

# Microeconomic Theory I: A Notebook

*With Jonathan Libgober*

**Sai Zhang**

Check my [Github Page](#), or [email me](#)!

September 1, 2021

---

## HERE WE GO!

This is my learning notebook of Microeconomic Theory I (Course number: ECON601 at USC Economics). As one of the core courses in an economic Ph.D. curriculum, Microeconomic Theory I is beyond important to my research. Therefore, I would love to use this notebook as a commitment mechanism, to document lecture notes, discuss session and office hour intuitions, reading summaries, my personal questions regarding the topics and more. By building a file from scratch, hopefull I could have a more systematic and sophisticated understanding on the content of this course.

I thank Prof. Jonathan Libgober at USC Economics for leading the discussion of the course and providing intuitive ways to understand microeconomic theory. Please check his webpage [here](#), he is such fun.

I also appreciate the time and effort my TA Qitong Wang put into this course, guiding me through discussing sessions and problem sets. When I have questions, he is always there to help.

Following the structure of the course, this notebook will cover three aspects of microeconomic theories: (a) individual decision making, (b) game theory, (c) mechanism design and contract theory. Apart from Jonathan's lecture notes, I will also summarize the reading materials, including: [Mas-Colell et al. \(1995\)](#)'s *Microeconomic Theory*, [Mailath \(2018\)](#)'s *Modelling Strategic Behavior*<sup>1</sup>, [Fudenberg and Tirole \(1991\)](#)'s *Game Theory*, [Myerson \(1991\)](#)'s *Game Theory: Analysis of Conflicts*, [Bolton and Dewatripont \(2005\)](#)'s *Contract Theory*, [Mailath and Samuelson \(2006\)](#)'s *Repeated Games and Reputation* and [Osborne and Rubinstein \(1994\)](#)'s *A Course in Game Theory*. Other materials will also be referred to along the way.

Building this notebook is truly a memorable journey for me. I would love to share this review and all the related materials to anyone that finds them useful. And unavoidably, I would make some typos and other minor mistakes (hopefully not big ones). So I'd really appreciate any correction. If you find any mistakes, please send the mistakes to this email address [saizhang.econ@gmail.com](mailto:saizhang.econ@gmail.com), BIG thanks in advance!

---

<sup>1</sup>Latest version (May 2021) available [here](#).

# Contents

<b>I</b>	<b>Individual Decision Making</b>	<b>4</b>
<b>1</b>	<b>Preferences and Choices, Utilities</b>	<b>5</b>
1.1	Preference Relations . . . . .	6
1.2	Choice Rules . . . . .	9
1.3	Linking Preferences with Choices . . . . .	10
1.4	Chap1Sec4 . . . . .	10
<b>2</b>	<b>Fundamentals of Consumer Theory</b>	<b>11</b>
<b>3</b>	<b>Lagrange Maximization and Duality</b>	<b>12</b>
<b>4</b>	<b>Monotone Comparative Statics</b>	<b>13</b>
<b>5</b>	<b>Expected Utility and Decisionmaking under Uncertainty</b>	<b>14</b>
<b>6</b>	<b>Aggregation and the Existence of a Representative Consumer</b>	<b>15</b>
<b>7</b>	<b>Producer Theory</b>	<b>16</b>
<b>8</b>	<b>Stochastic Choice</b>	<b>17</b>
<b>II</b>	<b>Game Theory</b>	<b>18</b>
<b>9</b>	<b>Nash Equilibrium and Bayesian Nash Equilibrium</b>	<b>19</b>
<b>10</b>	<b>Rationalizability and DOminant Strategies</b>	<b>20</b>
<b>11</b>	<b>Correlated Equilibrium</b>	<b>21</b>
<b>12</b>	<b>Dynamic Games and Refinements</b>	<b>22</b>
<b>13</b>	<b>Repeated Games/Folk Theorem</b>	<b>23</b>
<b>14</b>	<b>Recursive Methods in Repeated Games</b>	<b>24</b>

