

# Empirical Finance: A Review

*For Personal Reference*

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Inspired by the Course *Empirical Finance* at  
London Business School by *Dr. Svetlana Bryzgalova*

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## HERE WE GO!

Empirical finance is an absolutely fascinating field, with some of the most cutting-edge methodologies and the most exploratory techniques. Although it is not my speciality, I am always interested in this literature. During my pre-doc research fellowship at London Business School, I have had the privilege to study in the course *Financial Economics II: Empirical Finance*. The course instructor Dr. Svetlana Bryzgalova is absolutely one of the most brilliant scholars I have encountered. Thanks to her, I have got to understand this literature more systematically. In this (personal) review, I summarize the most influential and inspirational works in this field and organize them by different topics. The structure of this review resembles the structure of Dr. Bryzgalova's course, while adjusted according to my personal research interest. I intend to review classic works and discuss some potential directions of future study regarding my personal interest in Behavioral Economics, Game Theory and Network.

Since this review is tailored according to my own research interest and experience, I will not only summarize the theoretical perspectives of the studies, present their findings and discuss how they fit into the literature, but document my replication attempts and pseudo codes as well. All the codes related to this review can be found on [my Github page](#).

I thank Dr. Svetlana Bryzgalova for her valuable intuitions and impressive knowledge of the empirical finance literature. Building this review is truly a memorable journey for me. I would love to share this review and all the related materials to anyone that finds them useful. And unavoidably, I would make some typos and other minor mistakes (hopefully not big ones). So I'd really appreciate any correction. If you find any mistakes, please either set up a branch on Github or send the mistakes to this email address [saizhang.econ@gmail.com](mailto:saizhang.econ@gmail.com), BIG thanks in advance!

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# CHAPTER 1

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## TIME-SERIES PREDICTABILITY

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Every investor knows that trading in financial markets is to play games with time itself. Daily trades determine asset prices at every date and hence influence the random distribution of future prices as well as the initial level of prices. One would need "much more careful attention to the process by which both expected payoffs and required rates of return determine asset prices"<sup>1</sup>.

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<sup>1</sup>See [Campbell \(2017, p. 121\)](#)

In this chapter, I first, following [Campbell \(2017, Chapter 5\)](#), summarize models mapping cash flows and discount rates into prices using present value relations in Section 1.1. Then I discuss the early evidence for mean reversion in returns in Section 1.2. In Section 1.3, I examine the excess volatility puzzle in the predictability debate. To accomodate the stylized facts of time-series predictability, Section 1.4 presents two of the most influential approaches to decompose prices. In Section 1.5, I selectively summarize some researches from the so-called "Prediction Zoo", which satirically describes the floods of price predictors. Finally, I discuss the issues and extensions of time-series predictability in Section 1.6.

## 1.1 Concepts and models

In this section, I follow [Campbell \(2017, Chapter 5\)](#) and discuss some of the conceptual building blocks for the strand of time-series empirical finance literature.

### 1.1.1 Market Efficiency

An intuitive way of explaining *market efficiency* is that efficient markets are competitive and allow no easy ways to make economic profit. A more useful and testable definition was given by [Malkiel \(1989, p. 127\)](#):

The market is said to be efficient with respect to some information set  $\phi$ , if security prices would be unaffected by revealing that information to all participants.

Some event studies that measure market responses to news announcements can be interpreted as tests of market efficiency regarding the announced information, but in general, this definition is not easy to test. On the other hand, [Malkiel \(1989\)](#) gives a more testable alternative:

Efficiency with respect to an information set  $\phi$  implies that it is impossible to make economic profits by trading on the basis of  $\phi$ .

This is the idea behind an enormous literature in empirical asset pricing: if an economic model defines the equilibrium return as  $\Theta_{i,t}$ , then the null hypothesis is

$$R_{i,t+1} = \Theta_{i,t} + U_{i,t+1} \quad (1.1)$$

where  $U_{i,t+1}$  is a FAIR game regarding the information set at  $t$ , or  $\mathbb{E}(U_{i,t+1}|\phi_t) = 0$ . Notice that market efficiency is equivalent to rational expectations, one must test a model of expected returns as well when testing market efficiency. After defining a model of expected returns, the variables to be included in the information set must be specified. [Malkiel and Fama \(1970\)](#) define three forms of efficient market hypothesis and the corresponding information sets:

- the *weak form*: past returns

- the *semi-strong form*: publicly available information such as stock splits, dividends, or earnings
- the *strong form*: information available to some market participants, but NOT necessarily to all participants.

← this could be tested by using measurable actions (trades or portfolio holdings) of the potentially better informed agents

In the time-series literature, the simplest economic model is constant return:  $\Theta_{i,t} = \Theta$ . In Section 1.2, I summarize the early literature focusing on this model.

Market efficiency has been widely tested and debated, now the most accepted view of market efficiency hypothesis is that it is a useful benchmark but does not hold perfectly. The debates between long-term versus short-term efficiency, micro versus macro efficiency are still and will continue to be heated. Some noticeable alternative hypotheses are:

- *High-frequency noise*: market prices are contaminated by short-term noise, which can be caused by measurement errors or illiquidity (bid-ask bounce).
- *Imperfect information processing*: the market reacts sluggishly to information after its releasing
- *Persistent mispricing*: market prices deviate substantially from efficient levels in a LONG time
- *Disposition effect*: individual investors are more willing to sell winning stocks than losing stocks, see [Shefrin and Statman \(1985\)](#) for details.

### 1.1.2 Model: autocorrelation of returns

The most basic time-series test of market efficiency is to test "whether past deviations of returns from model-implied expected returns predict future return deviations" (See [Campbell, 2017](#), p. 124). The leading approach to do so is to test the autocorrelations.

Starting points:

1. The null hypothesis  $H_0$ : the stock returns are i.i.d.
2. The standard error for any single sample autocorrelation equals asymptotically  $1/\sqrt{T}$ , see [Box and Pierce \(1970\)](#) for a detailed discussion.
3. The standard error would be large, (0.1 if  $T = 100$ ), not so easy to detect small autocorrelation

Any autocorrelation test would have to solve these issues.

#### 1.1.2.1 Q-statistics

Because the stock returns are i.i.d. ( $H_0$ ), different autocorrelations are uncorrelated with one another. [Box and Pierce \(1970\)](#) calculates a sum of  $K$  squared sample autocorrelations:

$$Q_K = T \sum_{j=1}^K \hat{\rho}_j^2 \quad (1.2)$$

where  $\hat{\rho}_j = \text{Corr}(r_t, r_{t-j})$ .  $Q$  is asymptotically distributed  $\chi^2$  with  $K$  degrees of freedom.

**Pros:** It solves the problem of the large standard errors.

**Cons:** It does NOT use the sign of the autocorrelations (squared). What could happen is that the expected reutrns are not constant, instead, they are each individually small but all have the same sign.

### 1.1.2.2 Variance ratio

One way to take the sign of autocorrelations into consideration is the variance ratio statistic. This statistic was introduced to the finance literature by [Lo and MacKinlay \(1988\)](#) and [Poterba and Summers \(1988\)](#).

