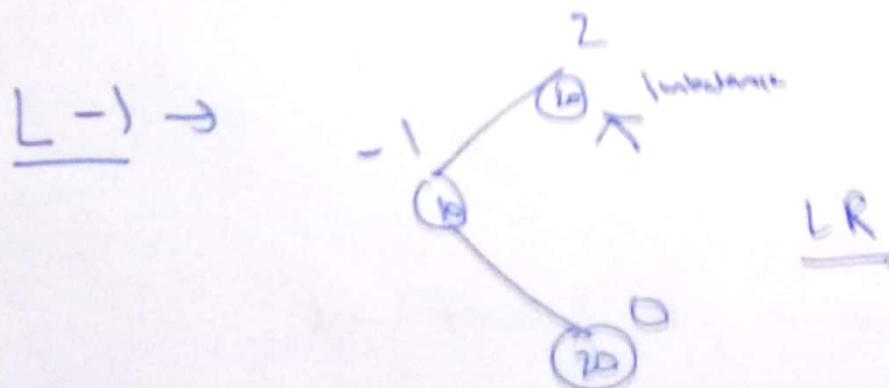
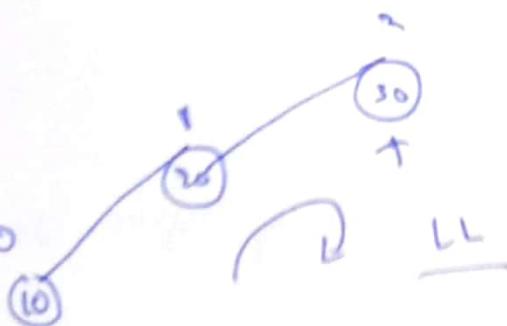
$\{0, 1, 2\}$

AVL  $\Rightarrow$  height balanced BST

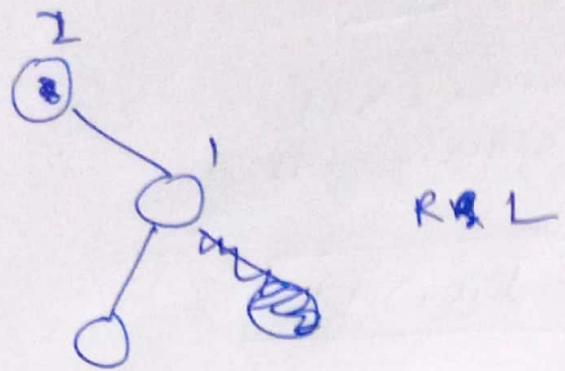
## Deletion

$L_1$   
 $L_{-1}$   
 $L_0$

$R_1$   
 $R_{-1}$   
 $R_0$

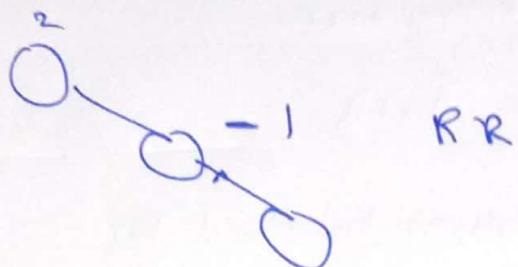


$R_1$



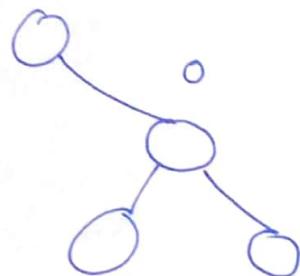
$R \rightarrow L$

$R_{-1}$



$R R$

$R_0$



$R_0 \& R_{-1}$

2-3 trees  $\Rightarrow$   
(B trees of degree 3)

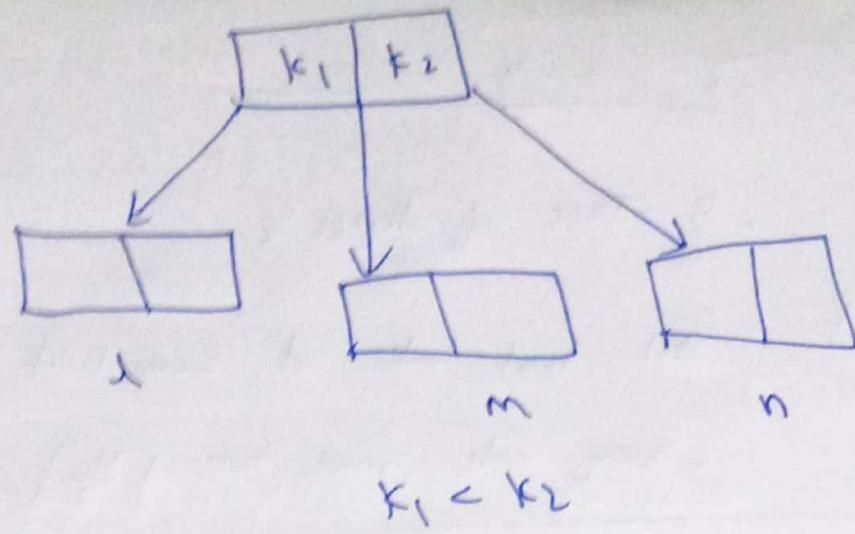
- Multiway Search Tree

- Degree

- B - Tree

- All leaf nodes are at same level

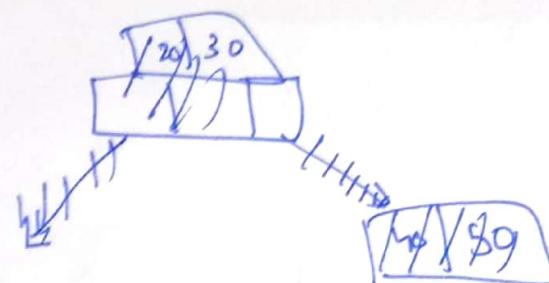
- Every node must have  $\left\lceil \frac{n}{2} \right\rceil = 2$  children



$l < k_1 \quad k_2 < n$

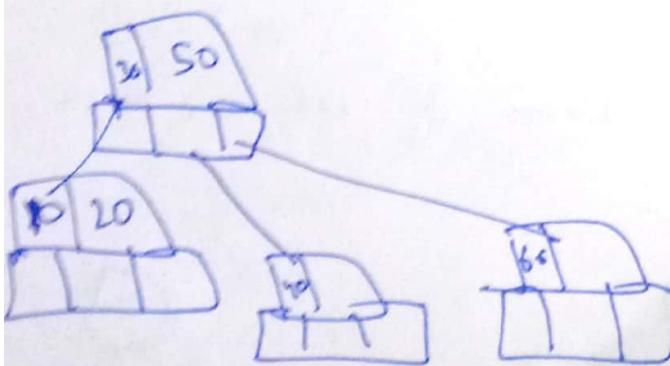
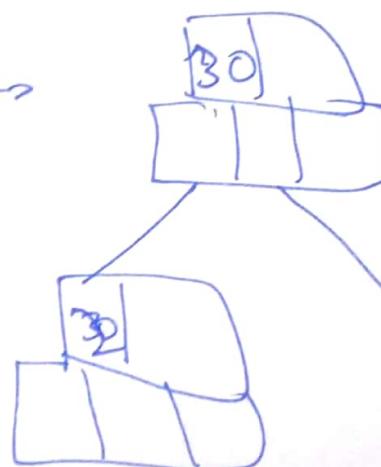
$k_1 < m < k_2$