<b>III Mechanism Design and Contract Theory</b>	<b>25</b>
15 Arrow's Theorem and Social Choice	26
16 Boundaries of the Firm and Coase's Theorem	27
17 Implementation Concepts	28
18 The Revelation Principle	29
19 Auctions and Optimal Auctions	30
20 Efficient Implementation	31
21 Moral Hazard	32
22 Full Implementation	33
Bibliography	34

**Part I**

**Individual Decision Making**

---

---

# CHAPTER 1

---

## PREFERENCES AND CHOICES, UTILITIES

### Contents

---

1.1	Preference Relations . . . . .	6
1.2	Choice Rules . . . . .	9
1.3	Linking Preferences with Choices . . . . .	10
1.4	Chap1Sec4 . . . . .	10

---

The first chapter summarizes the basic setting of individual decision making: preferences, choices and utilities. The main reference is Chapter 1 of [Mas-Colell et al. \(1995\)](#).

In this chapter, we will focus on 3 domains:

---

<b>choice</b>	given a set $A$ , what choice from $A$ is made
<b>preference</b>	given alternatives $x, y$ , which does the decision maker prefers
<b>utility</b>	given an object $X$ , how much does the DM likes $X$ (as a number)

---

The starting point of individual decision problem is a *set of possible (mutually exclusive) alternatives* from which the individual must choose. To model decision making process on this set of alternatives, one can:

- either start from the tastes, i.e., *preference relations* of individuals, and set up the patterns of decision making with preferences
- or, start from the actual actions of individuals, i.e. *choices*, to deduct a pattern of decision making

With this two major approaches in mind, we know what's coming: the *rationality* of preferences and the central assumption of choices, the *Weak Axiom of Revealed Preference (WARP)*. And of course, the two approaches and two basic assumptions are parallel, so we need to figure out how link the (underlying) preferences and (observed) choices.

## 1.1 Preference Relations

We start from the basic: *weak preference relation*,  $\succeq$ .

**Definition 1.1.1.** A weak preference relation  $\succeq$  on a set  $X$  is a subset of  $X \times X$ . If  $(x, y) \in \succeq \Rightarrow x$  is at least as good as  $y$ , written as  $x \succeq y$

A weak preference relation will induce two other types of relations on  $X$ :

**Definition 1.1.2.** With  $\succeq$  defined by Def. 1.1.1, we have

- the *strict preference relation*,  $>$  can be induced from  $\succeq$  as:  $x > y \Leftrightarrow x \succeq y \wedge y \not\succeq x$ , or in words,  $x$  is preferred to  $y$ .
- the *indifference relation*,  $\sim$  can be induced from  $\succeq$  as:  $x \sim y \Leftrightarrow x \succeq y \wedge y \succeq x$ , or in words,  $x$  is indifferent to  $y$ .

With the definition of these relations, we now define the central assumption of relations: *rationality*.

**Definition 1.1.3.** A weak preference relation  $\succeq$  is *rational* if it is:

- Complete:  $\forall x, y \in X, x \succeq y$  or  $y \succeq x$  or both
- Transitive:  $\forall x, y, z \in X, x \succeq y \wedge y \succeq z \Rightarrow x \succeq z$

How to understand them? They are both strong assumptions:

- Completeness of  $\succeq$  means it is well-defined between any two possible alternatives. From the perspective of an individual, completeness means that she will make choices, and only meditated choices.
- Transitivity of  $\succeq$  implies that the decision maker will not have a preference cycle, since whoever has a preference cycle would suffer economically for it<sup>1</sup>.

With the definition of rational  $\succeq$  in Def. 1.1.3, we can prove the following properties of  $>$  and  $\sim$  induced by  $\succeq$ :

**Theorem 1.1.1.** If  $\succeq$  is rational, then:

- i.  $>$  is irreflexive ( $x > x$  never holds) and transitive ( $x > y \wedge y > z \Rightarrow x > z$ )

Proof:

- irreflexive: by Def. 1.1.2,  $x > x \Rightarrow x \succeq x \wedge x \not\succeq x$ , self contradiction.
- transitive:  $x > y \Rightarrow x \succeq y \wedge y \not\succeq x$ ,  $y > z \Rightarrow y \succeq z \wedge z \not\succeq y$ . By transitivity of  $\succeq$ ,  $x \succeq y \wedge y \succeq z \Rightarrow x \succeq z$ . If  $z \succeq x$ , by transitivity of  $\succeq$  and  $x \succeq y$ , we would have  $z \succeq y$ , contradicting  $y > z$ . Therefore  $x \succeq z \wedge z \not\succeq x \Rightarrow x > z$ .

