Microeconomic Theory I: A Notebook

With Jonathan Libgober

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Check my Github Page, or email me!

September 3, 2021

HERE WE GO!

This is my learning notebook of Microeconomic Theory I (Course number: ECON601 at USC Economics). As one of the core courses in an economic Ph.D. curriculum, Microeconomic Theory I is beyond important to my research. Therefore, I would love to use this notebook as a commitment mechanism, to document lecture notes, discuss session and office hour intuitions, reading summaries, my personal questions regarding the topics and more. By building a file from scratch, hopefull I could have a more systematic and sophisticated understanding on the content of this course.

I thank Prof. Jonathan Libgober at USC Economics for leading the discussion of the course and providing intuitive ways to understand microeconomic theory. Please check his webpage here, he is such fun.

I also appreciate the time and effort my TA Qitong Wang put into this course, guiding me through discussing sessions and problem sets. When I have questions, he is always there to help.

Following the structure of the course, this notebook will cover three aspects of microeconomic theories: (a) individual decision making, (b) game theory, (c) mechanism design and contract theory. Apart from Jonathan's lecture notes, I will also summarize the reading materials, including: **?**'s *Microeconomic Theory*, **?**'s *Modelling Strategic Behavior*[1], **?**'s *Game Theory*, **?**'s *Game Theory: Analysis of Conflicts*, **?**'s *Contract Theory*, **?**'s *Repeated Games and Reputation* and **?**'s *A Course in Game Theory*. Other materials will also be referred to along the way.

Building this notebook is truly a memorable journey for me. I would love to share this review and all the related materials to anyone that finds them useful. And unavoidably, I would make some typos and other minor mistakes (hopefully not big ones). So I'd really appreciate any correction. If you find any mistakes, please send the mistakes to this email address saizhang.econ@gmail.com, BIG thanks in advance!

[1]Latest version (May 2021) available here.

Contents

Part I

Individual Decision Making

CHAPTER 1

PREFERENCES AND CHOICES, UTILITIES

Contents

The first chapter summarizes the basic setting of individual decision making: preferences, choices and utilities. The main reference is Chapter 1 of **?**.

In this chapter, we will focus on 3 domains:

choice	given a set A, what choice from A is made
preference	given alternatives x, y, which does the decision maker prefers
utility	given an object X, how much does the DM likes X (as a number)

The starting point of individual decision problem is a *set of possible (mutually exclusive) alternatives* from which the individual must choose. To model decision making process on this set of alternatives, one can:

- either start from the tastes, i.e., *preference relations* of individuals, and set up the patterns of decision making with preferences
- or, start from the actual actions of individuals, i.e. *choices*, to deduct a pattern of decision making

With this two major approaches in mind, we know what's coming: the ***rationality*** of preferences and the central assumption of choices, the ***Weak Axiom of Revealed Preference (WARP)***. And of course, the two approaches and two basic assumptions are parallel, so we need to figure out how link the (underlying) preferences and (observed) choices.

1.1 Preference Relations

We start from the basic: *weak preference relation*, $\succsim$.

Definition 1.1.1. A weak preference relation $\succsim$ on a set X is a subset of $X \times X$. If $(x, y) \in \succsim \Rightarrow x$ is at least as good as y, written as $x \succsim y$

A weak preference relation will induce two other types of relations on X:

Definition 1.1.2. With $\succsim$ defined by Def. **??**, we have

- the *strict preference relation*, $\succ$ can be induced from $\succsim$ as: $x \succ y \Leftrightarrow x \succsim y \wedge y \not\succsim x$, or in words, x if preferred to y.
- the *indifference relation*, $\sim$ can be induced from $\succsim$ as: $x \sim y \Leftrightarrow x \succsim y \wedge y \succsim x$, or in words, x is indifferent to y.

With the definition of these relations, we now define the central assumption of relations: ***rationality***.

Definition 1.1.3. A weak preference relation $\succsim$ is *rational* if it is:

- Complete: $\forall x, y \in X, x \succsim y$ or $y \succsim x$ or both
- Transitive: $\forall x, y, z \in X, x \succsim y \wedge y \succsim z \Rightarrow x \succsim z$

How to understand them? They are both strong assumptions:

- Completeness of $\succsim$ means it is well-defined between any two possible alternatives. From the perspective of an individual, completeness means that she will make choices, and only meditated choices.
- Transitivity of $\succsim$ implies that the decision maker will not have a preference cycle, since whoever has a preference cycle would suffer economically for it[1].

