Microeconomic Theory I: A Notebook

With Jonathan Libgober

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Check my Github Page, or email me!

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HERE WE GO!

This is my learning notebook of Microeconomic Theory I (Course number: ECON601 at USC Economics). As one of the core courses in an economic Ph.D. curriculum, Microeconomic Theory I is beyond important to my research. Therefore, I would love to use this notebook as a commitment mechanism, to document lecture notes, discuss session and office hour intuitions, reading summaries, my personal questions regarding the topics and more. By building a file from scratch, hopefull I could have a more systematic and sophisticated understanding on the content of this course.

I thank Prof. Jonathan Libgober at USC Economics for leading the discussion of the course and providing intuitive ways to understand microeconomic theory. Please check his webpage here, he is such fun.

I also appreciate the time and effort my TA Qitong Wang put into this course, guiding me through discussing sessions and problem sets. When I have questions, he is always there to help.

Following the structure of the course, this notebook will cover three aspects of microeconomic theories: (a) individual decision making, (b) game theory, (c) mechanism design and contract theory. Apart from Jonathan's lecture notes, I will also summarize the reading materials, including: Mas-Colell et al. (1995)'s *Microeconomic Theory*, Mailath (2018)'s *Modelling Strategic Behavior*[1], Fudenberg and Tirole (1991)'s *Game Theory*, Myerson (1991)'s *Game Theory: Analysis of Conflicts*, Bolton and Dewatripont (2005)'s *Contract Theory*, Mailath and Samuelson (2006)'s *Repeated Games and Reputation* and Osborne and Rubinstein (1994)'s *A Course in Game Theory*. Other materials will also be referred to along the way.

Building this notebook is truly a memorable journey for me. I would love to share this review and all the related materials to anyone that finds them useful. And unavoidably, I would make some typos and other minor mistakes (hopefully not big ones). So I'd really appreciate any correction. If you find any mistakes, please send the mistakes to this email address saizhang.econ@gmail.com, BIG thanks in advance!

[1] Latest version (May 2021) available here.

Contents

Part I

Individual Decision Making

CHAPTER 1

PREFERENCES AND CHOICES, UTILITIES

Contents

The first chapter summarizes the basic setting of individual decision making: preferences, choices and utilities. The main reference is Chapter 1 of Mas-Colell et al. (1995).

In this chapter, we will focus on 3 domains:

choice	given a set A, what choice from A is made
preference	given alternatives x, y, which does the decision maker prefers
utility	given an object X, how much does the DM likes X (as a number)

The starting point of individual decision problem is a *set of possible (mutually exclusive) alternatives* from which the individual must choose. To model decision making process on this set of alternatives, one can:

- either start from the tastes, i.e., *preference relations* of individuals, and set up the patterns of decision making with preferences
- or, start from the actual actions of individuals, i.e. *choices*, to deduct a pattern of decision making

With this two major approaches in mind, we know what's coming: the ***rationality*** of preferences and the central assumption of choices, the ***Weak Axiom of Revealed Preference (WARP)***. And of course, the two approaches and two basic assumptions are parallel, so we need to figure out how link the (underlying) preferences and (observed) choices.

1.1 Preference Relations

We start from the basic: *weak preference relation*, $\succsim$.

Definition 1.1. A weak preference relation $\succsim$ on a set X is a subset of $X \times X$. If $(x, y) \in \succsim \Rightarrow x$ is at least as good as y, written as $x \succsim y$

A weak preference relation will induce two other types of relations on X:

Definition 1.2. With $\succsim$ defined by Def. 1.1, we have

- the *strict preference relation*, $\succ$ can be induced from $\succsim$ as: $x \succ y \Leftrightarrow x \succsim y \wedge y \not\succsim x$, or in words, x if preferred to y.
- the *indifference relation*, $\sim$ can be induced from $\succsim$ as: $x \sim y \Leftrightarrow x \succsim y \wedge y \succsim x$, or in words, x is indifferent to y.

With the definition of these relations, we now define the central assumption of relations: ***rationality***.

Definition 1.3. A weak preference relation $\succsim$ is *rational* if it is:

- Complete: $\forall x, y \in X, x \succsim y$ or $y \succsim x$ or both
- Transitive: $\forall x, y, z \in X, x \succsim y \wedge y \succsim z \Rightarrow x \succsim z$

How to understand them? They are both strong assumptions:

- Completeness of $\succsim$ means it is well-defined between any two possible alternatives. From the perspective of an individual, completeness means that she will make choices, and only meditated choices.
- Transitivity of $\succsim$ implies that the decision maker will not have a preference cycle, since whoever has a preference cycle would suffer economically for it[1].