- ii.  $\sim$  is reflexive ( $x \sim x, \forall x$ ), transitive ( $x \sim y \wedge y \sim z \Rightarrow x \sim z$ ) and symmetric ( $x \sim y \Rightarrow y \sim x$ )

<sup>1</sup>There are 2 types of violations of transitivity: irrational and mechanical. Irrational violations are easy to understand: decision makers simply do not follow transitivity assumption, many reasons have been raised, including mental account, framing, menu effect, attraction effect, etc. Mechanical violations means that decision makers are "forced" to violate transitivity. One example of this type of violation is aggregation of considerations: decision makers may aggregate several sub-preferences as together to make the choice, leading to violation of transitivity. Another example is when the preference is only defined for differences above a certain level (problem of perceptible differences). See Mas-Colell et al. (1995, Page 7-8), Rubinstein (2012, Page 4-5) for details

Proof:

- reflexive: by completeness of  $\succeq$ ,  $\forall x, x \succeq x \Rightarrow x \sim x$
  - transitive:  $x \sim y \Rightarrow x \succeq y \wedge y \succeq x$ ,  $y \sim z \Rightarrow y \succeq z, z \succeq y$ , by the transitivity of  $\succeq$ , we have  $x \succeq z \wedge z \succeq x$ , hen  $x \sim z$
  - symmetric:  $x \sim y \Rightarrow x \succeq y \wedge y \succeq x \Leftrightarrow y \succeq x \wedge x \succeq y \Rightarrow y \sim x$
- iii.  $x > y \succeq z \Rightarrow x > z$

Proof:  $x > y \Rightarrow x \succeq y \wedge y \not\succeq x$ , hence  $x > y \succeq z \Rightarrow x \succeq z$ . If  $z \succeq x$ , by transitivity of  $\succeq$ ,  $y \succeq x$ , contradicting  $x > y$ . Therefore,  $z \not\succeq x$

We can also directly define a *rational*  $>$  (see [Kreps \(1990, Page 19-21\)](#)):

**Definition 1.1.4.** A strict preference relation  $>$  is rational if it is:

- asymmetric:  $\nexists x, y \in X$  s.t.  $x > y \wedge y > x$
- negatively transitive:  $x > y \Rightarrow \forall z \in X \setminus \{x, y\}, x > z \vee z > y \vee$  both.

With Def. 1.1.4 and Def. 1.1.3, we can prove that  $\succeq$  is rational iff  $>$  is rational:

**Theorem 1.1.2.**  $\succeq$  is rational  $\Leftrightarrow >$  is rational, specifically:

- $\succeq$  is complete  $\Leftrightarrow >$  is asymmetric
- $\succeq$  is transitive  $\Leftrightarrow >$  is negatively transitive

Now we prove this theorem:

**Step 1** proof  $\succeq$  is rational  $\Rightarrow >$  is rational

- **asymmetric**

if  $\exists x, y$  s.t.  $x > y$  and  $y > x$ , then by the definition of induced strict preference, the pair  $x, y$  must satisfy

$$\begin{cases} x \succeq y \text{ and } y \not\succeq x & (x > y) \\ y \succeq x \text{ and } x \not\succeq y & (y > x) \end{cases}$$

which is, by completeness of rational  $\succeq$ , impossible. Therefore, such pair  $x, y$  don't exist.  $>$  is proved to be asymmetric.