**The basic setting is:** for a holding period  $K$ , the log return of this entire period  $r_t(K)$  is the sum of all the one-period returns  $r_{t+i}$ :

$$r_t(K) \equiv r_t + r_{t+1} + \cdots + r_{t+K-1}$$

and the variance ratio over the period  $K$  would be defined as:

$$VR(K) = \frac{Var(r_t(K))}{K \cdot Var(r_t)}$$

If there are not autocorrelations, then the i.i.d. returns would have identical variance in each period from  $t$  to  $t + K$ , and  $Var(r_t(K)) = Var(r_t + \cdots + r_{t+K-1}) = Var(r_t) + \cdots + Var(r_{t+K-1}) = K \cdot Var(r_t)$ . Thus,  $VR(K) = 1$ . If we rewrite the definition of the variance ratio as:

$$VR(K) = \frac{Var(r_t(K))}{K \cdot Var(r_t)} = 1 + \underbrace{2 \sum_{j=1}^{K-1} \left(1 - \frac{j}{K}\right) \hat{\rho}_j}_{\text{weighted average of the first } K-1 \text{ sample autocorrelations}} \quad (1.3)$$

Then by comparing  $VR(K)$  with 1, we can deduct the direction of the autocorrelations:

$VR(K) > 1$	predominantly <b>positive</b> autocorrelations
$VR(K) = 1$	no autocorrelations
$VR(K) < 1$	predominantly <b>negative</b> autocorrelations: mean reversion

Notice that the weight term  $1 - \frac{j}{K}$  increases as  $j$  approaches  $K^2$ .

The asymptotic variance of the variance-ratio statistic, under  $H_0$  (i.i.d. returns), is:

$$Var(\hat{VR}(K)) = \frac{4}{T} \sum_{j=1}^{K-1} \left(1 - \frac{j}{K}\right)^2 = \frac{2(2K-1)(K-1)}{3KT} \xrightarrow{K \rightarrow \infty} \frac{4K}{3T} \quad (1.4)$$

<sup>2</sup>[Cochrane \(1988\)](#) showed that the estimator of  $VR(K)$  can be interpreted in terms of the frequency domain. It is asymptotically equivalent to  $2\pi$  times the normalized spectral density estimator at the zero frequency, which uses the Bartlett kernel.



When  $K \rightarrow \infty$ ,  $T \rightarrow \infty$ , and  $K/T \rightarrow 0$  (Priestley, 1981, p. 463), the true return process can be serially correlated and heteroskedastic, but the variance of the variance-ratio is still given as:

$$Var(\hat{VR}(K)) = \frac{4K}{3T} \cdot VR(K)^2 \quad (1.5)$$

Notice that this can be quite large with a large  $VR(K)$ . This is due to the fact that  $K/T \rightarrow 0$  is a dangerous assumption because in practice  $K$  is often large relative to the sample size. To tackle this, Lo and MacKinlay (1988) develop alternative asymptotics assuming  $K/T \rightarrow \delta$  where  $\delta > 0$ . Through Monte Carlo simulations, they demonstrated that this new distribution is a more robust approximation to the small-sample distribution of the VR statistic. Most current applications of the VR statistic cite  $K/T \rightarrow \delta > 0$  as the justification for using Monte Carlo distributions (i.e. set at  $K = \delta T$ ) as representative of the VR statistic's sampling distribution. Some recent challenges of this result are discussed in Section 1.1.3.

To accommodate  $r_t$ 's exhibiting conditional heteroskedasticity, Lo and MacKinlay (1988) proposed a heteroskedasticity-robust variance estimation of  $VR(K)$  as:

$$Var^*(\hat{VR}(K)) = 4 \sum_{j=1}^{K-1} \left(1 - \frac{j}{K}\right)^2 \cdot \frac{\sum_{t=j+1}^T (r_t - \bar{r})^2 (r_{t-j} - \bar{r})^2}{\left[\sum_{t=1}^T (r_t - \bar{r})^2\right]^2}$$

where  $\bar{r} = \frac{1}{T} \sum_{t=1}^T r_t$  is the estimated mean of returns.

### 1.1.2.3 Regression approach

Fama and French (1988) established a regression approach to test  $AR(K)$ . The basic idea is to regress the  $K$ -period return on the lagged  $K$ -period return:

$$r_t(K) = \alpha_K + \beta_K r_{t-K}(K) + \epsilon_t^K$$

The coefficient  $\beta_K$  would then be:

$$\beta_K = \frac{Cov[r_t(K), r_{t-K}(K)]}{Var[r_{t-K}(K)]} = 2 \left[ \frac{VR(2K)}{VR(K)} - 1 \right] = \frac{2 \sum_{j=1}^{K-1} \left( \frac{\min(j, 2K-j)}{K} \right) \rho_j}{VR(K)} \quad (1.6)$$

It is clear to see that:

$\beta_K > 0$	predominantly <b>positive</b> autocorrelations
$\beta_K = 0$	no autocorrelations
$\beta_K < 0$	predominantly <b>negative</b> autocorrelations: mean reversion

### 1.1.3 Extension: Other variance-ratio tests

As summarized by Charles and Darné (2009), the intuition behind the VR test is rather simple, but conducting a statistical inference using the VR test is less straightforward. In this bonus subsection, I briefly summarize some recent development of individual VR tests, multiple VR tests and bootstrapping VR tests. For more detailed discussion, see Charles and Darné (2009) for a review.

### 1.1.3.1 Individual VR tests

Conventional VR tests, such as the **Lo and MacKinlay** test, are asymptotic tests: their sampling distributions are approximated by their limiting distributions. In practice, the asymptotic theory provides a poor approximation to the small-sample distribution of the VR statistic, which impeded the use of the statistic. In general, the ability of the asymptotic distribution to approximate the finite-sample distribution depends crucially on the value of  $K$ . For a large  $K$  relative to  $T$ , **Lo and MacKinlay (1990a)** have proved that the VR statistics are severely biased and right skewed. Several alternative tests try to tackle this issue.

**Chen and Deo (2006) Test:** they suggested a simple power transformation of the VR statistic when  $K$  is NOT too large. This transformation is able to solve the right-skewness problem and robust to conditional heteroskedasticity. They showed that the transformed VR statistic leads to significant gain in power against mean reverting alternative. They define the VR statistic based on the periodogram and this new statistic is precisely the normalized discrete periodogram average estimate of the spectral density of a stationary process at the origin.

**Wright (2000) Test:** they proposed a non-parametric alternative using signs and ranks. This test outperforms the **Lo and MacKinlay** test in 2 ways:

- (1) As the rank and sign tests have an exact sampling distribution, there is no need to resort to asymptotic distribution approximation.
- (2) The tests may be more powerful against a wide range of models displaying serial correlation, including fractionally integrated alternatives.

The rank-based tests display low-size distortions under conditional heteroskedasticity. One thing to notice is that the sign test assumes a zero drift value, **Luger (2003)** extended this test with unknown drift.

**Choi (1999) Test:** To overcome the issue that the arbitrary and *ad hoc* choice of  $K$ , **Choi (1999)** proposed a data-dependent procedure to determine the optimal value of  $K$ . This test is based on frequency domain following **Cochrane (1988)**. However, instead of using Bartlett kernel as **Cochrane**, **Choi** employed the quadratic spectral kernel since it's optimal in estimating the spectral density at the zero frequency.