With the definition of rational $\succsim$ in Def. **??**, we can prove the following properites of $\succ$ and $\sim$ *induced* by $\succsim$:

Theorem 1.1.1. If $\succsim$ is rational, then:

i. $\succ$ is irreflexive ($x \succ x$ never holds) and transitive ($x \succ y \wedge y \succ z \Rightarrow x \succ z$)
   Proof:
   - irreflexive: by Def. **??**, $x \succ x \Rightarrow x \succsim x \wedge x \not\succsim x$, self contracdiction.
   - transitive: $x \succ y \Rightarrow x \succsim y \wedge y \not\succsim x$, $y \succ z \Rightarrow y \succsim z \wedge z \not\succsim y$. By transitivity of $\succsim$, $x \succsim y \wedge y \succsim z \Rightarrow x \succsim z$. If $z \succsim x$, by transitivity of $\succsim$ and $x \succsim y$, we would have $z \succsim y$, contradicting $y \succ z$. Therefore $x \succsim z \wedge z \not\succsim x \Rightarrow x \succ z$.
ii. $\sim$ is reflexive ($x \sim x, \forall x$), transitive ($x \sim y \wedge y \sim z \Rightarrow x \sim z$) and symmetric ($x \sim y \Rightarrow y \sim x$)

[1]There are 2 types of violations of transitivity: irrational and mechanical. Irrational violations are easy to understand: decision makers simply do not follow transivity assumption, many reasons have been raised, including mental account, framing, menu effect, attraction effect, etc. Mechanical violations means that decision makers are "forced" to violate transitivity. One example of this type of violation is aggregation of considerations: decision makers may aggregate several sub-preferences as together to make the choice, leading to violation of transitivity. Another example is when the preference is only defined for differences above a certain level (problem of perceptible differences). See **?**, Page 7-8, **?**, Page 4-5 for details

Proof:

- reflexive: by completeness of $\succsim$, $\forall x, x \succsim x \Rightarrow x \sim x$
- transitive: $x \sim y \Rightarrow x \succsim y \wedge y \succsim x$, $y \sim z \Rightarrow y \succsim z, z \succsim y$, by the transitivity of $\succsim$, we have $x \succsim z \wedge z \succsim x$, hen $x \sim z$
- symmetric: $x \sim y \Rightarrow x \succsim y \wedge y \succsim x \Leftrightarrow y \succsim x \wedge x \succsim y \Rightarrow y \sim x$

iii. $x \succ y \succsim z \Rightarrow x \succ z$

Proof: $x \succ y \Rightarrow x \succsim y \wedge y \not\succsim x$, hence $x \succ y \succsim z \Rightarrow x \succsim z$. If $z \succsim x$, by transitivity of $\succsim$, $y \succsim x$, contradicting $x \succ y$. Therefore, $z \not\succsim x$

We can also directly define a *rational* $\succ$ (see **?**, Page 19-21):

Definition 1.1.4. A strict preference ralation $\succ$ is rational if it is:

- asymmetric: $\nexists x, y \in X$ s.t. $x \succ y \wedge y \succ x$
- negatively transitive: $x \succ y \Rightarrow \forall z \in X \setminus \{x, y\}, x \succ z \vee z \succ y \vee$ both.

With Def. **??** and Def. **??**, we can prove that $\succsim$ is rational iff $\succ$ is rational:

Theorem 1.1.2. $\succsim$ is rational $\Leftrightarrow$ $\succ$ is rational, specifically:

- $\succsim$ is complete $\Leftrightarrow$ $\succ$ is asymmetric
- $\succsim$ is transitive $\Leftrightarrow$ $\succ$ is negatively transitive

Now we prove this theorem:

Step 1 proof $\succsim$ is rational $\Rightarrow$ $\succ$ is rational

- **asymmetric**

  if $\exists x, y$ s.t. $x \succ y$ and $y \succ x$, then by the definition of induced strict preference, the pair x, y must satisfy

$$\begin{cases} x \succsim y \text{ and } y \not\succsim x & (x \succ y) \\ y \succsim x \text{ and } x \not\succsim y & (y \succ x) \end{cases}$$

  which is, by completeness of rational $\succsim$, impossible. Therefore, such pair x, y don't exist. $\succ$ is proved to be asymetric.