- **negatively transitive**

First,  $\forall z \notin \{x, y\}$ , by completeness of rational  $\succeq$ , the relation between  $x$  and  $z$  is either  $x \succeq z$  or  $z \succeq x$ . Similarly, the relation between  $y$  and  $z$  is either  $y \succeq z$  or  $z \succeq y$ .

Second, given  $x > y$ ,  $x, y$  satisfies  $x \succeq y$  and  $y \not\succeq x$ .

Also, it is easy to prove that:  $x > y \wedge y \succeq z \Rightarrow x > z$ ,  $x > y \wedge z \succeq x \Rightarrow z > y$ ; and  $x > y \wedge z \sim x \Rightarrow z > y$ ,  $x > y \wedge y \sim z \Rightarrow x > z$

Now we have the following scenarios:

1. if  $z \succeq x$  and  $y \succeq z$ , by transitivity of rational  $\succeq$ ,  $y \succeq x$ , contradicting the definition of  $x > y$ . This scenario doesn't exist.
2. if  $x \succeq z$  and  $y \succeq z$ , since  $x > y$ , with the auxiliary result proved above, we have  $x > z$
3. if  $z \succeq x$  and  $z \succeq y$ , since  $x > y$ , with the auxiliary result proved above, we have  $z > y$
4. if  $x \succeq z$  and  $z \succeq y$ , since  $x > y$ , suppose:



- (a)  $z \succeq x$  as well, then  $x \sim z$ , in this case  $z > y$ ;  
 (b)  $z \not\succeq x$ , then  $x > z$   
 (c)  $y \succeq z$  as well, then  $y \sim z$ , in this case  $x > z$   
 (d)  $y \not\succeq z$ , then  $z > y$   
 therefore, a complete summary of (a) to (d) would give:

	$z \succeq x$	$z \not\succeq x$
$y \succeq z$	$z > y \ \& \ x > z$	$x > z$
$y \not\succeq z$	$z > y$	$x > z \ \& \ z > y$

Combining all above, we have proved negative transitivity of  $>$ .

With asymmetry and negative transitivity proved, we've proved that  $\succeq$  is rational  $\Rightarrow >$  is rational

**Step 2** proof  $>$  is rational  $\Rightarrow \succeq$  is rational.

- Complete: with a rational  $x > y$ , we know  $\nexists x, y$  s.t.  $x > y$  and  $y > x$  by asymmetry. Therefore,  $\forall x, y$ , we have two possibilities.
  - $x > y$  and  $y \not\succeq x$ , which would naturally induce a weak preference  $x \succeq y$
  - $y > x$  and  $x \not\succeq y$ , which would naturally induce a weak preference  $y \succeq x$
 therefore,  $\forall x, y$ , either  $x \succeq y$  or  $y \succeq x$  completeness of  $\succeq$  is proven.
- Transitive: with a rational  $x > y$ , negative transitivity gives  $\forall z \notin \{x, y\}$ , either  $x > z$ ,  $z > y$ , or both. By negative transitivity, we have:
  - $x > z$ : following same procedure, we know  $x \succeq z$ . If:
    - \*  $y \succeq z$ , since  $x > z \Rightarrow z \not\succeq x$ , by completeness we have  $x \succeq z$ , thus  $x \succeq y \wedge y \succeq z \Rightarrow x \succeq z$
    - \*  $z \succeq y$ , since  $x > y \Rightarrow x \not\succeq y$ , by completeness we have  $x \succeq y$ , thus  $x \succeq z \wedge z \succeq y \Rightarrow x \succeq y$
  - $z > y$ : again, we know  $z \succeq y$ . If:
    - \*  $x \succeq z$ , since  $x > y \Rightarrow y \not\succeq x$ , by completeness we have  $x \succeq y$ , thus  $z \succeq y \wedge x \succeq z \Rightarrow x \succeq y$
    - \*  $z \succeq x$ , with  $x \succeq y$ , suppose  $y \succeq z$ , this contradicts  $z > y$ , thus  $z \succeq x \wedge x \succeq y \Rightarrow z \succeq y$
  - $x > z$  and  $z > y$ : again we know  $x \succeq z$  and  $z \succeq y$ . Suppose  $y \succeq x$ , this contradicts  $x > y$ , therefore  $x \succeq z \wedge z \succeq y \Rightarrow x \succeq y$

In all three scenarios, transitivity is proved.