### 1.1.3.2 Multiple VR tests

All the tests above are individual tests, where the null hypothesis is tested for an individual value of  $K$ . However, to determine whether a time series is a random walk, we need to rule out all possibilities, meaning that for all values of  $K$ , the null hypothesis can not be rejected. It is necessary to conduct a joint test where a multiple comparison of VRs over a set of different time horizons is made. However, conducting separate individual tests for a number of  $K$  values may lead to over rejection of the null hypothesis. Several tests have been developed for this problem, with the joint null hypothesis  $H_0 : \forall K_i, V(K_i) = 1$  against the alternative  $H_1 : \exists K_i, V(K_i) \neq 1$

**Chow and Denning (1993) Test:** This test statistic is defined as

$$MVR_1 = \sqrt{T} \max_{1 \leq i \leq m} |M_1(K_i)|$$

where  $M_1(K_i) = \frac{VR(K_i)-1}{\sqrt{2(2K-1)(K-1)/3KT}}$ . This is based on the idea that the decision regarding the null hypothesis can be obtained from the maximum absolute value of the individual VR statistics. Then, they applied the Sidak probability inequality and give an upper bound to the critical values taken in the studentized maximum modulus (SMM) distribution. The statistic follows the SMM distribution with  $m$  and  $T$  degrees of freedom, where  $m$  is the number of  $K$  values. To accommodate heteroskedasticity, one can change  $M_1(K_i)$  into a heteroskedasticity-robust individual VR test.

**Whang and Kim (2003) Test:** They use a subsampling technique to develop a multiple VR test. When sample size ( $T$ ) is relatively small, this test outperforms the conventional VR tests, and shows little to no serious size distortions. The statistic is:

$$MVR_T = \sqrt{T} \max_{1 \leq i \leq m} |VR(K_i) - 1|$$

and the sampling distribution function for the  $MVR_T$  statistic is asymptotically a maximum of a multivariate normal vector with an unknown covariance matrix, which would be complicated to estimate. Therefore, they proposed to approximate the null distribution by means of the subsampling approach. For a subsample of size  $b$ :  $(x_t, \dots, x_{t-b+1})$  where  $t = 1, \dots, T - b + 1$ . The statistic  $MVR$ s calculated from all individual subsamples would generate a  $(1 - \alpha)$ th percentile for the  $100(1 - \alpha)\%$  critical value. To implement this subsampling technique, a choice of block length  $b$  must be made. **Whang and Kim (2003)** recommended the interval of  $(2.5T^{0.3}, 3.5T^{0.6})$ , but they also found that the size and power properties of their test are not sensitive to  $b$ .

**Belaire-Franch and Contreras (2004) Test:** This is a multiple rank and sign VR tests, an extension to the **Wright's** rank- and sign-based tests. The test is based on the definition of **Chow and Denning (1993)** procedure. The rank-based procedures are exact under the i.i.d. assumption whereas the sign-based procedures are exact under both the i.i.d. and martingale difference sequence assumption. They showed that rank-based tests are more powerful than their sign-based counterparts.

**Richardson and Smith (1991, Wald-Type Test):** They suggested a joint test based on the following Wald statistic:

$$MVR_{RS}(K) = T(\mathbf{VR} - \mathbf{1})'\Phi^{-1}(\mathbf{VR} - \mathbf{1})$$

where  $\mathbf{VR}$  is the  $K \times 1$  vector of sample  $K$  VRs,  $\mathbf{1}$  is the  $K \times 1$  unit vector;  $\Phi$  is the covariance matrix of  $\mathbf{VR}$ . This statistic  $MVR_{RS}(K)$  follows a  $\chi^2$  distribution with  $K$  degrees of freedom. One thing to remember about this test is that the VR tests are computed over Long lags with overlapping observations, the distribution of the VR test is NON-normal.

**Cecchetti and Lam (1994, Wald-Type Test):** They also developed a Wald-type multiple VR statistic that incorporates the correlations between VR statistics at various horizon and weights them according to their variances:

$$MVR_{CL}(K) = [\mathbf{VR}(K) - \mathbb{E}[\mathbf{VR}(K)]]'\Psi^{-1}(K)[\mathbf{VR}(K) - \mathbb{E}[\mathbf{VR}(K)]]$$

Again,  $\mathbf{VR}(K)$  is a vector of VR statistics and  $\Psi$  is a measure of the covariance matrix of  $\mathbf{VR}$ ; and again, this statistic follows a  $\chi^2$  distribution with  $K$  degrees of freedom.

However, after using Monte Carlo techniques to study the empirical distribution of  $MVR_{CL}(K)$ , they have found that it has large positive skewness, not  $\chi^2$ .

**Chen and Deo (2006, Wald-Type Test):** They proposed a joint VR test based on their individual power transformed VR statistic, also following a  $\chi^2$  distribution with  $K$  degrees of freedom. One feature sets the **Chen and Deo** test apart from the **Richardson and Smith** test and the **Cecchetti and Lam** test: this test is with ONE-sided alternative (i.e.,  $H_1 : \exists K_i \text{ s.t. } VR(K_i) < 1$ ).

### 1.1.3.3 Bootstrapping VR tests

Instead of using the subsampling method, some researchers proposed to employ a bootstrap method, which is distribution-free and can be used to estimate the sampling distribution of the VR statistic when the distribution of the original population is unknown.

**Kim (2006) Test:** **Kim (2006)** applied the wild bootstrap to the **Lo and MacKinlay** test and the **Chow and Denning** test in 3 stages:

- (1) From a bootstrap sample of  $T$  observations  $X_t^* = \eta_t X_t$ , where  $\eta_t$  is a random sequence with  $E(\eta) = 0$ ,  $E(\eta^2) = 1$  and  $t = 1, \dots, T$
- (2) For the bootstrap sample generated in (1), calculate  $VR^* = VR(X^*, K_i)$
- (3) Repeat (1) and (2) for a sufficient amount of times  $m$ , to form a bootstrap distribution of the test statistic  $\{VR(X^*, K_j; j)\}_{j=1}^m$

Conditional on  $X_t$ ,  $X_t^*$  is a serially uncorrelated sequence with zero mean and variance  $X_t^2$ . Thus, wild bootstrapping approximates the sampling distributions under the null hypothesis, which is a desirable property for a bootstrap test. To perform the test, a specific form of  $\eta_t$  should be chosen. **Kim (2006)** recommended using the standard normal distribution for  $\eta_t$ .

**Malliaropulos and Priestley (1999) Test:** They used a weighted bootstrap method proposed by **Wu et al. (1986)**, which is heteroskedasticity-robust and done by resampling normalized returns instead of actual returns. The bootstrap scheme can be summarized in 4 steps:

- (1) For each  $t$ , draw a weighting factor  $z_t^*$  with replacement from the empirical distribution of normalized returns  $z_t = (r_t - \bar{r})/\sigma(r)$ , where  $t = 1, \dots, T$ ,  $\bar{r} = E(r)$ ,  $\sigma(r) = Var(r)$ .
- (2) Form the bootstrap sample of  $T$  observations  $\tilde{r}_t^* = z_t^* r_t$
- (3) Calculate the VR statistic  $VR^*(K)$  from the sample  $r_t^*$
- (4) Repeat steps (1) and (2)  $M$  times, obtaining  $VR^*(K; m)_{1 \leq m \leq M}$

Using this procedure, resampling from normalized returns, the weighted bootstrap method accounts for the possible non-constancy of the variance of returns. This weighted bootstrap scheme is designed to overcome the difficulty that resampling methods may generate data that are less dependent than the original data. One thing to notice is that **Malliaropulos and Priestley's** method is not asymptotically pivotal and not supported by any asymptotic theory or Monte Carlo evidence to evaluate its properties, unlike **Kim's** method.