- **negatively transitive**

  First, $\forall z \notin \{x, y\}$, by completeness of rational $\succsim$, the relation between x and z is either $x \succsim z$ or $z \succsim x$. Similarly, the relation between y and z is either $y \succsim z$ or $z \succsim y$.

  Second, given $x \succ y$, x, y satisfies $x \succsim y$ and $y \not\succsim x$.

  Also, it is easy to prove that: $x \succ y \wedge y \succsim z \Rightarrow x \succ z, x \succ y \wedge z \succsim x \Rightarrow z \succ y$; and $x \succ y \wedge z \sim x \Rightarrow z \succ y, x \succ y \wedge y \sim z \Rightarrow x \succ z$

  Now we have the following scenarios:

  1. if $z \succsim x$ and $y \succsim z$, by transitivity of rational $\succsim$, $y \succsim x$, contradicting the definition of $x \succ y$. This scenario doesn't exist.
  2. if $x \succsim z$ and $y \succsim z$, since $x \succ y$, with the auxilary result proved above, we have $x \succ z$
  3. if $z \succsim x$ and $z \succsim y$, since $x \succ y$, with the auxilary result proved above, we have $z \succ y$
  4. if $x \succsim z$ and $z \succsim y$, since $x \succ y$, suppose:

(a) $z \gtrsim x$ as well, then $x \sim z$, in this case $z \succ y$;
(b) $z \not\gtrsim x$, then $x \succ z$
(c) $y \gtrsim z$ as well, then $y \sim z$, in this case $x \succ z$
(d) $y \not\gtrsim z$, then $z \succ y$
therefore, a complete summary of (a) to (d) would give:

	$z \gtrsim x$	$z \not\gtrsim x$
$y \gtrsim z$	$z \succ y$ & $x \succ z$	$x \succ z$
$y \not\gtrsim z$	$z \succ y$	$x \succ z$ & $z \succ y$

Combining all above, we have proved negative transitivity of $\succ$.

With asymmetry and negative transitivity proved, we've proved that $\gtrsim$ is rational $\Rightarrow \succ$ is rational

Step 2 proof $\succ$ is rational $\Rightarrow \gtrsim$ is rational.

- Complete: with a rational $x \succ y$, we know $\nexists x, y$ s.t. $x \succ y$ and $y \succ x$ by asymmetry. Therefore, $\forall x, y$, we have two possibilities.
  - $x \succ y$ and $y \not\succ x$, which would naturally induce a weak preference $x \gtrsim y$
  - $y \succ x$ and $x \not\succ y$, which would naturally induce a weak preference $y \gtrsim x$

  therefore, $\forall x, y$, either $x \gtrsim y$ or $y \gtrsim x$ completeness of $\gtrsim$ is proven.
- Transitive: with a rational $x \succ y$, negative transivity gives $\forall z \notin \{x, y\}$, either $x \succ z, z \succ y$, or both. By negative transitivity, we have:
  - $x \succ z$: following same procedure, we know $x \gtrsim z$. If:
    * $y \gtrsim z$, since $x \succ z \Rightarrow z \not\gtrsim x$, by completeness we have $x \gtrsim z$, thus $x \gtrsim y \wedge y \gtrsim z \Rightarrow x \gtrsim z$
    * $z \gtrsim y$, since $x \succ y \Rightarrow x \not\gtrsim y$, by completeness we have $x \gtrsim y$, thus $x \gtrsim z \wedge z \gtrsim y \Rightarrow x \gtrsim y$
  - $z \succ y$: again, we know $z \gtrsim y$. If:
    * $x \gtrsim z$, since $x \succ y \Rightarrow y \not\gtrsim x$, by completeness we have $x \gtrsim y$, thus $z \gtrsim y \wedge x \gtrsim z \Rightarrow x \gtrsim y$
    * $z \gtrsim x$, with $x \gtrsim y$, suppose $y \gtrsim z$, this contradicts $z \succ y$, thus $z \gtrsim x \wedge x \gtrsim y \Rightarrow z \gtrsim y$
  - $x \succ z$ and $z \succ y$: again we know $x \gtrsim z$ and $z \gtrsim y$. Suppose $y \gtrsim x$, this contradicts $x \succ y$, therefore $x \gtrsim z \wedge z \gtrsim y \Rightarrow x \gtrsim y$

  In all three scenarios, transitivity is proved.

