

Microeconomic Theory I: A Notebook

With Jonathan Libgober

Sai Zhang

Check my [Github Page](#), or [email me](#)!

September 2, 2021

HERE WE GO!

This is my learning notebook of Microeconomic Theory I (Course number: ECON601 at USC Economics). As one of the core courses in an economic Ph.D. curriculum, Microeconomic Theory I is beyond important to my research. Therefore, I would love to use this notebook as a commitment mechanism, to document lecture notes, discuss session and office hour intuitions, reading summaries, my personal questions regarding the topics and more. By building a file from scratch, hopefull I could have a more systematic and sophisticated understanding on the content of this course.

I thank Prof. Jonathan Libgober at USC Economics for leading the discussion of the course and providing intuitive ways to understand microeconomic theory. Please check his webpage [here](#), he is such fun.

I also appreciate the time and effort my TA Qitong Wang put into this course, guiding me through discussing sessions and problem sets. When I have questions, he is always there to help.

Following the structure of the course, this notebook will cover three aspects of microeconomic theories: (a) individual decision making, (b) game theory, (c) mechanism design and contract theory. Apart from Jonathan's lecture notes, I will also summarize the reading materials, including: [Mas-Colell et al. \(1995\)](#)'s *Microeconomic Theory*, [Mailath \(2018\)](#)'s *Modelling Strategic Behavior*¹, [Fudenberg and Tirole \(1991\)](#)'s *Game Theory*, [Myerson \(1991\)](#)'s *Game Theory: Analysis of Conflicts*, [Bolton and Dewatripont \(2005\)](#)'s *Contract Theory*, [Mailath and Samuelson \(2006\)](#)'s *Repeated Games and Reputation* and [Osborne and Rubinstein \(1994\)](#)'s *A Course in Game Theory*. Other materials will also be referred to along the way.

Building this notebook is truly a memorable journey for me. I would love to share this review and all the related materials to anyone that finds them useful. And unavoidably, I would make some typos and other minor mistakes (hopefully not big ones). So I'd really appreciate any correction. If you find any mistakes, please send the mistakes to this email address saizhang.econ@gmail.com, BIG thanks in advance!

¹Latest version (May 2021) available [here](#).

Contents

I	Individual Decision Making	4
1	Preferences and Choices, Utilities	5
1.1	Preference Relations	6
1.2	Choice Rules	9
1.3	Linking Preferences with Choices	10
1.4	Chap1Sec4	11
2	Fundamentals of Consumer Theory	12
3	Lagrange Maximization and Duality	13
4	Monotone Comparative Statics	14
5	Expected Utility and Decisionmaking under Uncertainty	15
6	Aggregation and the Existence of a Representative Consumer	16
7	Producer Theory	17
8	Stochastic Choice	18
II	Game Theory	19
9	Nash Equilibrium and Bayesian Nash Equilibrium	20
10	Rationalizability and DOminant Strategies	21
11	Correlated Equilibrium	22
12	Dynamic Games and Refinements	23
13	Repeated Games/Folk Theorem	24
14	Recursive Methods in Repeated Games	25

III Mechanism Design and Contract Theory	26
15 Arrow's Theorem and Social Choice	27
16 Boundaries of the Firm and Coase's Theorem	28
17 Implementation Concepts	29
18 The Revelation Principle	30
19 Auctions and Optimal Auctions	31
20 Efficient Implementation	32
21 Moral Hazard	33
22 Full Implementation	34
Bibliography	35

Part I

Individual Decision Making

CHAPTER 1

PREFERENCES AND CHOICES, UTILITIES

Contents

1.1	Preference Relations	6
1.2	Choice Rules	9
1.3	Linking Preferences with Choices	10
1.4	Chap1Sec4	11

The first chapter summarizes the basic setting of individual decision making: preferences, choices and utilities. The main reference is Chapter 1 of [Mas-Colell et al. \(1995\)](#).