Q. Keys  $\rightarrow 20, 30, 40, 50, 60, 10, 15, 70, 80$

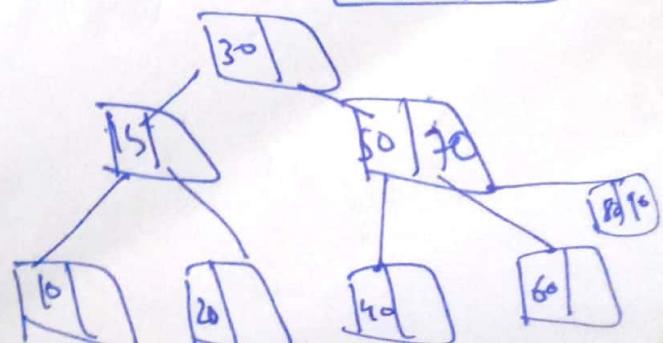


$20 \quad 30$  40

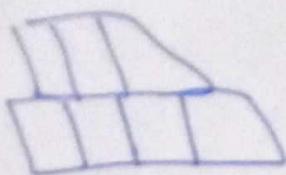
$\rightarrow$



$\rightarrow$



2 - 3 - 4



- B-Tree of degree 4
- All leaf nodes at same level
- Every node must have  $\lceil \frac{4}{2} \rceil = 2$

### Red-Black Tree

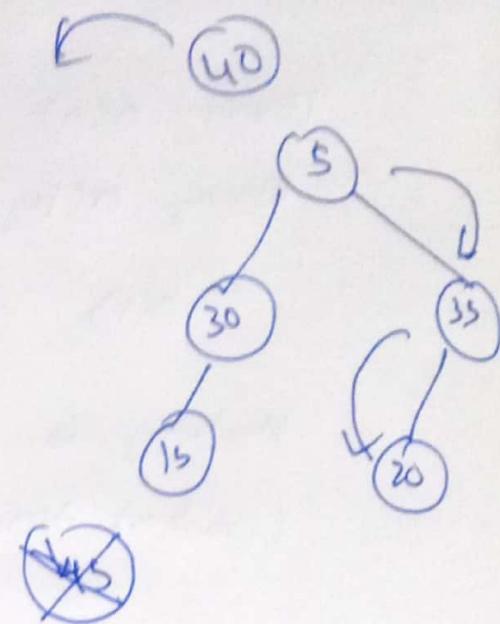
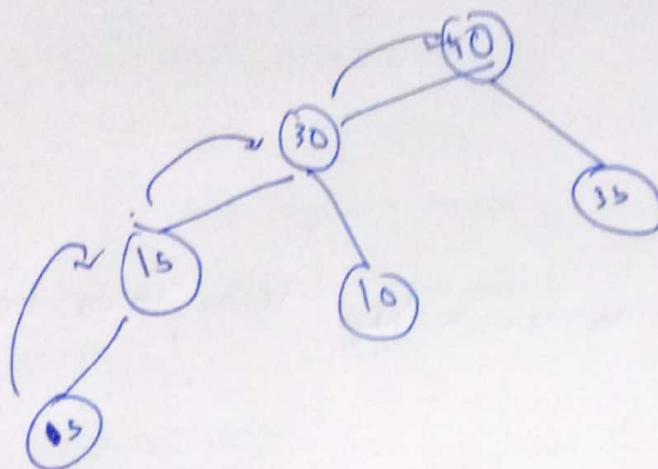
- Height balanced BST similar to 2-3-4 Tree
- Every node is either Red or Black.
- Root  $\rightarrow$  Black
- NULL
- Number of Black on paths from root to leaf are same.
- NO two red nodes should be there
- height  $\log n \leq h \leq 2 \log n$ 
  - less than AVL
- Parent & children of red are black

10 20 30



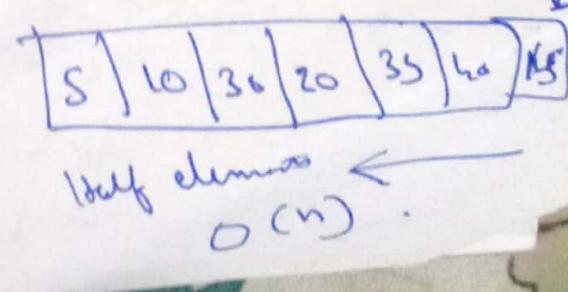
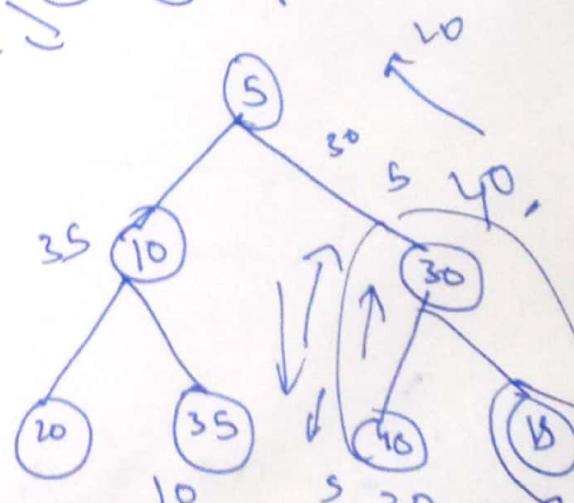
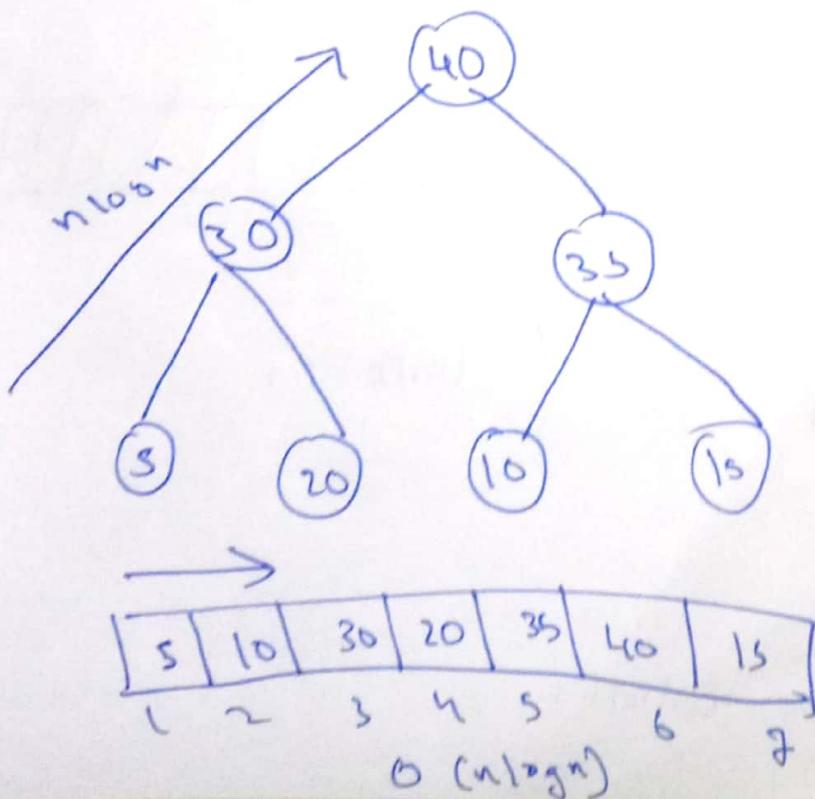
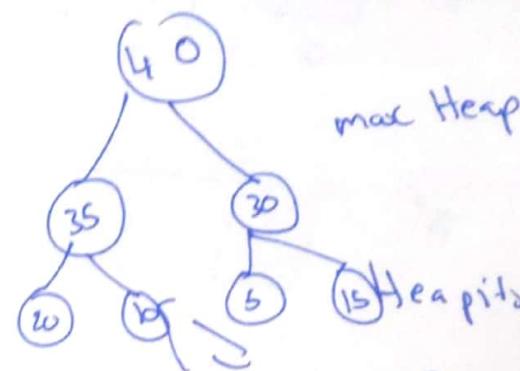
max Heap min Heap