## 1.2 Autocorrelations in returns: empirical evidence

With the models introduced in Section 1.1.2, the mean reversion in stock returns has been examined empirically over the years. The common ground is that there are autocorrelations, but the directions of autocorrelations are indefinite across different settings.

A brief summary is listed below, and the details are discussed in the corresponding sub-sections.

### 1.2.1 Mean Riverse: Negative autocorrelations

In daily, weekly and even monthly data, individual stocks have small negative autocorrelations, a.k.a., mean reverse. Mean reversion implies that stocks are less risky for long-run investors, thus inducing investors with a longer investment horizon (e.g. younger investors, university endowments etc.) to allocate (on average) a larger share of their portfolio to the risky asset. Furthermore, mean-reverse can also justify the strategies of purchasing securities that have recently declined in value. It could also partially explain why economic agents could be reluctant to tie decisions to current market values.

Given the observation of mean reserve, if stock returns are i.i.d. over time, a mean-variance investor will choose a constant stock/bond allocation that does NOT depend on his investment horizons as a result. In the benchmark model, investors' horizon plays no role.

Empricially, a mean-reverting component of stock prices tends to induce negative autocorrelation in returns. Several representative studies are summarized and discussed below, most of them focusing on the long-horizon returns. I also summarize a theoretical explanation of this observation.

#### 1.2.1.1 Fama and French (1988)

Using stock returns of 1926-1985 sample period, **Fama and French** presented evidence of large negative autocorrelations for return horizons beyond a year. Their proposition is that a slowly decaying component of prices induces negative AR in returns. This decay is slow such that for daily or weekly holding periods, it will not have a significant effect.

First, the authors present a simple model for stock prices:

$$p_t = q_t + z_t, \quad q_t = q_{t-1} + \mu + \eta_t$$

where price  $p_t$  is the sum of a random walk  $q_t$  (with  $\mu$  is expected drift,  $\eta_t$  is white noise) and a stationary component  $z_t$ .  $z_t = \phi z_{t-1} + \epsilon_t$  is the driving force of ARs of prices. Then the continuously compounded return from  $t$  to  $t+T$  is:  $r_{t,t+T} = p_{t+T} - p_t = [q_{t+T} - q_t] + [z_{t+T} - z_t]$ . We know  $q_t$  is a random walk,  $z_t$  is stationary, thus for T-period

changes in  $z_t$ , the AR(1) is:

$$\rho_T = \frac{Cov[z_{t+T} - z_t, z_t - z_{t-T}]}{Var[z_{t+T} - z_t]}$$

which is just the slope of regressing  $z_{t+T} - z_t$  on  $z_t - z_{t-T}$ . When  $T$  increases,  $Cov(z_t, z_{t+T})$  approaches 0, thus:

$$\rho_T = \frac{-\sigma^2(z) + 2Cov(z_t, z_{t+T} - Cov(z_t, z_{t+2T}))}{2\sigma^2(z) - 2Cov(z_{t+T}, z_t)} \xrightarrow{T \rightarrow \infty} -0.5$$

If  $z_t$  is an AR(1), i.e.  $z_{t+1} = \phi z_t$ , then  $E_t[z_{t+T}, z_t] = (\phi^T - 1)z_t$ , and  $Cov[z_{t+T} - z_t, z_t, z_{t-T}] = -(\phi^T - 1)^2 \sigma^2(z) = Var(E_t[z_{t+T} - z_t])$ . Thus,  $\rho^T = -\frac{Var(E_t[z_{t+T} - z_t])}{Var(z_{t+T} - z_t)}$ .

Now, We cannot observe  $z_t$ , instead, we use observed returns to infer its properties. If changes in the random walk  $q_t$  and stationary components  $z_t$  are uncorrelated, then the slope of regression  $r_{t,t+T}$  on  $r_{t-T,t}$  is:

$$\beta_T = \frac{Cov[r_{t,t+T}, r_{t-T,t}]}{\sigma^2[r_{t-T,t}]} = \frac{\rho_T Var[z_{t+T} - z_t]}{Var[z_{t+T} - z_t] + Var[q_{t+T} - q_t]} \simeq \frac{-Var(E_t[z_{t+T} - z_t])}{\sigma^2(r_{t-T}, t_t)}$$

With this equation, we can deduct that if the price:

- Does not have a stationary component ( $\forall t, z_t = 0$ ):  $\beta_T = 0$ , no ARs
- Does not have a random walk ( $\forall t, q_t = 0$ ):  $\beta_T = \rho_T \xrightarrow{T \rightarrow \infty} -0.5$
- Has both  $z_t$  and  $q_t$ :  $\beta_T = \rho_T \cdot \frac{1}{1 + Var[q_{t+T} - q_t]/Var[z_{t+T} - z_t]}$ ,  $z_t$  pushes  $\beta_T$  to -0.5 while  $q_t$  pushes it to 0. Notice that  $Var[z_{t+T} - z_t] \xrightarrow{T \rightarrow \infty} 2\sigma^2(z)$ ,  $Var[q_{t+T} - q_t] \xrightarrow{T \rightarrow \infty} \infty$ , eventually,  $\beta_T \rightarrow 0$ .

With this fairly simple model, **Fama and French** predict that if stock prices have both random-walk and slowly decaying stationary components, the slopes of regressing  $r_{t,t+T}$  on  $r_{t-T,t}$  might form a U-shaped pattern: starting around 0 for short horizons, become more negative as  $T$  increases, then moving back to 0.

To support this proposition, **Fama and French** examined returns for industry portfolios and size-based portfolios. Industry and size are dimensions to capture differences in return behavior, which will generate different mix of  $q_t$  and  $z_t$ , leading to different return patterns. They used NYSE-listed stocks' monthly returns from 1926 to 1985 for the analysis. They run OLS (and bias-adjusted OLS) regressions  $r_{t+T,t} = \alpha_t + \beta_t r_{t,t-T} + \epsilon_{t,t+T}$  for market capitalization based decile portfolios, SIC 17-industry portfolios, equal- and value-weighted NYSE market portfolios. I have replicated the size-based portfolio and market portfolio results with updated data till 2020. Yearly return window  $T$  is chosen as  $T \in \{1, \dots, 10\}$ , or 12 to 120 months.