In this chapter, we will focus on 3 domains:

choice	given a set A , what choice from A is made
preference	given alternatives x, y , which does the decision maker prefers
utility	given an object X , how much does the DM likes X (as a number)

The starting point of individual decision problem is a *set of possible (mutually exclusive) alternatives* from which the individual must choose. To model decision making process on this set of alternatives, one can:

- either start from the tastes, i.e., *preference relations* of individuals, and set up the patterns of decision making with preferences
- or, start from the actual actions of individuals, i.e. *choices*, to deduct a pattern of decision making

With this two major approaches in mind, we know what's coming: the *rationality* of preferences and the central assumption of choices, the *Weak Axiom of Revealed Preference (WARP)*. And of course, the two approaches and two basic assumptions are parallel, so we need to figure out how link the (underlying) preferences and (observed) choices.

1.1 Preference Relations

We start from the basic: *weak preference relation*, \succeq .

Definition 1.1.1. A weak preference relation \succeq on a set X is a subset of $X \times X$. If $(x, y) \in \succeq \Rightarrow x$ is at least as good as y , written as $x \succeq y$

A weak preference relation will induce two other types of relations on X :

Definition 1.1.2. With \succeq defined by Def. 1.1.1, we have

- the *strict preference relation*, $>$ can be induced from \succeq as: $x > y \Leftrightarrow x \succeq y \wedge y \not\succeq x$, or in words, x is preferred to y .
- the *indifference relation*, \sim can be induced from \succeq as: $x \sim y \Leftrightarrow x \succeq y \wedge y \succeq x$, or in words, x is indifferent to y .

With the definition of these relations, we now define the central assumption of relations: *rationality*.

Definition 1.1.3. A weak preference relation \succeq is *rational* if it is:

- Complete: $\forall x, y \in X, x \succeq y$ or $y \succeq x$ or both
- Transitive: $\forall x, y, z \in X, x \succeq y \wedge y \succeq z \Rightarrow x \succeq z$

How to understand them? They are both strong assumptions:

- Completeness of \succeq means it is well-defined between any two possible alternatives. From the perspective of an individual, completeness means that she will make choices, and only meditated choices.
- Transitivity of \succeq implies that the decision maker will not have a preference cycle, since whoever has a preference cycle would suffer economically for it¹.

With the definition of rational \succeq in Def. 1.1.3, we can prove the following properties of $>$ and \sim induced by \succeq :

Theorem 1.1.1. If \succeq is rational, then:

- i. $>$ is irreflexive ($x > x$ never holds) and transitive ($x > y \wedge y > z \Rightarrow x > z$)

Proof:

- irreflexive: by Def. 1.1.2, $x > x \Rightarrow x \succeq x \wedge x \not\succeq x$, self contradiction.
- transitive: $x > y \Rightarrow x \succeq y \wedge y \not\succeq x, y > z \Rightarrow y \succeq z \wedge z \not\succeq y$. By transitivity of $\succeq, x \succeq y \wedge y \succeq z \Rightarrow x \succeq z$. If $z \succeq x$, by transitivity of \succeq and $x \succeq y$, we would have $z \succeq y$, contradicting $y > z$. Therefore $x \succeq z \wedge z \not\succeq x \Rightarrow x > z$.

- ii. \sim is reflexive ($x \sim x, \forall x$), transitive ($x \sim y \wedge y \sim z \Rightarrow x \sim z$) and symmetric ($x \sim y \Rightarrow y \sim x$)

¹There are 2 types of violations of transitivity: irrational and mechanical. Irrational violations are easy to understand: decision makers simply do not follow transitivity assumption, many reasons have been raised, including mental account, framing, menu effect, attraction effect, etc. Mechanical violations means that decision makers are "forced" to violate transitivity. One example of this type of violation is aggregation of considerations: decision makers may aggregate several sub-preferences as together to make the choice, leading to violation of transitivity. Another example is when the preference is only defined for differences above a certain level (problem of perceptible differences). See Mas-Colell et al. (1995, Page 7-8), Rubinstein (2012, Page 4-5) for details

Proof:

- reflexive: by completeness of \succeq , $\forall x, x \succeq x \Rightarrow x \sim x$
 - transitive: $x \sim y \Rightarrow x \succeq y \wedge y \succeq x$, $y \sim z \Rightarrow y \succeq z, z \succeq y$, by the transitivity of \succeq , we have $x \succeq z \wedge z \succeq x$, hen $x \sim z$
 - symmetric: $x \sim y \Rightarrow x \succeq y \wedge y \succeq x \Leftrightarrow y \succeq x \wedge x \succeq y \Rightarrow y \sim x$
- iii. $x > y \succeq z \Rightarrow x > z$