With completeness and transitivity proved, we've proved that $\succ$ is rational $\Rightarrow \gtrsim$ is rational.

Notice that negative positivity in Def. **??**, is logically equivalent to its *contrapositive*: $\exists z \in X \setminus \{x, y\}$ s.t. $x \not\succ z \wedge z \not\succ y \Rightarrow x \not\succ y$. This is percisely why the definition is called negative transitivity.

1.2 Choice Rules

Next, we approach the theory of decision making from choice behavior itself. Formally, choice behavior is represented by means of a *choice structure* $(\mathcal{B}, C(\cdot))$. Now, we define choice structure $(\mathcal{B}, C(\cdot))$:

Definition 1.2.1. A choice structure $(\mathcal{B}, C(\cdot))$ has two ingredients:

- $\mathcal{B} \subset \mathcal{P}(X) \setminus \emptyset$, where $\mathcal{P}(X)$ is the power set of X. This means, every element $B \in \mathcal{B}$ is a subset of X[2].
- $C(\cdot)$ is a *choice rule correspondence* that assigns a nonempty set of chosen elements $C(B) \subset B, \forall B \in \mathcal{B}$[3].

Now we discuss the CORE assumption in this section: the Weak Axiom of Revealed Preference (WARP):

Definition 1.2.2. A choice set $(\mathcal{B}, C(\cdot))$ satisfies WARP if:

- $\forall B, B'$ and $x, y \in B \cap B', x \in C(B), y \in C(B') \Rightarrow x \in C(B')$

Or in words, WARP requires that if x is chosen from some alternatives where y is also available, then there can be NO budget set containing both x and y but only y is chosen.

Following WARP, define the *reveal preference relation* $\succsim^*$ as:

Definition 1.2.3. Given a choice structure $(\mathcal{B}, C(\cdot))$, $x \succsim^* y \Leftrightarrow \exists B \in \mathcal{B}$ s.t. $x, y \in B \wedge x \in C(B)$

In words, x is revealed at least as good as y.

With revealed preference defiend, we can rephrase WARP as: *If x is revealed at least as good as y, then y **cannot** be revealed preferred to x.* Hence, $\succsim^*$ is not symmetric.

One thing to remember is that $\succsim^*$ need not be either complete or transitive. For $\succsim^*$ to be comparable, for a $B \in \mathcal{B}$ and $x, y \in B$, we must have either $x \in C(B)$, $y \in C(B)$ or both.

An example is:

Example 1.2.1. Consider a choice structure $(\mathcal{B}, C(\cdot))$ from $X = \{x, y, z\}$, where $\mathcal{B} = \{\{x, y\}, \{x, y, z\}\}$. Under WARP, $C\{x, y\} = \{x\} \Rightarrow y \notin C\{x, y, z\}$. BUT, we can have $z \in C(\{x, y, z\})$.

This is why the induced preference is called *revealed*: you don't know what else is going on.

[2]The elements $B \in \mathcal{B}$ are so-called *budget sets*. The budget sets in $\mathcal{B}$ should be thought of as an exhaustive listing of all the choice experiments that can be achieved, but it is possible that some subsets of X are not achievable.

[3]The choice set $C(B)$ can contain a single element, which is the choice among the alternatives in B. BUT, $C(B)$ can contain multiple elements, then elements of $C(B)$ are the *acceptable alternatives* in B.

1.3 Linking Preferences with Choices

Now we have two major approaches of decision making process: preference relations in Section **??** and choice rules in Section **??**, what we need to do is to link them. This linkage will emerge when we examine two central assumptions: **rationality** and **WARP**. So the major question here is:

$$\textbf{rational} \succsim \overset{???}{\Longleftrightarrow} (\mathcal{B}, C(\cdot)) \textbf{ satisfies WARP}$$

And the answer is: *YES!* but not exactly. Now let's dig in.

Rational $\succsim \Rightarrow (\mathcal{B}, C(\cdot))$ satisfies WARP

First, **rational** $\succsim \Rightarrow$ $(\mathcal{B}, C(\cdot))$ **satisfies WARP** is a big YES. To prove this, we need to define *induced choice correspondence*:

> **Definition 1.3.1.** Given a **rational** $\succsim$ on X, if the decision maker faces a nonempty subset of alternatives $B \subset X$, by maximizing her preference, she would choose any one of the elements in the *induced choice correspondence*: $C^*(B, \succsim) = \{x \in B : x \succsim y, \forall y \in B\}$

The induced choice correspondence $C^*(B, \succsim)$ has an important property:

> **Theorem 1.3.1.** if X is finite, $C^*(B, \succsim)$ will be **nonempty**.