Heapify



generally

$O(n \log n)$



## Hashing Technique

Keys : 8, 3, 6, 10, 15, 18, 4

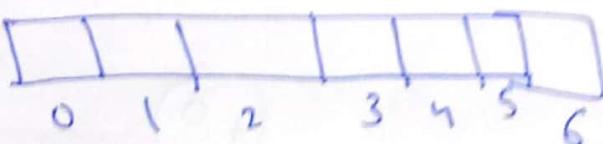
Linear  $O(n)$

Binary  $O(\log n)$

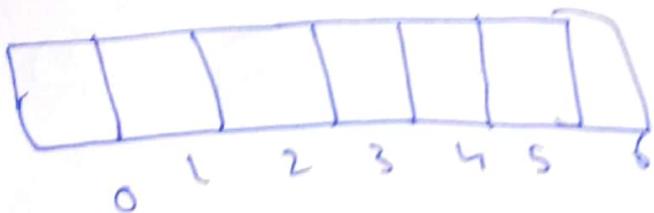
•  $O(1)$

Hashing is used to search key less than  $\log n$   
(constant time)

A



A



$$h(x) = x \mod 10 \quad (\text{collision})$$

ideal

one-one

one -

many - one collision.

\* Open Hashing →

- Chaining

\* Closed Hashing →

- Open Addressing

- ① Linear Probing

- ② Quadratic Probing

- ③ Double Hashing

Chaining →

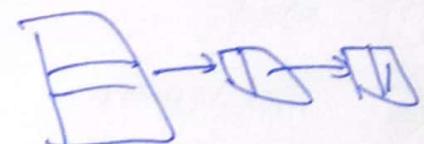
$$\lambda = \frac{\text{no. of keys}}{\text{size}}$$

$$h(x) = x \% 10$$

Succesful Search →

$$O(1) \quad t = 1 + \frac{\lambda}{2}$$

Avg.  
time



Avg. unsuccessful Search

$$t = 1 + \lambda$$

Linear Probing →

$$h(x) = (h(x) + i) \% 10 \quad \text{where } i = 0, 1, 2, 3$$

$$\text{Succesful } \sum_{i=0}^m \ln\left(\frac{1}{1-i}\right)$$

$$\frac{1}{1-\lambda} \quad \text{unsuccessful search}$$

Linear probing has problem of clustering

### Quadratic probing

$$h(x) = (h(x) + f(i)) \% 10$$

Double hashing →

$$h'(x) = (h_1(x) + i * h_2(x)) \% 10$$

$$h_1(x) = x \% 10$$

$$h_2(x) = 7 - (x \% 7)$$

Hash Functions →

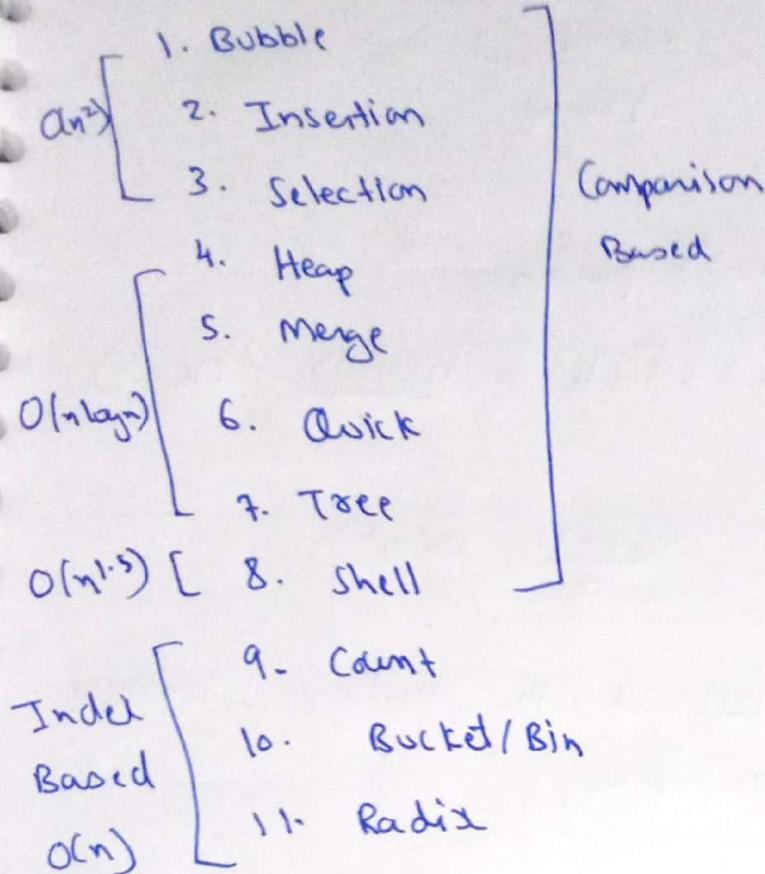
- ① mod  $h(x) = (x \% \text{size}) + 1$
- ② mid square  $h(x) = x^2 \quad (121) \quad \dots \square \dots$
- ③ folding  $ABC \quad \dots \square \dots$   
char →  $65 + 66 + 67$   
 $= X98$   
skip carry  
or take mod.

$$\begin{array}{r} 12 \\ 33 \\ 47 \\ \hline 92 = 9 + 2 = 11 \end{array}$$

if not  
there  
we can  
perform  
mod

we can design our fn but hash fn  
should always give same result.