Proof: $x > y \Rightarrow x \succeq y \wedge y \not\succeq x$, hence $x > y \succeq z \Rightarrow x \succeq z$. If $z \succeq x$, by transitivity of \succeq , $y \succeq x$, contradicting $x > y$. Therefore, $z \not\succeq x$

We can also directly define a *rational* $>$ (see [Kreps \(1990, Page 19-21\)](#)):

Definition 1.1.4. A strict preference relation $>$ is rational if it is:

- asymmetric: $\nexists x, y \in X$ s.t. $x > y \wedge y > x$
- negatively transitive: $x > y \Rightarrow \forall z \in X \setminus \{x, y\}, x > z \vee z > y \vee$ both.

With Def. 1.1.4 and Def. 1.1.3, we can prove that \succeq is rational iff $>$ is rational:

Theorem 1.1.2. \succeq is rational $\Leftrightarrow >$ is rational, specifically:

- \succeq is complete $\Leftrightarrow >$ is asymmetric
- \succeq is transitive $\Leftrightarrow >$ is negatively transitive

Now we prove this theorem:

Step 1 proof \succeq is rational $\Rightarrow >$ is rational

- **asymmetric**

if $\exists x, y$ s.t. $x > y$ and $y > x$, then by the definition of induced strict preference, the pair x, y must satisfy

$$\begin{cases} x \succeq y \text{ and } y \not\succeq x & (x > y) \\ y \succeq x \text{ and } x \not\succeq y & (y > x) \end{cases}$$

which is, by completeness of rational \succeq , impossible. Therefore, such pair x, y don't exist. $>$ is proved to be asymmetric.

- **negatively transitive**

First, $\forall z \notin \{x, y\}$, by completeness of rational \succeq , the relation between x and z is either $x \succeq z$ or $z \succeq x$. Similarly, the relation between y and z is either $y \succeq z$ or $z \succeq y$.

Second, given $x > y$, x, y satisfies $x \succeq y$ and $y \not\succeq x$.

Also, it is easy to prove that: $x > y \wedge y \succeq z \Rightarrow x > z$, $x > y \wedge z \succeq x \Rightarrow z > y$; and $x > y \wedge z \sim x \Rightarrow z > y$, $x > y \wedge y \sim z \Rightarrow x > z$

Now we have the following scenarios:

1. if $z \succeq x$ and $y \succeq z$, by transitivity of rational \succeq , $y \succeq x$, contradicting the definition of $x > y$. This scenario doesn't exist.
2. if $x \succeq z$ and $y \succeq z$, since $x > y$, with the auxiliary result proved above, we have $x > z$
3. if $z \succeq x$ and $z \succeq y$, since $x > y$, with the auxiliary result proved above, we have $z > y$
4. if $x \succeq z$ and $z \succeq y$, since $x > y$, suppose:

- (a) $z \succeq x$ as well, then $x \sim z$, in this case $z > y$;
 - (b) $z \not\succeq x$, then $x > z$
 - (c) $y \succeq z$ as well, then $y \sim z$, in this case $x > z$
 - (d) $y \not\succeq z$, then $z > y$
- therefore, a complete summary of (a) to (d) would give:

	$z \succeq x$	$z \not\succeq x$
$y \succeq z$	$z > y \ \& \ x > z$	$x > z$
$y \not\succeq z$	$z > y$	$x > z \ \& \ z > y$

Combining all above, we have proved negative transitivity of $>$.

With asymmetry and negative transitivity proved, we've proved that \succeq is rational $\Rightarrow >$ is rational

Step 2 proof $>$ is rational $\Rightarrow \succeq$ is rational.