With completeness and transitivity proved, we've proved that  $>$  is rational  $\Rightarrow \succeq$  is rational.

Notice that negative positivity in Def. 1.1.4, is logically equivalent to its *contrapositive*:  $\exists z \in X \setminus \{x, y\}$  s.t.  $x \not\succeq z \wedge z \not\succeq y \Rightarrow x \not\succeq y$ . This is precisely why the definition is called negative transitivity.

## 1.2 Choice Rules

Next, we approach the theory of decision making from choice behavior itself. Formally, choice behavior is represented by means of a *choice structure*  $(\mathcal{B}, C(\cdot))$ . Now, we define choice structure  $(\mathcal{B}, C(\cdot))$ :

**Definition 1.2.1.** A choice structure  $(\mathcal{B}, C(\cdot))$  has two ingredients:

- $\mathcal{B} \subset \mathcal{P}(X) \setminus \emptyset$ , where  $\mathcal{P}(X)$  is the power set of  $X$ . This means, every element  $B \in \mathcal{B}$  is a subset of  $X$ <sup>2</sup>.
- $C(\cdot)$  is a *choice rule correspondence* that assigns a nonempty set of chosen elements  $C(B) \subset B, \forall B \in \mathcal{B}$ <sup>3</sup>.

Now we discuss the CORE assumption in this section: the Weak Axiom of Revealed Preference (WARP):

**Definition 1.2.2.** A choice set  $(\mathcal{B}, C(\cdot))$  satisfies WARP if:

- $\forall B, B'$  and  $x, y \in B \cap B', x \in C(B), y \in C(B') \Rightarrow x \in C(B')$

Or in words, WARP requires that if  $x$  is chosen from some alternatives where  $y$  is also available, then there can be NO budget set containing both  $x$  and  $y$  but only  $y$  is chosen.

Following WARP, define the *reveal preference relation*  $\succeq^*$  as:

**Definition 1.2.3.** Given a choice structure  $(\mathcal{B}, C(\cdot))$ ,  $x \succeq^* y \Leftrightarrow \exists B \in \mathcal{B}$  s.t.  $x, y \in B \wedge x \in C(B)$

In words,  $x$  is revealed at least as good as  $y$ .

With revealed preference defiend, we can rephrase WARP as: *If  $x$  is revealed at least as good as  $y$ , then  $y$  **cannot** be revealed preferred to  $x$ .* Hence,  $\succeq^*$  is not symmetric.

One thing to remember is that  $\succeq^*$  need not be either complete or transitive. For  $\succeq^*$  to be comparable, for a  $B \in \mathcal{B}$  and  $x, y \in B$ , we must have either  $x \in C(B)$ ,  $y \in C(B)$  or both.

An example is:

**Example 1.2.1.** Consider a choice structure  $(\mathcal{B}, C(\cdot))$  from  $X = \{x, y, z\}$ , where  $\mathcal{B} = \{\{x, y\}, \{x, y, z\}\}$ . Under WARP, we can NOT have  $C(\{x, y, z\}) = \{x, y\}$  and  $C\{x, y\} = \{x\}$  at the same time.  $C(\{x, y, z\}) = \{x, y\}$  means that  $x \succeq^* y, x \succeq^* z, y \succeq^* x, y \succeq^* z$ ;  $C(\{x, y\}) = \{x\}$  means that  $y \not\succeq^* x$ . Under WARP, the contradiction can not

<sup>2</sup>The elements  $B \in \mathcal{B}$  are so-called *budget sets*. The budget sets in  $\mathcal{B}$  should be thought of as an exhaustive listing of all the choice experiments that can be achieved, but it is possible that some subsets of  $X$  are not achievable.

<sup>3</sup>The choice set  $C(B)$  can contain a single element, which is the choice among the alternatives in  $B$ . BUT,  $C(B)$  can contain multiple elements, then elements of  $C(B)$  are the *acceptable alternatives* in  $B$ .