A brief proof of this proposition is: If X is finite, B is finite as well. We will prove by induction. Starting from $|B| = 1$, the only element of B is in $C^*(B, \succsim)$. Now suppose $C^*(B, \succsim)$ is nonempty when $|B_n| = n$, let $x^* \in C^*(B_n, \succsim)$; when $|B_{n+1}| = n + 1$, let the $n + 1$th element y ($\{y\} = B_{n+1} \setminus B_n$). By the completeness of a rational $\succsim$, either $y \succsim x^*$ or $x^* \succsim y$:

i. $y \succsim x^*$: since $x^* \in C^*(B_n, \succsim) \Rightarrow x^* \succsim x, \forall x \in B_n$. By transitivity of $\succsim$, $y \succsim x, \forall \in B_n$. By completeness, $y \succsim y$ as well. Hence, $y \in C^*(B_{n+1}, \succsim)$.
ii. $x^* \succsim y$: since $x^* \in C^*(B_n, \succsim) \Rightarrow x^* \succsim x, \forall x \in B_n$, hence $x^* \succsim x, \forall x \in B_n \cup y \Rightarrow x^* \in C^*(B_{n+1}, \succsim)$

Notice that when B is finite, a stronger condition of $\succsim$ being acyclic and complete is equilavent to an induced choice rule $C^*(B, \succsim) \neq \varnothing$:

> **Theorem 1.3.2.** For a finite B, $\succsim$ is complete and **acyclic** $\Leftrightarrow C^*(B, \succsim) \neq \varnothing$

$\succsim$ is acyclic mean that: $b_1 \succsim b_2, b_2 \succsim b_3, \cdots, b_{n-1} \succsim b_n \Rightarrow b_n \not\succsim b_1$. An example of transitive but not *acyclic* relations is indifference $\sim$: $a_1 \sim a_2 \sim \cdots \sim a_n \Rightarrow a_n \sim a_1$. A brief proof of Theorem **??** is:

i. acyclic $\succsim \Rightarrow C^*(B, \succsim) \neq \varnothing$: Suppose if $C^*(B, \succsim) = \varnothing$, for $b_1 \in B, b_1 \notin C^*(B, \succsim) \Rightarrow \exists b_2$ s.t. $b_2 \succsim b_1$. Continue this process, we can generate a sequence of $\cdots \succsim b_2 \succsim b_1$, since B is finite, this sequence must end at b_n. If $\succsim$ is acyclic, $b_1 \not\succsim b_n$, this gives $b_n \succ b_1$, which would mean b_n must be in $C^*(B, \succsim)$, contradicting.
ii. $C^*(B, \succsim) \neq \varnothing \Rightarrow$ acyclic $\succsim$: Suppose $\succsim$ is not acyclic, then there exists $b_1 \succsim b_2 \succsim \cdots \succsim b_n \succsim b_1$, then for set $B = \{b_1, b_2, \cdots, b_n\}$, $\nexists b^*$ s.t. $b^* \succsim b_i \forall b_i \in B$, i.e., $C^*(B, \succsim) = \varnothing$.

With induced choice correspondence $C^*(B, \succsim)$ defined and non-emptyness proved, we can then say:

Theorem 1.3.3. If $\succsim$ is a rational preference relation, then the choice structure generated by $\succsim$, $(\mathcal{B}, C^*(\cdot, \succsim))$, satisfies WARP

We can prove this theorem quite easily: $\forall B, B'$ suppose we have $x, y \in B \cap B'$ and $x \in C^*(B, \succsim), y \in C^*(B', \succsim)$, then $x \succsim a, \forall a \in B$ and $y \succsim b, \forall b \in B'$. Naturally, we have $x \succsim y$ since $y \in B$. By rationality (transitivity) of $\succsim$, we have $x \succsim y \succsim b, \forall b \in B'$, which means $x \in C^*(B', \succsim)$. This is precisely the definition of WARP

$(\mathcal{B}, C(\cdot))$ satisfies WARP $\Rightarrow$ Rational $\succsim$

The proof of this direction is more subtle, and is NOT necessarily a yes. Again, we start from a auxilary definition:

Definition 1.3.2. For a choice structure $(\mathcal{B}, C(\cdot))$, a rational preference relation $\succsim$ **rationalizes** $C(\cdot)$ relative to $\mathcal{B}$ if $C(B) = C^*(B, \succsim), \forall B \in \mathcal{B}$.