## Sorting Techniques →



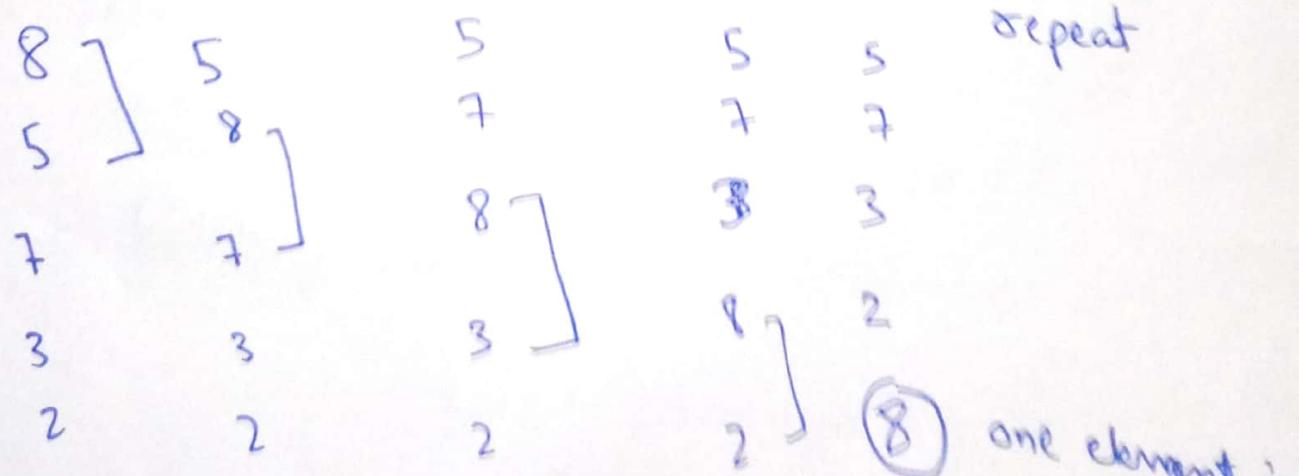
Criteria for Analysis →

- ① Number of Comparisons
- ② Number of swaps
- ③ Adaptive
- ④ Stable
- ⑤ Extra Memory

## Bubble Sort →

A	8	5	7	3	2
	0	1	2	3	4

1st Pass



4 comp

4 swap

one element is sorted.

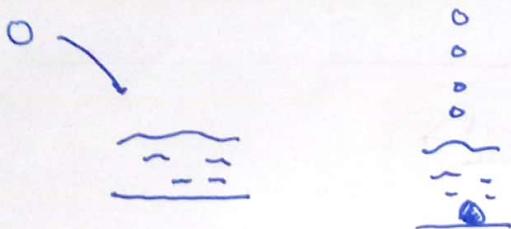
2 <sup>nd</sup> pass	3 <sup>rd</sup> pass	4 <sup>th</sup> pass
3 Comp	2 Comp	1 Comp
3 Swap	2 Swap	1 Swap

No. of passes :  $4 = (n-1)$  passes

No. of Comparisons :  $1+2+3+4 \rightarrow \frac{n(n-1)}{2} \quad O(n^2)$

Max. No. of swaps :  $\frac{n(n-1)}{2} \quad O(n^2)$

Stone is heavier, will settle at the bottom, bubbles are lighter they will raise up



void BubbleSort (int A[], int n) {

```
for (i=0; i<n-1; i++) { int flag; flag=0;
    for (j=0; j<n-1-i; j++) { if (A[j] > A[j+1])
        swap (A[j], A[j+1]) flag=1;
    }
}
```

$\downarrow \downarrow \downarrow$   
if ( $flag == 0$ ) break;

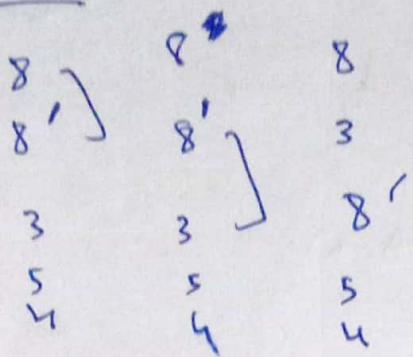
Adaptive  $\Rightarrow$  there can be cases when no swaps  
 $j \rightarrow 2$  (passes)

$O(n)$  3 (not done)  
5 (sorted) - (i.e. program is already  
7 (can be used to detect sorted  
8

min  $O(n)$

max  $O(n^2)$

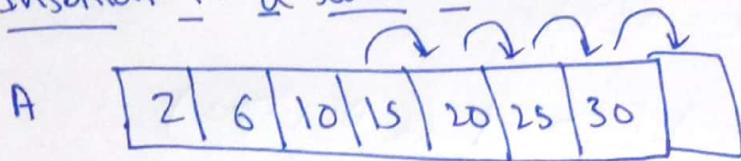
Stable



Insertion Sort  $\rightarrow$

Insertion in a sorted list  $\rightarrow$

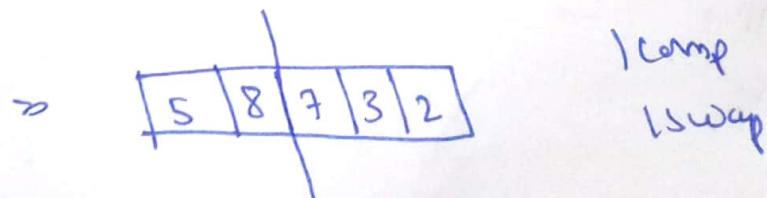
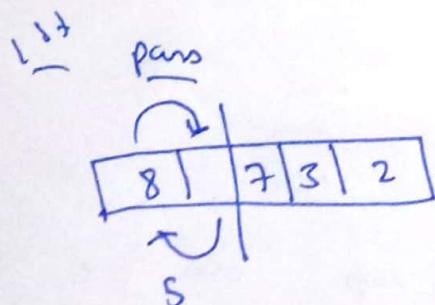
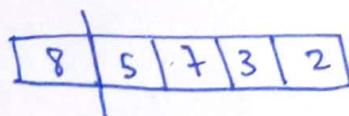
$O(1) - O(n)$



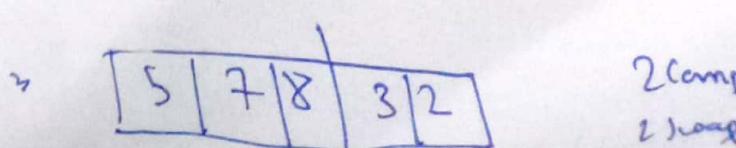
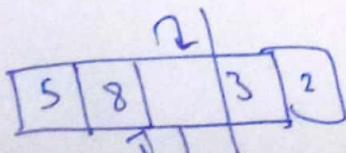
ele = 12      similarly, in linked list

Inserting element in sorted position in an array.

Insertion Sort  $\rightarrow$



2<sup>nd</sup> pass



3 <sup>rd</sup> pass	4 <sup>th</sup> pass	No. of passes $\rightarrow (n-1)$ passes
3 Comp	4 Comp	No. of comp. $\rightarrow \frac{n(n-1)}{2}$
3 pass	x pass	No. of swap $\rightarrow O(n^2)$

void insertionSort (int A[], int n)

{ for (i=1; i<n; i++)

{ j = i-1;

x = A[i];

while (j > -1 && A[j] > x)

{ A[j+1] = A[j];

j--;

$O(n^2)$

A[j+1] = x;

}

Analysis  $\rightarrow$

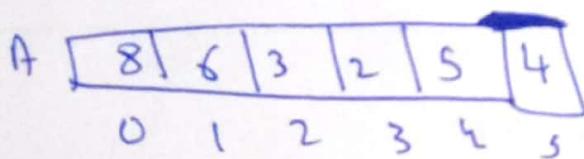
Adaptive  $\rightarrow$  By nature  
Stable.