The left panel of Figure 1.1 replicates the Hansen-Hodrick adjusted OLS regression results. The general trend resembles **Fama and French**'s findings in two aspects: First, the OLS  $\beta$ s are generally negative, reaching minimum values for 3-5-year returns and becoming less negative for return horizons beyond 5 years. This U-shaped pattern, as explained by **Fama and French**, is consistent with the hypothesis that prices have stationary components (leading to negative  $\beta$ s, i.e., mean-reverse) and random-walk



components that eventually dominating long-horizon returns ( $\beta = 0$ ). Second, portfolio heterogeneity is presented. Equal-weighted market portfolio resembles smaller firms, meanwhile, value-weighted market portfolio resembles larger firms. In the right panel, I have applied Newey-West standard error adjustment. Results are similar.

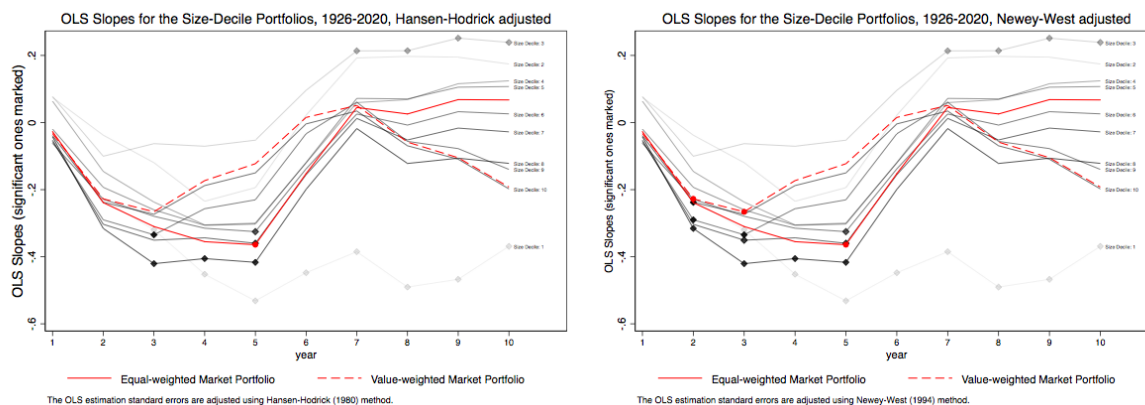


Figure 1.1: OLS Slopes for Size and Market Portfolios, 1926-2020

I also split the time window into 1926-1985 and 1986-2020. While most of the observations resemble the pooled results, two interesting results emerge: First, although the negativity of  $\beta$ s and the U-shaped patterns are still observed, the nature of the return structure has changed significantly after 1985. The difference between returns of different size-decile portfolios has become bigger. Second, after 1985, the mean reverse of returns appears to be in a larger scale for the equal-weighted market index, suggesting a divergence between small and large firms. But in general, the conclusion of U-shaped retrun horizon and mean-reverse still holds.

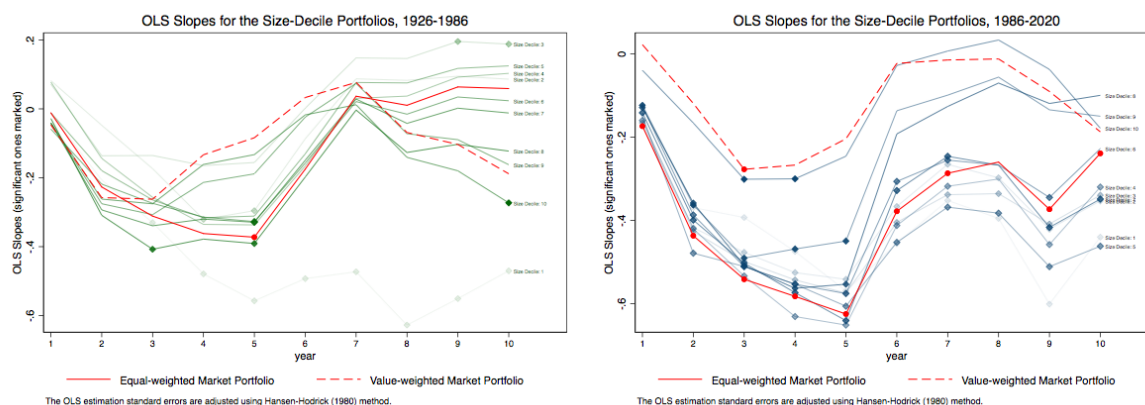


Figure 1.2: OLS Slopes for Size and Market Portfolios, 1926-1985 and 1986-2020

In the paper, **Fama and French** also simulated mimic properties of NYSE monthly returns 10000 times to obtain sampling distributions of *betas*. They simulate two models with 0 true slope: 1) a random walk in the log price with normal (0,1) monthly return, 2) a random walk where return variances change every 2 years to approxi-

mate the real variance changes; and 2 models with both a random-walk and an AR1 component, which contain either constant or changing variances as well.

Notice that the estimation this finite sample suffers from biased standard error: OLS standard errors are wrong due to overlapping observations, GMM (Newey-West) standard errors are wrong due to the small sample size (for the 10-year returns, there are only 6 truly independent observations from 1926 to 1985).

### 1.2.1.2 Poterba and Summers (1988)

In this paper, **Poterba and Summers** used mean-variance ratio tests to present evidence of mean reversion in stock prices of NYSE market, individual securities and global markets. Remember that Equation 1.3 gives the summary that a variance ratio  $VR(K)$  that is **less than** 1 indicates mean reverse.

Their analysis were done in 2 major steps:

- Examine the "power" of VR tests:
  - Return variance decomposition: They used the model that stock prices ( $p_t$ ) have two components: permanent  $p_t^*$  ( $p_t^* - p_{t-1}^* = \epsilon_t$ ) and transitory AR(1)  $u_t$  ( $u_t = \rho_1 u_{t-1} + v_t$ ). Return  $\Delta p_t$  is then  $\Delta p_t = \epsilon_t + \frac{1-L}{1-\rho_1 L} v_t$ . If  $\epsilon_t$  and  $v_t$  are independent,  $\Delta p_t$  follows an ARMA(1,1) process. Then  $\rho_1$  determines the persistence of the transitory component, the relative magnitude of  $\sigma_\epsilon^2$  and  $\sigma_v^2$ , defined as  $\delta$ , determines the importance of the two components in return movements.  $\delta$  and  $\rho_1$  will determine the return series.<sup>3</sup>
  - Monte-Carlo experiments: They generated 25000 sequences of 720 returns, with each return sequence generated by drawing 720 pairs of standard normal variates ( $\sigma_\epsilon^2, \sigma_v^2$ ). They set  $\sigma_\epsilon^2 = 1$ , thus the variance of returns is  $1 + \frac{2\sigma_v^2}{1+\rho_1}$ , and the "importance" of the transitory component  $p_t^*$  is  $\delta = \frac{2\sigma_v^2}{(1+\rho_1)+2\sigma_v^2}$ . Replication

### 1.2.1.3 De Bondt and Thaler (1985)

### 1.2.1.4 Pástor and Stambaugh (2012)

## 1.2.2 Positive autocorrelations

Due to the positive cross-autocorrelations among individual stocks, **Stock indexes** have predominantly positive high-frequency autocorrelations. Below, I summarize 2 papers in detail on this subject.