With the definition of rational $\succsim$ in Def. 1.3, we can prove the following properites of $\succ$ and $\sim$ *induced* by $\succsim$:

Theorem 1.1. If $\succsim$ is rational, then:

i. $\succ$ is irreflexive ($x \succ x$ never holds) and transitive ($x \succ y \wedge y \succ z \Rightarrow x \succ z$)
Proof:
   - irreflexive: by Def. 1.2, $x \succ x \Rightarrow x \succsim x \wedge x \not\succsim x$, self contracdiction.
   - transitive: $x \succ y \Rightarrow x \succsim y \wedge y \not\succsim x$, $y \succ z \Rightarrow y \succsim z \wedge z \not\succsim y$. By transitivity of $\succsim$, $x \succsim y \wedge y \succsim z \Rightarrow x \succsim z$. If $z \succsim x$, by transitivity of $\succsim$ and $x \succsim y$, we would have $z \succsim y$, contradicting $y \succ z$. Therefore $x \succsim z \wedge z \not\succsim x \Rightarrow x \succ z$.

ii. $\sim$ is reflexive ($x \sim x, \forall x$), transitive ($x \sim y \wedge y \sim z \Rightarrow x \sim z$) and symmetric ($x \sim y \Rightarrow y \sim x$)

[1] There are 2 types of violations of transitivity: irrational and mechanical. Irrational violations are easy to understand: decision makers simply do not follow transivity assumption, many reasons have been raised, including mental account, framing, menu effect, attraction effect, etc. Mechanical violations means that decision makers are "forced" to violate transitivity. One example of this type of violation is aggregation of considerations: decision makers may aggregate several sub-preferences as together to make the choice, leading to violation of transitivity. Another example is when the preference is only defined for differences above a certain level (problem of perceptible differences). See Mas-Colell et al. (1995, Page 7-8), Rubinstein (2012, Page 4-5) for details

Proof:
- reflexive: by completeness of $\succsim$, $\forall x, x \succsim x \Rightarrow x \sim x$
- transitive: $x \sim y \Rightarrow x \succsim y \wedge y \succsim x$, $y \sim z \Rightarrow y \succsim z, z \succsim y$, by the transitivity of $\succsim$, we have $x \succsim z \wedge z \succsim x$, hen $x \sim z$
- symmetric: $x \sim y \Rightarrow x \succsim y \wedge y \succsim x \Leftrightarrow y \succsim x \wedge x \succsim y \Rightarrow y \sim x$

iii. $x \succ y \succsim z \Rightarrow x \succ z$
Proof: $x \succ y \Rightarrow x \succsim y \wedge y \not\sim x$, hence $x \succ y \succsim z \Rightarrow x \succsim z$. If $z \succsim x$, by transitivity of $\succsim$, $y \succsim x$, contradicting $x \succ y$. Therefore, $z \not\sim x$

We can also directly define a *rational* $\succ$ (see Kreps (1990, Page 19-21)):

Definition 1.4. A strict preference ralation $\succ$ is rational if it is:
- asymmetric: $\nexists x, y \in X$ s.t. $x \succ y \wedge y \succ x$
- negatively transitive: $x \succ y \Rightarrow \forall z \in X \setminus x, y, x \succ z \vee z \succ y \vee$both.

With Def. 1.4 and Def. 1.3, we can prove that $\succsim$ is rational iff $\succ$ is rational.

1.2 Choice Rules

something

1.3 Linking Preferences with Choices

1.4 Chap1Sec4

CHAPTER 2

FUNDAMENTALS OF CONSUMER THEORY

CHAPTER 3

LAGRANGE MAXIMIZATION AND DUALITY

CHAPTER 4

MONOTONE COMPARATIVE STATICS

CHAPTER 5

EXPECTED UTILITY AND DECISIONMAKING UNDER UNCERTAINTY

CHAPTER 6

AGGREGATION AND THE EXISTENCE OF A REPRESENTATIVE CONSUMER

CHAPTER 7

PRODUCER THEORY

CHAPTER 8

STOCHASTIC CHOICE

Part II

Game Theory

CHAPTER 9

NASH EQUILIBRIUM AND BAYESIAN NASH EQUILIBRIUM

CHAPTER 10

RATIONALIZABILITY AND DOMINANT STRATEGIES

CHAPTER 11

CORRELATED EQUILIBRIUM

CHAPTER 12

DYNAMIC GAMES AND REFINEMENTS

CHAPTER 13

REPEATED GAMES/FOLK THEOREM

CHAPTER 14

RECURSIVE METHODS IN REPEATED GAMES

Part III

Mechanism Design and Contract Theory

CHAPTER 15

ARROW'S THEOREM AND SOCIAL CHOICE

CHAPTER 16

BOUNDARIES OF THE FIRM AND COASE'S THEOREM

CHAPTER 17

IMPLEMENTATION CONCEPTS

CHAPTER 18

THE REVELATION PRINCIPLE

CHAPTER 19

AUCTIONS AND OPTIMAL AUCTIONS

CHAPTER 20

EFFICIENT IMPLEMENTATION

CHAPTER 21

MORAL HAZARD

CHAPTER 22

FULL IMPLEMENTATION

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