- Complete: with a rational $x > y$, we know $\nexists x, y$ s.t. $x > y$ and $y > x$ by asymmetry. Therefore, $\forall x, y$, we have two possibilities.
 - $x > y$ and $y \not\succeq x$, which would naturally induce a weak preference $x \succeq y$
 - $y > x$ and $x \not\succeq y$, which would naturally induce a weak preference $y \succeq x$
 therefore, $\forall x, y$, either $x \succeq y$ or $y \succeq x$ completeness of \succeq is proven.
- Transitive: with a rational $x > y$, negative transitivity gives $\forall z \notin \{x, y\}$, either $x > z$, $z > y$, or both. By negative transitivity, we have:
 - $x > z$: following same procedure, we know $x \succeq z$. If:
 - * $y \succeq z$, since $x > z \Rightarrow z \not\succeq x$, by completeness we have $x \succeq z$, thus $x \succeq y \wedge y \succeq z \Rightarrow x \succeq z$
 - * $z \succeq y$, since $x > y \Rightarrow x \not\succeq y$, by completeness we have $x \succeq y$, thus $x \succeq z \wedge z \succeq y \Rightarrow x \succeq y$
 - $z > y$: again, we know $z \succeq y$. If:
 - * $x \succeq z$, since $x > y \Rightarrow y \not\succeq x$, by completeness we have $x \succeq y$, thus $z \succeq y \wedge x \succeq z \Rightarrow x \succeq y$
 - * $z \succeq x$, with $x \succeq y$, suppose $y \succeq z$, this contradicts $z > y$, thus $z \succeq x \wedge x \succeq y \Rightarrow z \succeq y$
 - $x > z$ and $z > y$: again we know $x \succeq z$ and $z \succeq y$. Suppose $y \succeq x$, this contradicts $x > y$, therefore $x \succeq z \wedge z \succeq y \Rightarrow x \succeq y$

In all three scenarios, transitivity is proved.

With completeness and transitivity proved, we've proved that $>$ is rational $\Rightarrow \succeq$ is rational.

Notice that negative positivity in Def. 1.1.4, is logically equivalent to its *contrapositive*: $\exists z \in X \setminus \{x, y\}$ s.t. $x \not\succeq z \wedge z \not\succeq y \Rightarrow x \not\succeq y$. This is precisely why the definition is called negative transitivity.

1.2 Choice Rules

Next, we approach the theory of decision making from choice behavior itself. Formally, choice behavior is represented by means of a *choice structure* $(\mathcal{B}, C(\cdot))$. Now, we define choice structure $(\mathcal{B}, C(\cdot))$:

Definition 1.2.1. A choice structure $(\mathcal{B}, C(\cdot))$ has two ingredients:

- $\mathcal{B} \subset \mathcal{P}(X) \setminus \emptyset$, where $\mathcal{P}(X)$ is the power set of X . This means, every element $B \in \mathcal{B}$ is a subset of X ².
- $C(\cdot)$ is a *choice rule correspondence* that assigns a nonempty set of chosen elements $C(B) \subset B, \forall B \in \mathcal{B}$ ³.

Now we discuss the CORE assumption in this section: the Weak Axiom of Revealed Preference (WARP):

Definition 1.2.2. A choice set $(\mathcal{B}, C(\cdot))$ satisfies WARP if:

- $\forall B, B'$ and $x, y \in B \cap B', x \in C(B), y \in C(B') \Rightarrow x \in C(B')$

Or in words, WARP requires that if x is chosen from some alternatives where y is also available, then there can be NO budget set containing both x and y but only y is chosen.

Following WARP, define the *reveal preference relation* \succeq^* as:

Definition 1.2.3. Given a choice structure $(\mathcal{B}, C(\cdot))$, $x \succeq^* y \Leftrightarrow \exists B \in \mathcal{B}$ s.t. $x, y \in B \wedge x \in C(B)$

In words, x is revealed at least as good as y .

With revealed preference defiend, we can rephrase WARP as: *If x is revealed at least as good as y , then y **cannot** be revealed preferred to x .* Hence, \succeq^* is not symmetric.

One thing to remember is that \succeq^* need not be either complete or transitive. For \succeq^* to be comparable, for a $B \in \mathcal{B}$ and $x, y \in B$, we must have either $x \in C(B), y \in C(B)$ or both.

An example is:

Example 1.2.1. Consider a choice structure $(\mathcal{B}, C(\cdot))$ from $X = \{x, y, z\}$, where $\mathcal{B} = \{\{x, y\}, \{x, y, z\}\}$. Under WARP, $C\{x, y\} = \{x\} \Rightarrow y \notin C\{x, y, z\}$. BUT, we can have $z \in C(\{x, y, z\})$.