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<sup>3</sup>This model was introduced by **Summers (1986)**.  $\Delta p_t = (p_t^* - p_{t-1}^*) + (u_t - u_{t-1})$ . After a backward induction, the AR(1)  $u_t$  can be written as  $u_t = \sum_{j=0}^{\infty} \rho_1^j v_{t-j}$ . If we define  $v_t = v_{t-1}/L$ , then we get  $u_t - u_{t-1} = \frac{1-L}{1-\rho_1 L} v_t$ .



### 1.2.2.1 Lo and MacKinlay (1988, 1990a)

Lo and MacKinlay (1988, 1990a) presented evidence that broad market indexes have predominantly positive high-frequency autocorrelations. In 1988, they found a 30% AR(1) with weekly returns of *September 1962 to December 1985*, while higher-order ARs are also positive although smaller in magnitude. In 1990a, they found similar results with the sample period extended to *December 1987*. They proposed that the equal-weighted index autocorrelation could be rewritten into the sum of own-autocovariances and cross-autocovariances of the component securities. Given that the autocorrelations of individual stock returns are generally negative, they deducted that the cross-autocovariances must be positive and large enough to exceed the sum of the negative own-autocovariances. They built the following model and showed the importance of the forecastability *across* securities for contrarian profits:

Consider a stylized contrarian investment strategy: buy stocks at time  $t$  that were losers at time  $t - k$  and sell stocks at  $t$  that were winners at  $t - k$ . This strategy can be formally written as

$$\omega_{i,t}(k) = -\frac{1}{N}(R_{i,t-k} - R_{m,t-k}), \quad i = 1, \dots, N$$

where  $R_{m,t-k} = \frac{1}{N} \sum_{i=1}^N R_{i,t-k}$  is the market (equal-weighted) index return.

By construction,  $\vec{\omega}_t(k) \equiv [\omega_{1,t}(k), \dots, \omega_{N,t}(k)]'$  is an arbitrage portfolio since the weights sum to zero. Such a strategy is designed to take advantage of stock market overreactions since the stocks whose returns deviate more from the market index return will be given higher weights (more positive for huge losers and vice versa). Profit generated from this strategy is  $\pi_t(k) = \sum_{i=1}^N \omega_{i,t}(k) R_{i,t}$ , rewrite this profit, get:

$$\pi_t(k) = \sum_{i=1}^N \omega_{i,t}(k) R_{i,t} = -\frac{1}{N} \sum_{i=1}^N (R_{i,t-k} - R_{m,t-k}) R_{i,t} = -\frac{1}{N} \sum_{i=1}^N R_{i,t-k} R_{i,t} + R_{m,t-k} R_{m,t}$$

take expectation, get:

$$\begin{aligned} E[\pi_t(k)] &= -\frac{1}{N} \sum_{i=1}^N E[R_{i,t-k} R_{i,t}] + E[R_{m,t-k} R_{m,t}] \\ &= -\frac{1}{N} \sum_{i=1}^N (Cov[R_{i,t-k}, R_{i,t}] + \mu_i^2) + (Cov[R_{m,t-k}, R_{m,t}] + \mu_m^2) \end{aligned}$$

where  $\mu_m \equiv E[R_{m,t}] = \mu' \iota / N$ . Reorganizing this equation into 3 components:

$$\begin{aligned}
 E[\pi_t(k)] &= -\frac{1}{N} \text{tr}(\Gamma_k) - \frac{1}{N} \sum_{i=1}^N \mu_i^2 + \frac{\iota' \Gamma_k \iota}{N^2} + \mu_m^2 \\
 &= \underbrace{\left\{ \frac{\iota' \Gamma_k \iota}{N^2} - \frac{1}{N} \text{tr}(\Gamma_k) \right\}}_{\mathbf{L}_k} - \underbrace{\left\{ \frac{1}{N} \sum_{i=1}^N (\mu_i - \mu_m)^2 \right\}}_{\mathbf{O}_k} \\
 &= \underbrace{\frac{1}{N^2} [\iota' \Gamma_k \iota - \text{tr}(\Gamma_k)]}_{\mathbf{C}_k} + \underbrace{\left( -\frac{N-1}{N} \right) \text{tr}(\Gamma_k)}_{\mathbf{O}_k} - \underbrace{\frac{1}{N} \sum_{i=1}^N (\mu_i - \mu_m)^2}_{\sigma^2(\mu)} \quad (1.7)
 \end{aligned}$$

where  $\text{tr}(\cdot)$  indicates the trace operator (sum of diagonal elements).

The three components are:

- :  $\mathbf{C}_k$  depends ONLY on the off-diagonals of the auto-covariance matrix  $\Gamma_k$
- :  $\mathbf{O}_k$  depends ONLY on the diagonals of the auto-covariance matrix  $\Gamma_k$
- :  $\sigma^2(\mu)$  is independent of  $\Gamma_k$

Or, cross-autocovariances dictate  $\mathbf{C}_k$ , own-autocovariances dictate  $\mathbf{O}_k$ , this is the separation needed.

With this decomposition, the authors explained that the profitability of such a contrarian strategy could be perfectly consistent with a **positively autocorrelated market index** and **negatively autocorrelated individual security returns**.

They also presented 5 illustrative scenarios:

#### A. I.I.D. returns

- $\forall k, \Gamma_k = 0, L_k = C_k = O_k = 0$ , thus,  $E[\pi_t(k)] = -\sigma^2(\mu) \leq 0$ .
- **Intuition:** When returns do follow random walks, any cross-sectional variation in expected returns would generate negative expected profits when trading with a contrarian strategy. Since the strategy is reduced to shorting the higher and buying the lower mean return securities. BUT,  $\sigma^2(\mu)$  is generally small.

#### B. Stock market overreaction

- **Assumption:** negative self-autocorrelations and zero cross-autocorrelations, i.e., the diagonal elements of  $\Gamma_k$  are negative, the non-diagonal elements are 0. Thus, ignoring the small  $\sigma^2(\mu)$ , the expected profit is

$$E[\pi_t(k)] \simeq L_k = O_k = -\left(\frac{N-1}{N}\right) \text{tr}(\Gamma_k) = -\left(\frac{N-1}{N}\right) \sum_{i=1}^N \gamma_{ii}(k) > 0$$

- **Intuition:** In a overreacting market, "what goes up must come down" and vice versa. Thus, a contrarian investment strategy is profitable on average. A special case is the model of "fads": the sum of a random walk and an AR(1)<sup>4</sup>.

<sup>4</sup>Notice that only AR(1) satisfies this conclusion.

### C. White noise and lead-lag relations

- **Assumption:**  $\forall i \in [1, \dots, N]$ , the return is given by:  $R_{i,t} = \mu_i + \beta_i \Lambda_{t-i} + \epsilon_{i,t}$ , where  $\beta_i > 0$ ,  $\Lambda$  is a serially independent common factor with zero mean and variance  $\sigma_\Lambda^2$ ,  $\epsilon_{i,t}$  is assumed to be both cross-sectionally and serially independent. When  $k < N$ , the autocovariance matrix  $\Gamma_k$  has zeros in all entries except along the  $k$ th superdiagonal. On the  $k$ th superdiagonal, the elements are  $\gamma_{i,i+k} = \sigma_\Lambda^2 \beta_i \beta_{i+k}$ , the profit is:

$$E[\pi_t(k)] \simeq L_k = C_k = \frac{\sigma_\Lambda^2}{N^2} \sum_{i=1}^{N-k} \beta_i \beta_{i+k} > 0$$

- **Intuition:** This is an artifact of the dependence of the  $i$ th security's return on a lagged common factor, where the lag is determined by  $i$ . Notice that the returns are serially independent, but stock  $i$ 's returns can be predicted with past returns of stock  $j$ , where  $j < i$ . With this cross-correlation, a contrarian strategy can still profit, as long as the cross autocovariances are sufficiently large.