min time  $O(n)$  max time  $O(n^2)$

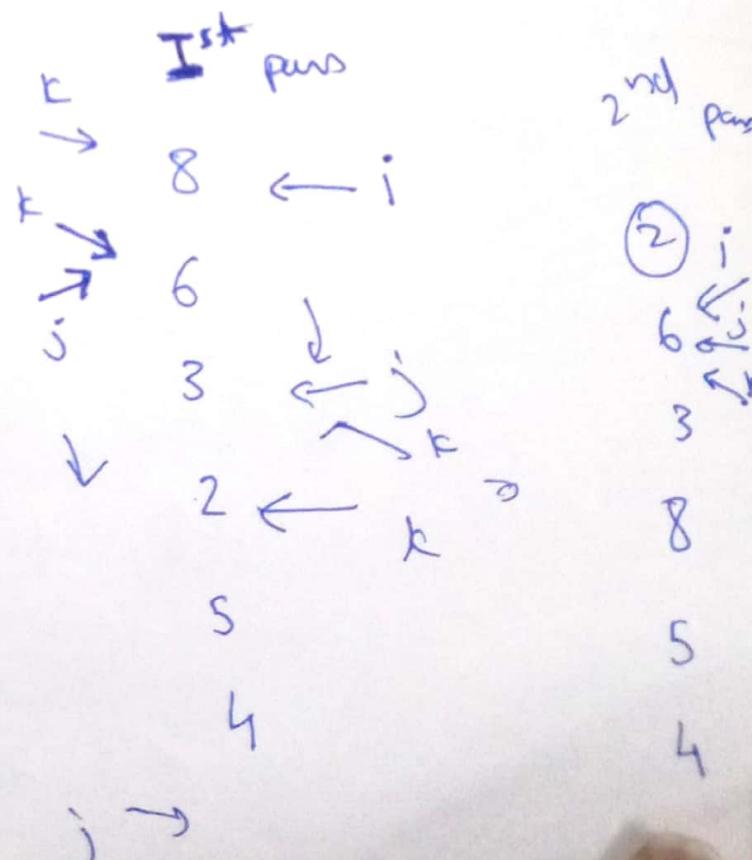
min Jumps  $O(1)$  max swap  $O(n^2)$

	Bubble Sort	Inversion Sort
min Comp	$O(n)$	$O(n)$
max Comp	$O(n^2)$	$O(n^2)$
min Swap	$O(1)$	$O(1)$
max Swap	$O(n^2)$	$O(n^2)$
Adaptive	✓	✓ → these are only adaptive sorts
Stable	✓	✓ only 3 sorts are stable
Linked List	No	Yes
k passes	Yes (2 passes 2 largest element)	No

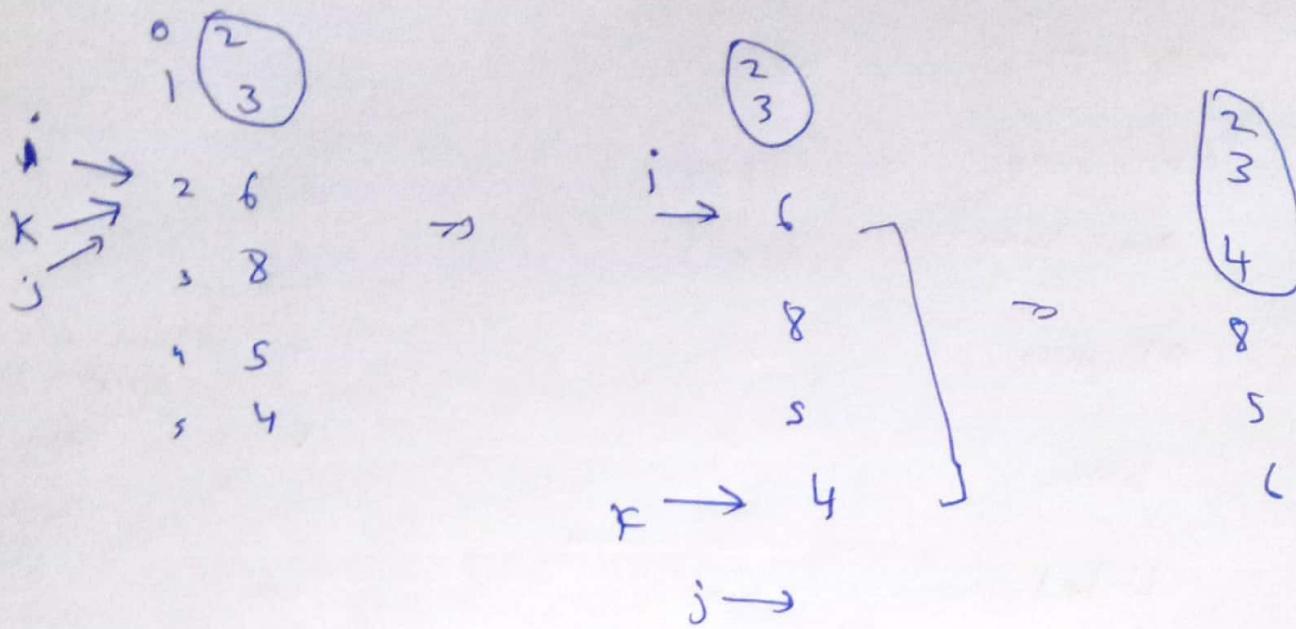
### Selection Sort $\Rightarrow$



j will find minimum element & if any element is found, bring j to that position.



III<sup>rd</sup> pass



Comp.

5      4      3      2      1

Swaps

1      1      1      1      1

min swaps.

No. of Comp. =  $O(n^2)$

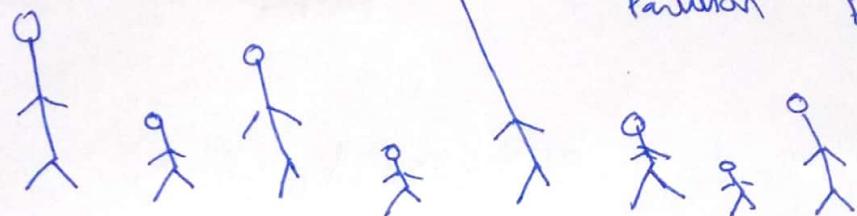
```
void selectionSort (int A[], int n)
{
    int i;
    for (i=0; i<n-1; i++)
    {
        for (j=k=i; j<n; j++)
        {
            if (A[j] < A[k])
                k=j;
        }
        swap (A[i], A[k]);
    }
}
```