This is why the induced preference is called *revealed*: you don't know what else is going on.

²The elements $B \in \mathcal{B}$ are so-called *budget sets*. The budget sets in \mathcal{B} should be thought of as an exhaustive listing of all the choice experiments that can be achieved, but it is possible that some subsets of X are not achievable.

³The choice set $C(B)$ can contain a single element, which is the choice among the alternatives in B . BUT, $C(B)$ can contain multiple elements, then elements of $C(B)$ are the *acceptable alternatives* in B .

1.3 Linking Preferences with Choices

Now we have two major approaches of decision making process: preference relations in Section 1.1 and choice rules in Section 1.2, what we need to do is to link them. This linkage will emerge when we examine two central assumptions: **rationality** and **WARP**. So the major question here is:

$$\text{rational} \stackrel{???}{\Leftrightarrow} (\mathcal{B}, C(\cdot)) \text{ satisfies WARP}$$

And the answer is: YES! but not exactly. Now let's dig in.

Rational $\Rightarrow (\mathcal{B}, C(\cdot))$ satisfies WARP

First, **rational $\Rightarrow (\mathcal{B}, C(\cdot))$ satisfies WARP** is a big YES. To prove this, we need to define *induced choice correspondence*:

Definition 1.3.1. Given a **rational** \succsim on X , if the decision maker faces a nonempty subset of alternatives $B \subset X$, by maximizing her preference, she would choose any one of the elements in the *induced choice correspondence*: $C^*(B, \succsim) = \{x \in B : x \succsim y, \forall y \in B\}$

The induced choice correspondence $C^*(B, \succsim)$ has an important property:

Theorem 1.3.1. if X is finite, $C^*(B, \succsim)$ will be **nonempty**.

A brief proof of this proposition is: If X is finite, B is finite as well. We will prove by induction. Starting from $|B| = 1$, the only element of B is in $C^*(B, \succsim)$. Now suppose $C^*(B, \succsim)$ is nonempty when $|B_n| = n$, let $x^* \in C^*(B_n, \succsim)$; when $|B_{n+1}| = n + 1$, let the $n + 1$ th element y ($\{y\} = B_{n+1} \setminus B_n$). By the completeness of a rational \succsim , either $y \succsim x^*$ or $x^* \succ y$:

- i. $y \succsim x^*$: since $x^* \in C^*(B_n, \succsim) \Rightarrow x^* \succsim x, \forall x \in B_n$. By transitivity of \succsim , $y \succsim x, \forall x \in B_n$. By completeness, $y \succsim y$ as well. Hence, $y \in C^*(B_{n+1}, \succsim)$.
- ii. $x^* \succ y$: since $x^* \in C^*(B_n, \succsim) \Rightarrow x^* \succsim x, \forall x \in B_n$, hence $x^* \succ x, \forall x \in B_n \cup y \Rightarrow x^* \in C^*(B_{n+1}, \succsim)$

Notice that when B is finite, a stronger condition of \succsim being acyclic and complete is equivalent to an induced choice rule $C^*(B, \succsim) \neq \emptyset$:

Theorem 1.3.2. For a finite B , \succsim is complete and **acyclic** $\Leftrightarrow C^*(B, \succsim) \neq \emptyset$

\succsim is acyclic mean that: $b_1 \succ b_2, b_2 \succ b_3, \dots, b_{n-1} \succ b_n \Rightarrow b_n \not\succ b_1$. An example of transitive but not *acyclic* relations is indifference \sim : $a_1 \sim a_2 \sim \dots \sim a_n \Rightarrow a_n \sim a_1$. A brief proof of Theorem 1.3.2 is:

- i. **acyclic $\Rightarrow C^*(B, \succsim) \neq \emptyset$** : Suppose if $C^*(B, \succsim) = \emptyset$, for $b_1 \in B, b_1 \notin C^*(B, \succsim) \Rightarrow \exists b_2$ s.t. $b_2 \succ b_1$. Continue this process, we can generate a sequence of $\dots \succ b_2 \succ b_1$, since B is finite, this sequence must end at b_n . If \succsim is acyclic, $b_1 \not\succ b_n$, this gives $b_n \succ b_1$, which would mean b_n must be in $C^*(B, \succsim)$, contradicting.
- ii. **$C^*(B, \succsim) \neq \emptyset \Rightarrow$ acyclic \succsim** : Suppose \succsim is not acyclic, then there exists $b_1 \succ b_2 \succ \dots \succ b_n \succ b_1$, then for set $B = \{b_1, b_2, \dots, b_n\}$, $\nexists b^* \in B$ s.t. $b^* \succ b_i \forall b_i \in B$, i.e., $C^*(B, \succsim) = \emptyset$.

With induced choice correspondence $C^*(B, \succsim)$ defined and non-emptiness proved, we can then say:

Theorem 1.3.3. If \succsim is a rational preference relation, then the choice structure generated by \succsim , $(\mathcal{B}, C^*(\cdot, \succsim))$, satisfies WARP

1.4 Chap1Sec4

CHAPTER 2

FUNDAMENTALS OF CONSUMER THEORY

CHAPTER 3

LAGRANGE MAXIMIZATION AND DUALITY

CHAPTER 4

MONOTONE COMPARATIVE STATICS

CHAPTER 5

EXPECTED UTILITY AND DECISIONMAKING UNDER UNCERTAINTY

CHAPTER 6

AGGREGATION AND THE EXISTENCE OF A REPRESENTATIVE CONSUMER

CHAPTER 7

PRODUCER THEORY

CHAPTER 8

STOCHASTIC CHOICE

Part II

Game Theory

CHAPTER 9

NASH EQUILIBRIUM AND BAYESIAN NASH EQUILIBRIUM

CHAPTER 10

RATIONALIZABILITY AND DOMINANT STRATEGIES

CHAPTER 11

CORRELATED EQUILIBRIUM

CHAPTER 12

DYNAMIC GAMES AND REFINEMENTS

CHAPTER 13

REPEATED GAMES/FOLK THEOREM

CHAPTER 14

RECURSIVE METHODS IN REPEATED GAMES

Part III

Mechanism Design and Contract Theory

CHAPTER 15

ARROW'S THEOREM AND SOCIAL CHOICE

CHAPTER 16

BOUNDARIES OF THE FIRM AND COASE'S THEOREM

CHAPTER 17

IMPLEMENTATION CONCEPTS

CHAPTER 18

THE REVELATION PRINCIPLE

CHAPTER 19

AUCTIONS AND OPTIMAL AUCTIONS

CHAPTER 20

EFFICIENT IMPLEMENTATION

CHAPTER 21

MORAL HAZARD

CHAPTER 22

FULL IMPLEMENTATION

BIBLIOGRAPHY

- Patrick Bolton and Mathias Dewatripont. *Contract Theory*, volume 1. The MIT Press, 2005. URL <https://ideas.repec.org/b/mtp/titles/0262025760.html>.
- Drew Fudenberg and Jean Tirole. *Game Theory*. MIT Press, 1991.
- David M. Kreps. *A Course in Microeconomic Theory*. Princeton University Press, 1990.
- George J Mailath. *Modeling Strategic Behavior: A Graduate Introduction to Game Theory and Mechanism Design*, volume 6. World Scientific, 2018.
- George J. Mailath and Larry Samuelson. *Repeated Games and Reputations: Long-Run Relationships*. Oxford University Press, 2006. URL <https://ideas.repec.org/b/oxp/obooks/9780195300796.html>.
- Andreu Mas-Colell, Michael Dennis Whinston, et al. *Microeconomic theory*, volume 1. New York: Oxford university press, 1995.
- Roger B. Myerson. *Game Theory: Analysis of Conflict*. Harvard University Press, 1991. ISBN 9780674341166. URL <http://www.jstor.org/stable/j.ctvjsf522>.
- Martin J. Osborne and Ariel Rubinstein. *A Course in Game Theory*. The MIT Press, 1994. URL <https://ideas.repec.org/b/mtp/titles/0262650401.html>.
- Ariel Rubinstein. *Lecture Notes in Microeconomic Theory: The Economic Agent - Second Edition*. Princeton University Press, rev - revised, 2 edition, 2012.