In words, if for all budget sets $B \in \mathcal{B}$, the choices generated by a rational $\succsim$, is just the choice rule $C(\cdot)$, $C(\cdot)$ is rationalized by $\succsim$. This is, in a sense, constructing an explanation of decision making behavior with preferences.

We already proved that $C^*(B, \succsim)$ satisfies WARP, which means that if a rationalizing preference relation to exist, WARP must be satisfied. However, if WARP is satisfied, a rationalizing preference relation does **NOT** necessarily exist.[4] Intuitiviely, more budget sets $B \in \mathcal{B}$ would mean that, to satisfy WARP, choice behavior would be restricted more, and it is easier to be self-contradicting. Therefore, to pin down a rational preference relation to rationalize $C(\cdot)$ relative to $\mathcal{B}$, we need to put some **restrictions on** $\mathcal{B}$.

Theorem 1.3.4. If $(\mathcal{B}, C(\cdot))$ is a choice structure that:

i. WARP is satisfied
ii. $\mathcal{B}$ includes **all** subsets of X of **up to 3** elements

then there exists a rational preference relation $\succsim$ s.t. $C(B) = C^*(B, \succsim), \forall B \in \mathcal{B}$. And this rational $\succsim$ is the **only** preference relation that can rationalize $(\mathcal{B}, C(\cdot))$.

Now let's prove it, by examing the natural candidate for a rationalizing preference relation: the **revealed preference relation** $\succsim^*$:

Step 1 Prove that $\succsim^*$ is rational

- Completeness: By (ii) of Def.**??**, all binary subsets of X are in $\mathcal{B}$. Hence, $\{x, y\} \in \mathcal{B}$. For this binary menu, $C(\{x, y\})$ must contain either x or y, therefore, $x \succsim^* y$ or $y \succsim^* x$ or both. Completeness proved.
- Transitivity: $\forall \{x, y, z\} \in \mathcal{B}$, $C(\{x, y, z\}) \neq \emptyset$. Suppose $x \succsim^* y, y \succsim^* z$, which implies that $x \in C(\{x, y\}), y \in C(\{y, z\})$, we then have three cases for $C(\{x, y, z\})$:
  a. $x \in C(\{x, y, z\})$, WARP gives that $x \in C(\{x, z\}) \Rightarrow x \succsim^* z$

[4] A simple example is: $X = \{x, y, z\}, \mathcal{B} = \{\{x, y\}, \{y, z\}, \{x, z\}\}$. Since $\mathcal{B}$ contains 3 binary menus, the choice structure $C(\{x, y\}) = \{x\}, C(\{y, z\}) = \{y\}, C(\{x, z\}) = \{z\}$ vacuously satisfy WARP. But, this choice structure cannot be rationalized since it contradicts transitivity.

b. $y \in C(\{x, y, z\})$, we have $x \in C(\{x, y\})$. WARP gives $x \in C(\{x, y, z\})$ $\Rightarrow x \succsim^* z$

c. $z \in C(\{x, y, z\})$, we have $y \in C(\{y, z\})$. WARP gives $y \in C(\{x, y, z\})$, and $x \in C(\{x, y\})$, WARP gives $x \in C(\{x, y, z\}) \Rightarrow x \succsim^* z$

Hence, $x \succsim^* y, y \succsim^* z \Rightarrow x \succsim^* z$

Step 2 Prove that $\succsim^*$ rationalizes $C(\cdot)$ on $\mathcal{B}$

Now, we need to show $\forall B \in \mathcal{B}, C(B) = C^*(B, \succsim^*)$. Logically, this means the revealed preference $\succsim^*$ inferred from $C(\cdot)$ actually generates $C(\cdot)$. Formally, we prove it in 2 steps:

a. Suppose $x \in C(B)$, which means that $\forall y \in B, x \succsim^* y$ (by Def.**??**), hence $x \in C^*(B, \succsim^*)$ (by Def.**??**). This proves $C(B) \subseteq C^*(B, \succsim^*)$

b. Suppose $x \in C^*(B, \succsim^*)$, which means that $\forall y \in B, x \succsim^* y$ (by Def.**??**). Therefore, $\forall y \in B$, there must exist a set $B_y \in \mathcal{B}$ s.t. $x, y \in B_y \Rightarrow x \in C(B_y)$. Since $C(B) \neq \emptyset$, suppose $z \in C(B)$, since $x \in C(B_z)$, WARP implies that $x \in C(B)$. This proves $C^*(B, \succsim^*) \subseteq C(B)$

Together, we have $C(B) = C^*(B, \succsim^*)$.