Adaptive	Stable.
2 $\leftarrow^i_k$	8
4	3
8	8'
10	4
12	2
15	7

2  
3  
5  
8'  
4  
8  
7

X

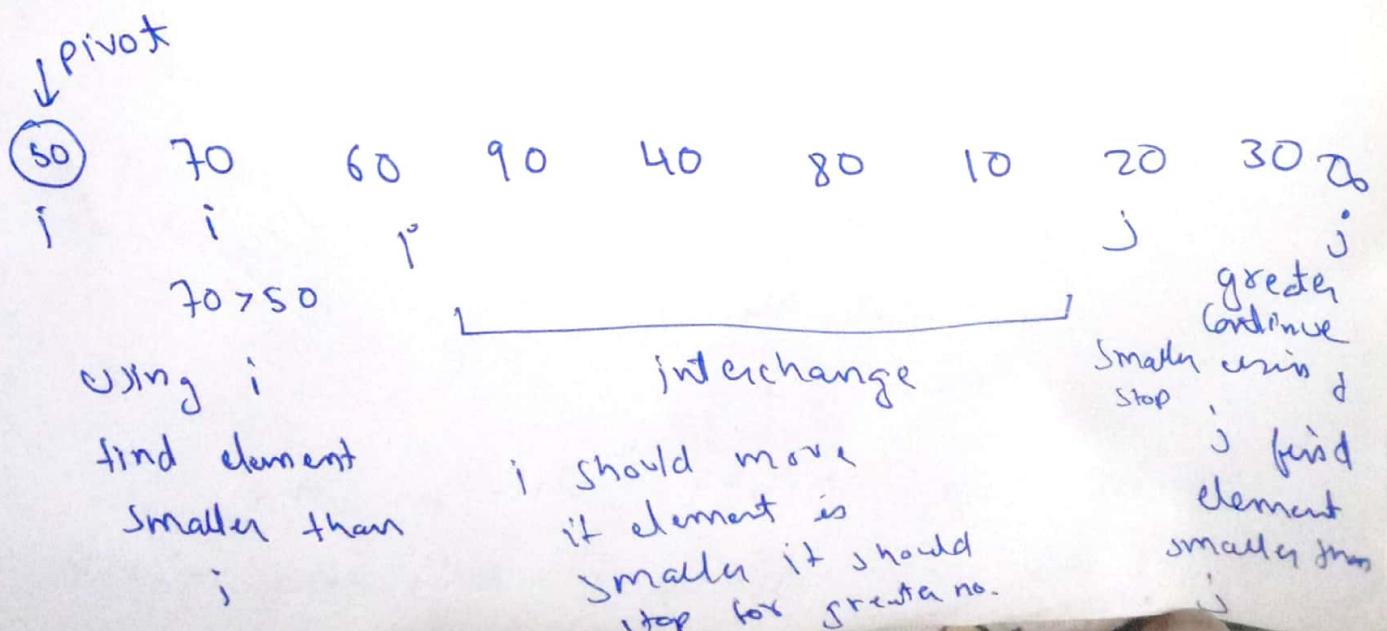
### Quick Sort

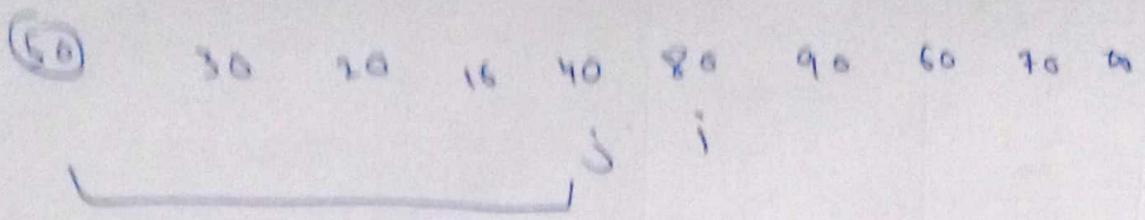


Selection  
Partition

Exchange/  
Exchange sort)

- (10) 30 20 70 40 90 80
- 80 70 40 30 20 10 (90)
- (40 30 20) (50) (90 70 80)





80  
 (40 30 20 10) (50) (80 90 60 70)  
 j  
 Partitioning position

```

int partition (int A[], int l, int h)
{
    int pivot = A[l];
    int i=l, j=h;
    do {
        do { i++; } while (A[i] <= pivot);
        do { j--; } while (A[j] > pivot);
        if (i < j)
            swap (A[i], A[j]);
    } while (i < j);
    swap (A[i], A[j]);
    return i;
}
    
```

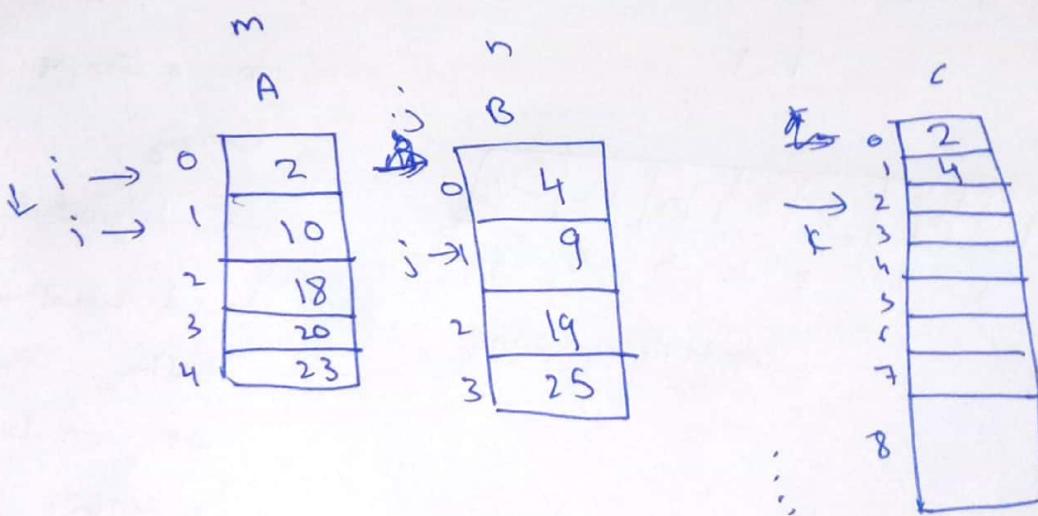
worst case  $O(n^2)$  already sorted

Best case  $O(n \log n)$  Partitioning middle

## Merging

- ① Merging 2 list
- ② Merging 2 list in single array
- ③ Merging multiple list

Merging is a process of combining two sorted lists into single sorted list.



```
void merge (int A[], int B[], int m, int n)
{ int i, j, k;
  i = j = k = 0;
  while (i < m && j < n)
  { if (A[i] < B[j])
      { C[k++] = A[i++]; }
    else
      { C[k++] = B[j++]; }
```

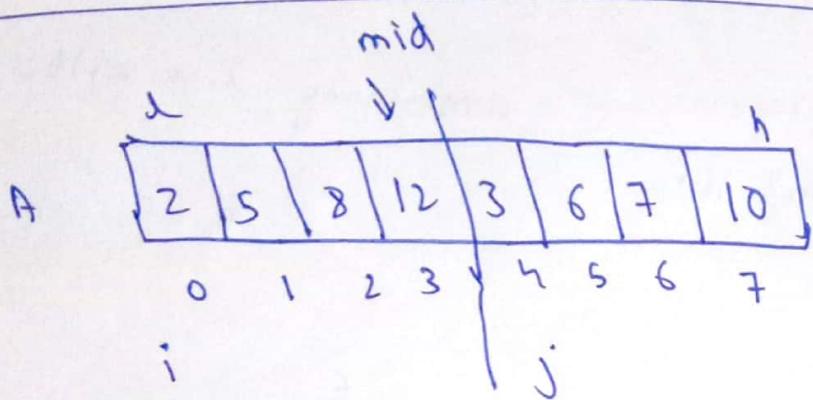
```
for ( ; i < m ; i++ )
```

```
    C[k + i] = A[i];
```

```
for ( ; j < n ; j++ )
```

```
    C[k + j] = B[j];
```