### D. Nonsynchronous trading and lead-lag effects<sup>5</sup>

- **Assumption:**

"Virtual" return: the returns for security  $i \in [1, \dots, N]$  are generated by:  $R_{i,t} = \mu_i + \beta_i \Lambda_t + \epsilon_{i,t}$ .  $\Lambda_t$  is some zero-mean, i.i.d. common factor,  $\epsilon_{i,t}$  is zero-mean, serially and cross-sectionally independent. But this time the returns are *unobservable*, i.e., "virtual".

Non-trade: in each period  $t$ , security  $i$  has an i.i.d. probability of  $p_i$  to be NOT traded. If not traded, a security's *observed* return  $R_{i,t}^o$  is 0 while its true return is still given by  $R_{i,t} = \mu_i + \beta_i \Lambda_t + \epsilon_{i,t}$ .

Observed return: The observed return of security  $i$  in period  $t$  is the sum of its virtual returns of all the past **consecutive non-trading** periods. Formally,  $R_{i,t}^o = \sum_{k=0}^{\infty} X_{i,t}(k) R_{i,t-k}$ . The weights  $X_{i,t}(k) \equiv (1 - \delta_{i,t}) \delta_{i,t-1} \cdots \delta_{i,t-k}$ , where  $\delta_i$  is the (i.i.d.) non-trading indicator.  $X_{i,t}(k)$  is also an indicator:

$$X_{i,t}(k) = \begin{cases} 1 & i \text{ is traded at } t, \text{ but not in any of the } k \text{ previous periods} \\ 0 & \text{otherwise} \end{cases}$$

Nontrading duration: the nontrading duration, or the number of past consecutive periods that security  $i$  is not traded, is  $\tilde{k}_{i,t} \equiv \sum_{k=1}^{\infty} \left( \prod_{j=1}^k \delta_{i,t-j} \right)$ , its expectation is  $E[\tilde{k}_{i,t}] = \frac{p_i}{1-p_i}$ .

Portfolio return: for an equal-weighted portfolio of securities with common nontrading probability  $p_\kappa$ , the observed return to portfolio can be approximated as  $R_{port,t}^o \rightarrow \mu_{port} + (1 - p_{port}) \beta_{port} \sum_{k=0}^{\infty} p_{port}^k \Lambda_{t-k}$ , where  $\beta_{port}$  is the average  $\beta$  of the securities. Then the observed return of the portfolio over  $q$  periods is  $R_{port,T}^o(q) \equiv \sum_{t=(T-1)q+1}^{Tq} R_{port,t}^o$ .

<sup>5</sup>See [Lo and MacKinlay \(1990b\)](#) for a detailed discussion.

- **Intuition:**

The "nontrading" problem aims to fix one problem: the prices of distinct securities are mistakenly assumed to be sampled simultaneously. Prices actually happen in different periods, but are treated as if they were observed at the same time. The "power" of a stock on others is related to how frequently it is traded: For a more frequently traded portfolio  $a$ , and a less frequently traded portfolio  $b$ ,  $R_{a,t-1}$  predicts  $R_{b,t}$  better than  $R_{b,t-1}$  predicts  $R_{a,t}$ . HOWEVER! This cannot fully explain the magnitude of weekly cross-autocorrelations.

E. Positively dependent common factor and bid-ask spread

- **Assumption:**

$R_{i,t}$  as the sum of: (a) a positively autocorrelated common factor, (b) idiosyncratic white noise, (c) a bid-ask spread process. Formally,

$$R_{i,t} = \mu_i + \beta_i \Lambda_t + \eta_{i,t} + \epsilon_{i,t}$$

where  $E[\Lambda_t] = 0$ ,  $E[\Lambda_{t-k}\Lambda_t] \equiv \gamma_\lambda(k) > 0$  (positively autocorrelated common factor),  $E[\epsilon_{i,t}] = E[\eta_{i,t}] = 0$  (idiosyncratic noise),  $Var[\epsilon_{i,t}] = \sigma_i^2$ .

The bid-ask spread has a AR(1) as  $E[\eta_{i,t-1}\eta_{i,t}] = -s_i^2/4$ , where  $s_i$  is the percentage bid-ask spread. All higher-order ARs and all cross-correlations are zero.

The autocovariance matrices are given by:

$$\Gamma_k = \begin{cases} \gamma_\lambda(1)\vec{\beta}\vec{\beta}' - \frac{1}{4}diag[s_1^2, \dots, s_N^2], & k = 1 \\ \gamma_\lambda(k)\vec{\beta}\vec{\beta}', & k > 1 \end{cases}$$

where  $\vec{\beta} \equiv [\beta_1, \dots, \beta_N]'$ . The autocovariance matrices are all symmetric, which is the signature departing this model from the lead-lag process. The profitability is:

$$L_k = \begin{cases} -\frac{\gamma_\lambda(1)}{N} \sum_{i=1}^N (\beta_i - \frac{\sum_{i=1}^N \beta_i}{N})^2 + \frac{N-1}{N^2} \sum_{i=1}^N \frac{s_i^2}{4}, & k = 1 \\ -\frac{\gamma_\lambda(k)}{N} \sum_{i=1}^N (\beta_i - \frac{\sum_{i=1}^N \beta_i}{N})^2, & k > 1 \end{cases}$$

- **Intuition:**

This model, developed by [Roll \(1984\)](#), will yield a positively autocorrelated market index since the white-noise and bid-ask components will be averaged out, leaving the common factor  $\Lambda_t$ . However, for individual securities, the bid-ask spread could dominate the common factor, yielding negative autocorrelations.

The positive profit of a contrarian strategy on returns generated by this model is solely due to the bid-ask spread (or the negative AR). In other words, if the portfolio is weighted using lags 2 or higher, the strategy will only generate negative profit.

With the profit decomposed in Equation (1.7), [Lo and MacKinlay](#) used 551 stocks that have no missing weekly returns from Jul 6, 1962 to Dec 31, 1987. They made the estimates for individual stocks and for 3 size-sorted quintiles. Their results can be summarized as:

1. From lag 1 to even lag 4, the estimated expected profits  $\hat{E}[\pi_t(k)]$  are positive. This contradicts the prediction of the common factor plus bid-ask spread model (positive only at lag 1, negative for higher lags).
2. Although  $\hat{E}[\pi_t(k)]$  is significant even at lag 4, the two components of the profits  $\hat{C}_k$  and  $\hat{O}_k$  are NOT significant. This suggests that  $\hat{C}_k$  and  $\hat{O}_k$  are negatively correlated and cancelling each other's noise. Since both  $\hat{C}_k$  and  $\hat{O}_k$  are functions of second moments and co-moments, the correlations are perhaps a result of co-skewness or kurtosis.
3. The cross-autocorrelation matrices for the size-sorted quintiles show that current returns of smaller stocks are correlated with past returns of large stocks. This asymmetry of autocorrelation matrices implies the autocovariance matrix  $\Gamma_k$  is also asymmetric, contradicting the bid-ask spread model again.
4. the cross-autocorrelation matrices also contradicts, at least partially, the non-trading model since they require an unrealistically high probability of non-trading.