## 1.3 Linking Preferences with Choices

Now we have two major approaches of decision making process: preference relations in Section 1.1 and choice rules in Section 1.2, what we need to do is to link them. This linkage will emerge when we examine two central assumptions: **rationality** and **WARP**.

## 1.4 Chap1Sec4

---

---

## CHAPTER 2

---

### FUNDAMENTALS OF CONSUMER THEORY

---

---

## CHAPTER 3

---

### LAGRANGE MAXIMIZATION AND DUALITY

---

---

## CHAPTER 4

---

### MONOTONE COMPARATIVE STATICS

---

---

## CHAPTER 5

---

### EXPECTED UTILITY AND DECISIONMAKING UNDER UNCERTAINTY

---

---

## CHAPTER 6

---

### AGGREGATION AND THE EXISTENCE OF A REPRESENTATIVE CONSUMER



---

---

# CHAPTER 7

---

## PRODUCER THEORY

---

---

## CHAPTER 8

---

### STOCHASTIC CHOICE

# **Part II**

## **Game Theory**

---

---

## CHAPTER 9

---

### NASH EQUILIBRIUM AND BAYESIAN NASH EQUILIBRIUM

---

---

## CHAPTER 10

---

### RATIONALIZABILITY AND DOMINANT STRATEGIES

---

---

# CHAPTER 11

---

## CORRELATED EQUILIBRIUM

---

---

## CHAPTER 12

---

### DYNAMIC GAMES AND REFINEMENTS

---

---

## CHAPTER 13

---

### REPEATED GAMES/FOLK THEOREM



---

---

## CHAPTER 14

---

### RECURSIVE METHODS IN REPEATED GAMES

## **Part III**

# **Mechanism Design and Contract Theory**

---

---

## CHAPTER 15

---

### ARROW'S THEOREM AND SOCIAL CHOICE

---

---

## CHAPTER 16

---

### BOUNDARIES OF THE FIRM AND COASE'S THEOREM

---

---

## CHAPTER 17

---

### IMPLEMENTATION CONCEPTS

---

---

## CHAPTER 18

---

### THE REVELATION PRINCIPLE

---

---

## CHAPTER 19

---

### AUCTIONS AND OPTIMAL AUCTIONS

---

---

## CHAPTER 20

---

### EFFICIENT IMPLEMENTATION



---

---

## CHAPTER 21

---

### MORAL HAZARD

---

---

## CHAPTER 22

---

### FULL IMPLEMENTATION

---

## BIBLIOGRAPHY

- Patrick Bolton and Mathias Dewatripont. *Contract Theory*, volume 1. The MIT Press, 2005. URL <https://ideas.repec.org/b/mtp/titles/0262025760.html>.
- Drew Fudenberg and Jean Tirole. *Game Theory*. MIT Press, 1991.
- David M. Kreps. *A Course in Microeconomic Theory*. Princeton University Press, 1990.
- George J Mailath. *Modeling Strategic Behavior: A Graduate Introduction to Game Theory and Mechanism Design*, volume 6. World Scientific, 2018.
- George J. Mailath and Larry Samuelson. *Repeated Games and Reputations: Long-Run Relationships*. Oxford University Press, 2006. URL <https://ideas.repec.org/b/oxp/obooks/9780195300796.html>.
- Andreu Mas-Colell, Michael Dennis Whinston, et al. *Microeconomic theory*, volume 1. New York: Oxford university press, 1995.
- Roger B. Myerson. *Game Theory: Analysis of Conflict*. Harvard University Press, 1991. ISBN 9780674341166. URL <http://www.jstor.org/stable/j.ctvjsf522>.
- Martin J. Osborne and Ariel Rubinstein. *A Course in Game Theory*. The MIT Press, 1994. URL <https://ideas.repec.org/b/mtp/titles/0262650401.html>.
- Ariel Rubinstein. *Lecture Notes in Microeconomic Theory: The Economic Agent - Second Edition*. Princeton University Press, rev - revised, 2 edition, 2012.