3

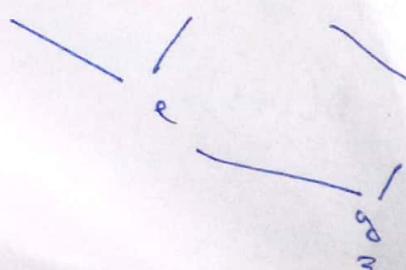


for merging  
into a single  
array,  
we definitely  
need another  
array than  
we can copy  
elements in  
earlier array

#### 4-way Merging

A      B      C      D

a      b      c      d



or  
2-way  
merging

#### m-way merging

e  
any  
pattern

```

void merge (int A[], int l, int mid, int h)
{
    int i, j, k;
    int B[h+1];
    for i = l; j = mid + 1; k = l;
    while (i <= mid && j <= h)
    {
        if (A[i] < A[j])
            B[k++] = A[i++];
        else
            B[k++] = A[j++];
    }
    for ( ; i < m; i++)
        B[k++] = A[i];
    for ( ; j < n; j++)
        B[k+1] = B[j];
    for (i=l; i<=h; i++)
        A[i] = B[i];
}

```

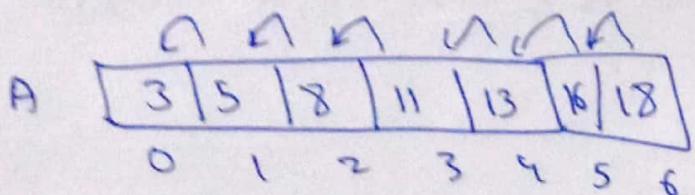
```

void MergeSort (int A[], int l, int h)
{
    if (l < h)
    {
        int mid =  $\lfloor (l+h)/2 \rfloor$ ;
        MergeSort (A, l, mid);
        MergeSort (A, mid+1, h);
        merge (A, l, mid, h);
    }
}

```

$n \log n$

Shell Sort  $\rightarrow$  (Insertion sort used in shell sort)



$$gap = \left\lfloor \frac{n}{2} \right\rfloor$$

Successive division

$n \log n$  each pass  
Scan =  
1st pass  
no. of passes

Normally gaps are taken from prime numbers  
11 elements 7 5 2

$$O(n^{3/2}) \quad n^{1.5} < n^2 \quad n^{5/3} \quad n^{2+66}$$

$$O(n \log n) \quad O(n^{1.5}) \quad O(n^{1.66})$$

adaptive

Don't take extra space.

Only merge sort in comparison based sort takes auxiliary array.

Heap Sort  
 $(O(n \log n))$

Heapsort  
 $O(n)$ )

# include <stdio.h>

void Insert(int H[], int n)

{ int i = n, temp;

```

temp = A[i];
while (i>1 && temp > A[i/2])
{
    A[i] = A[i/2];
    i=i/2;
}
A[i] = temp;

}

void int Delete (int A[], int n);

int main ()
{
    int H[] = {0, 2, 5, 8, 9, 4, 10, 73};
    for (i=2; i<=7; i++)
        Insert (H, i);
    printDelete (1, 7);
    return 0;
}

int Delete (int A[], int n)
{
    int i, j; int temp, val;
    x = A[n];
    val = A[1];
    A[1] = A[n];
    i = 1; j = i*2;
    while (j < n)
    {
        if (A[j+1] > A[j]) j += 1;
        if (A[i] < A[j]) {
            temp = A[i];
            A[i] = A[j];
            A[j] = temp;
            i = 2*j;
            j = 2*j + 3;
        }
    }
    return val;
}

```

## Index Based Sorting →

① Count Sort →

→	6	3	9	10	15	6	8	12	3	6
	0	1	2	3	4	5	6	7	8	9

→	0	0	0	2	0	0	3	0	1	1	1	0	1	0	0	1
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

→	3	3	6	6	6	8	9	10	12	15
---	---	---	---	---	---	---	---	----	----	----

→	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

$O(m+n)$

$O(n)$

Void CountSort (int A[], int n)

{ int max; }

int \*C;

max = findMax(A, n);

C = new int [max+1];

for (i=0; i < max+1; i++)

C[i] = 0;

```
for (i=0; i<n; i++)  
    C[A[i]]++;
```

i=0; j=0;

while (i < max + 1)

{ i++ (C[i] > 0)

{ A[j+i] = i

{ C[i] --;

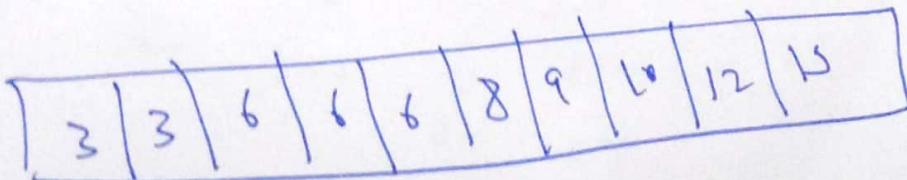
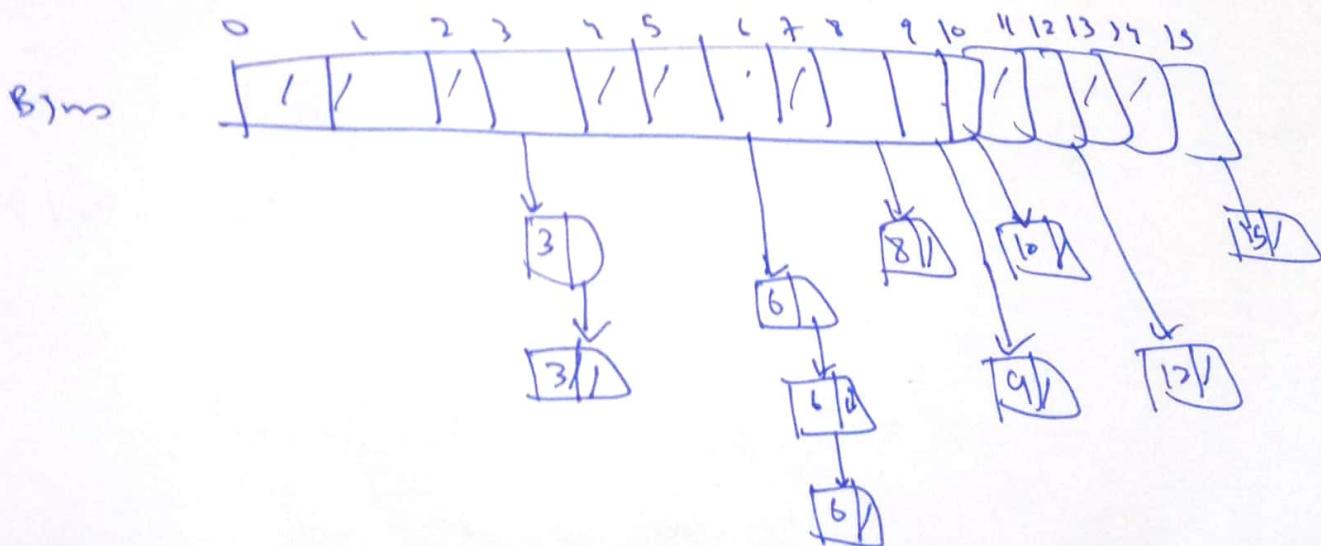
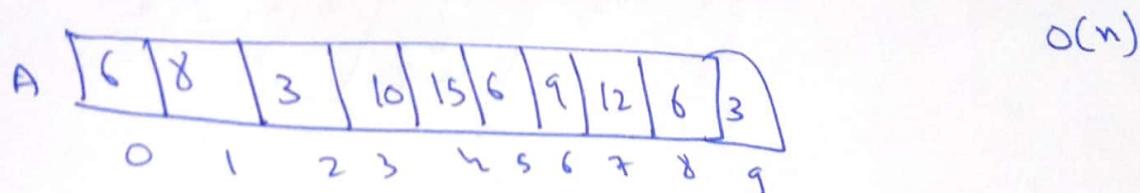
}

