2) Part A

if $g = h * f$

Apply Fourier transform

$\Rightarrow \quad G = H \cdot F$

$\Rightarrow \quad F = G/H \quad$ when $H(u) \neq 0$

$\quad\quad\quad\quad c(u)$ otherwise.

Depending on whether h is 1st or 2nd or nth order derivative the boundary conditions change.

$$f(x) = \underbrace{IFT(F_1)(x) + \sum_{\text{when } H(u)=0} c(u) \cdot e^{iux}}$$

where $F_1 = F$ when $H(u) \neq 0$

$\quad\quad\quad 0$ otherwise.

same as IFT(F).

Determine $c(u)$ by putting boundary conditions

↳ Boundary condition here would be pixel intensity at pixels or equivalent information for nth order derivative

Part B:

$g_x = h_x * f \quad g_y = h_y * f$

Apply Fourier transform

$G_x = H_x F \quad G_y = H_y F.$

$$\therefore F = \begin{cases} \dfrac{G_x}{H_x} = \dfrac{G_y}{H_y} & \text{when } H_x, H_y \neq 0 \\ \dfrac{G_y}{H_y} & H_x = 0,\ H_y \neq 0 \\ \dfrac{G_x}{H_x} & H_y = 0,\ H_x \neq 0 \\ c(u,v) & H_x = H_y = 0. \end{cases}$$

Define $F_1 = \begin{cases} 0 & H_x = H_y = 0 \\ F & \text{otherwise.} \end{cases}$

clearly $f(x,y) = IFT(F_1)(x,y) + \sum_{H_x = H_y = 0} e^{i(ux+vy)} \cdot c(u,v)$

Determine $c(u,v)$ by putting boundary conditions.

→ We need boundary conditions as we loose information while convolving with gradient kernal (could be any other kernal as well)

↳ Boundary condition here would be pixel intensity at one pixel. (say (m,n))

↳ As this can be thought of as many rows differentiated or coloumns, from 1st part we can determine pixel intensities of the pirticular row "m" & from this using y-gradients we can determine all the other pixel intensites, which is same as finding all the Fourier coefficients.