Step 3 Prove $\succsim^*$ is the unique choice

Since $\mathcal{B}$ includes all two-element subsets of X, the choice behavior in $C(\cdot)$ completely determines the pairwise preference relations over X of any rationalizing preference.

Now, it is **proved**! Notice that the main assumption(restriction) here is $\mathcal{B}$ **includes all subsets of** X **of up to 3 elements**, this gives completeness, which is fundamental.

Two things to keep in mind

We have proved the twoway links of preferences and choices:

- Rational $\succsim \Rightarrow (\mathcal{B}, C^*(\cdot, \succsim))$ satisfies WARP (see Thm.**??**)
- A WARP-satisfying, up-to-3-element $(\mathcal{B}, C(\cdot))$ can be uniquely rationalized by a rational $\succsim$ (see Thm.**??**)

However, there are still something to keep in mind.

First, for a given choice structure $(\mathcal{B}, C(\cdot))$, there my be **more than one** rationalizing preference relation $\succsim$ in general. Here is the simplest example: For $X = \{x, y\}, \mathcal{B}\{\{x\}, \{y\}\}$ and the choice structure $C(\{x\}) = \{x\}, C(\{y\} = \{y\})$. In this case, **ANY** relation preference relation of X can rationalize $C(\cdot)$ This is related to both Def.**??** and (ii) of Thm.**??**. Thm.**??** gives that if $\mathcal{B}$ contains **ALL binary** menus of X, then there could be at most one rationalizing preference relation.

Second, the restriction for WARP$\Rightarrow$ rational $\succsim$, namely $\mathcal{B}$ containing all subsets of up to 3 elements, is too strong. For many economic problems, we will not consider all possible subsets, or limit ourselves to up-to-3-element ones. A strengthened version of WARP will be introduced later for that purpose.

Finally, up till now, we define a rationalizing preference as one: $C(B) = C * (B, \succsim)$ (Def.**??**). A common alternative would be to require only $C(B) \subset C^*(B, \succsim)$: if $C(B)$ is a **subset** of the most preferred choices generated by $\succsim$, i.e., $C^*(B, \succsim)$, for all $B \in \mathcal{B}$.

1.4 Chap1Sec4

CHAPTER 2

FUNDAMENTALS OF CONSUMER THEORY

CHAPTER 3

LAGRANGE MAXIMIZATION AND DUALITY

CHAPTER 4

MONOTONE COMPARATIVE STATICS

CHAPTER 5

EXPECTED UTILITY AND DECISIONMAKING UNDER UNCERTAINTY

CHAPTER 6

AGGREGATION AND THE EXISTENCE OF A REPRESENTATIVE CONSUMER

CHAPTER 7

PRODUCER THEORY

CHAPTER 8

STOCHASTIC CHOICE

Part II

Game Theory

CHAPTER 9

NASH EQUILIBRIUM AND BAYESIAN NASH EQUILIBRIUM

CHAPTER 10

RATIONALIZABILITY AND DOMINANT STRATEGIES

CHAPTER 11

CORRELATED EQUILIBRIUM

CHAPTER 12

DYNAMIC GAMES AND REFINEMENTS

CHAPTER 13

REPEATED GAMES/FOLK THEOREM

CHAPTER 14

RECURSIVE METHODS IN REPEATED GAMES

Part III

Mechanism Design and Contract Theory

CHAPTER 15

ARROW'S THEOREM AND SOCIAL CHOICE

CHAPTER 16

BOUNDARIES OF THE FIRM AND COASE'S THEOREM

CHAPTER 17

IMPLEMENTATION CONCEPTS

CHAPTER 18

THE REVELATION PRINCIPLE

CHAPTER 19

AUCTIONS AND OPTIMAL AUCTIONS

CHAPTER 20

EFFICIENT IMPLEMENTATION

CHAPTER 21

MORAL HAZARD

CHAPTER 22

FULL IMPLEMENTATION