else { i++ }

}

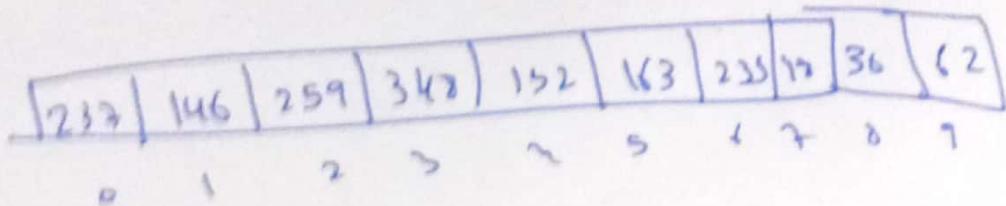
,

Bucket / Bin Sort →

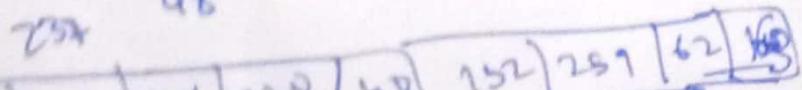
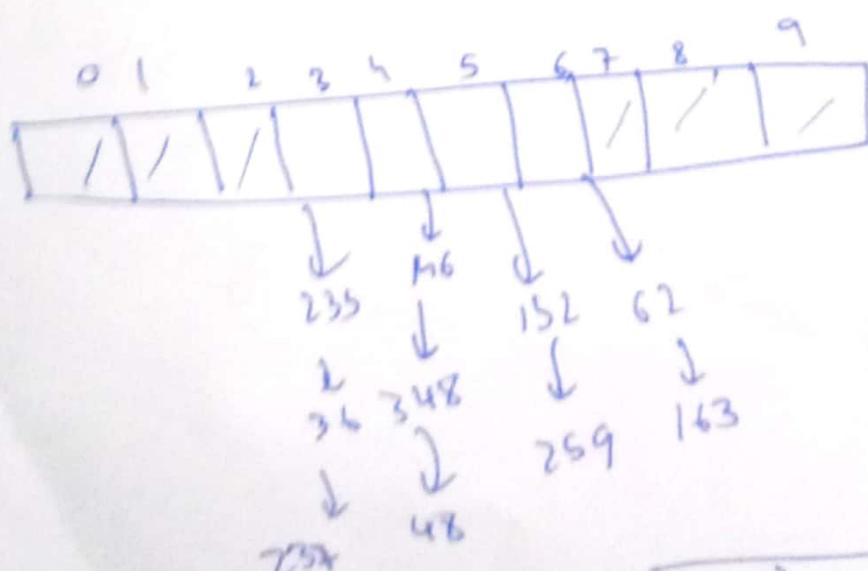
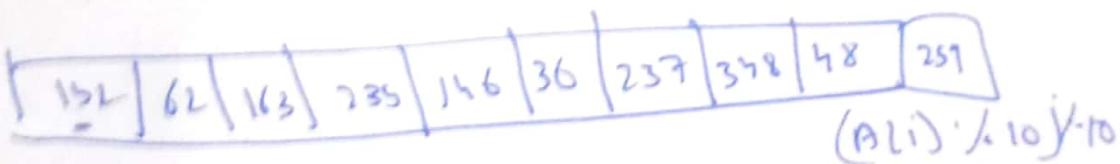
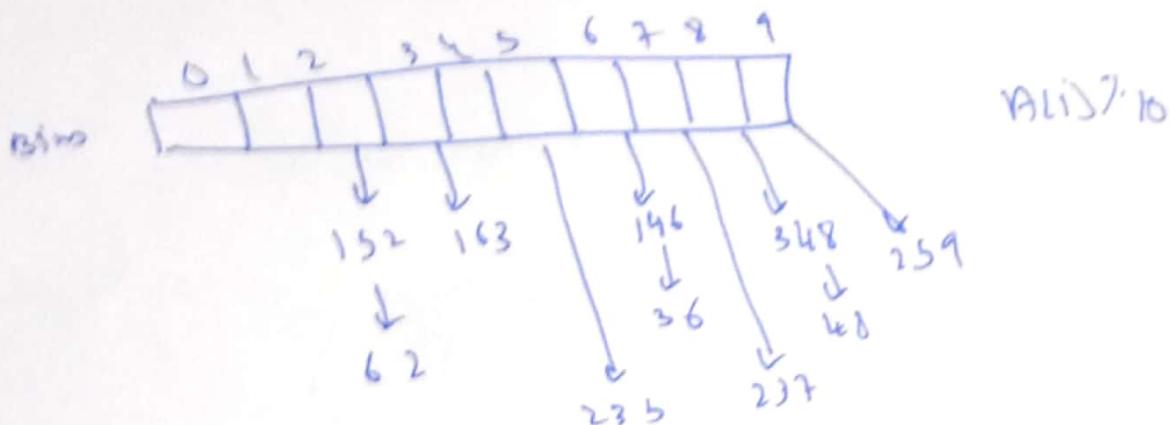


## Radix Sort

In quick sort, we were taking array of bins of length of length  $\sqrt{n}$

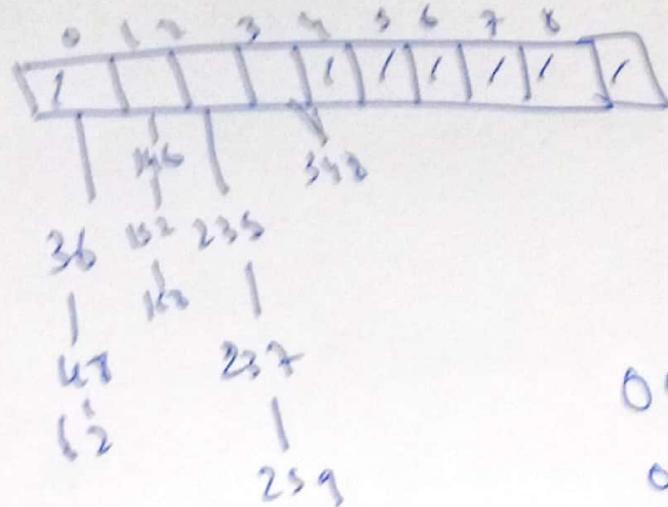


we won't take  $348$  bins, we'll take  $10$  bins



$$(M_1/160) \cdot 1.10$$

3rd year



36 48 62 146 152 168 235 237 259 362