### 1.2.2.2 Froot and Perold (1995)

Froot and Perold (1995) examined the positive autocorrelations of high-frequency index returns with various market indexes and trade frequencies. The authors linked the positive correlations to an information-based explanation: slow dissemination and inefficient processing of market-wide information. They build a simple model and show the index's theoretical autocorrelations fall with an increase in the dissemination speed of market-wide information. Furthermore, they attribute this decrease of positive index autocorrelations entirely to cross-stock autocorrelations. Self-autocorrelations of stocks are relatively persistent.

#### A brief summary of the model:

1.  $N$  stocks, managed by risk-neutral specialists individually. The true value is  $V_t^i \equiv V_t + \xi_t^i$ , where  $V_t$  and  $\xi_t^i$  follow independent random walks where the innovations are i.i.d. normal:  $\Delta V_t = u_t \sim \mathcal{N}(0, \sigma_u^2)$ ,  $\Delta \xi_t^i = e_t^i \sim \mathcal{N}(0, \sigma_e^2)$
2. Trading cost and technical delays of information dissemination:
  - delayed observation of value:  $V_t^i$  is observed at  $t + 1$
  - instant observation of order flow:  $F_t^i = u_t + e_t^i + v_t^i$ , where  $u_t + e_t^i$  is an informed-traders' component, the i.i.d.  $v_t^i \sim \mathcal{N}(0, \sigma_v^2)$  is the "liquidity" traders' component.
  - optimal price:  $P_t^i = \lambda F_t^i + V_{t-1}^i$  where  $\lambda = \frac{\sigma_u^2 + \sigma_e^2}{\sigma_u^2 + \sigma_e^2 + \sigma_v^2}$  is the OLS estimator of  $u_t + e_t^i$  on  $F_t^i$ .
  - price change from  $t - 1$  to  $t$ :  $\Delta P_t^i = \lambda(u_t + e_t^i + \Delta v_t^i) + (1 - \lambda)(u_{t-1} + e_{t-1}^i)$
3. Self-autocorrelations and cross-autocorrelations:
  - Self-autocovariances:  $Cov(\Delta P_t^i, \Delta P_{t-1}^i) = 0$
  - Cross-autocovariances:  $Cov(\Delta P_t, \Delta P_{t-1}) = \frac{N-1}{N}(1 - \lambda)\lambda\sigma_u^2 > 0$
4. Fast market-wide information dissemination:
  - $V_t$  is observable at  $t$  instead of  $t + 1$ :  $V_t$  trade is costless, so traders would earn

positive net profits unless innovation in  $V_t$  are fully incorporated in current prices. A futures market might serve as a *billboard*, making the current value publicly observable.

- optimal price:  $P_t^i = \lambda'(F_t^i - u_t) + V_{t-1}^i + u_t$ , where  $\lambda' = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_v^2}$
- price change of stock  $i$  at  $t$ :  $\Delta P_t^i = \lambda'(e_t^i + \Delta v_t^i) + (1 - \lambda')e_{t-1}^i + u_t$
- cross-stock autocovariances:  $Cov(\Delta P_t, \Delta P_{t-1}) = \frac{N-1}{N}Cov(u_t, u_{t-1}) = 0$ , it disappears!

### Empirical evidence

- **Main results:** **Froot and Perold (1995)** first examine a very short-horizon return series: 15-minute returns of S&P 500 cash index from 1983-02 to 1989-12. They found 2 main results:
  1. There are significant positive autocorrelations for 15-minute index returns.
  2. The positive autocorrelations declined/disappeared in the 1980s.

They rule out two alternative interpretations: **bid-ask bounce** and **nontrading effect** as the driving force of this decrease in autocorrelation (in other words, separate overreaction from the **information dissemination**).

- **Bid-ask bounce:** As described above, bid-ask bounce could lower the level of AR coefficients. In 1980s, bid-ask spread increased, leading to an increase in bounce; Investors tended to trade more frequently in portfolios, leading to greater synchronisness of buys and sells, leading to higher bid-ask bounce. These two facts could be linked to the decline/disappearance of the positive ARs.

To rule this explanation out, **Froot and Perold** examined higher-order ARs. Bid-ask bounce would lead to equal-size reduction in all ARs, instead of just AR(1). The empirical evidence proved otherwise.

They also separated bid-ask component from the last-trade index. They define  $L_t$  as the index of last-trade prices and  $M_t$  the index of extant midquotes for each last-trade price, the returns of them as  $l_t = \ln(L_t/L_{t-1})$  and  $m_t = \ln(M_t/M_{t-1})$ .  $L_t - M_t$  measures the distance between the last trade index and the log bid-ask error  $l_t - m_t \simeq \Delta(L_t - M_t)$  as  $\epsilon_t$ .

$\frac{Cov(l_t, l_{t-1}) - Cov(m_t, m_{t-1})}{Var(l_t)} = \frac{Cov(\epsilon_t, \epsilon_{t-1})}{Var(l_t)} + \frac{Cov(m_t, \epsilon_{t-1})}{Var(l_t)} + \frac{Cov(m_{t-1}, \epsilon_t)}{Var(l_t)}$  can be used to measure bid-ask effects. They report that the bid-ask component only accounts for 1/3 of the change in the AR of last trade index.

← If  $\epsilon_t > 0$ , there is a greater fraction of buys at time  $t$  then at time  $t - 1$

When considering the three sub-components individually, they report:

- $\frac{Cov(\epsilon_t, \epsilon_{t-1})}{Var(l_t)}$ , a pure measure of Roll-type bounce, declines marginally from 1983 to 1988, while the self-autocovariance  $\frac{\sum_i (\omega^i)^2 Cov(e_t^i, e_{t-1}^i)}{Var(l_t)}$  of the index (squared-weighted sum of self-ARs of all stocks) increases. This means that bid-ask bounce in individual stocks becomes less important and the average bid-ask spread narrows, while cross-correlation in bid-ask bounce becomes more negative and portfolio trading increases. But in general, the number is too small to explain the change in the correlation of the last-trade index.
- $\frac{Cov(m_t, \epsilon_{t-1})}{Var(l_t)}$  measures the correlation between past increases in buys/sells and current increases/decreases in the midquote index, this measure is large and



positive empirically but converging to 0 over time. 2 possible explanations are considered:

- (1) **eating-through-the-order-book (ETOB)**: Suppose there is a limit order at the ask price and more limit orders with higher prices, then buy orders come in and deals are made firstly at the ask price and then move to higher price orders gradually. This way, when buy order flow is positively autocorrelated (for example, big trades are broken up and executed sequentially), an increase in buy order tends to forecast an increase in the ask price, therefore, an increase in the future midquote index.
- (2) **sluggish-response-to-order-flow (SRFI)**: The order-flow information of a given stock is incorporated into quotes for other stocks slowly over time. For example, a buy order at the ask of GM stock (increases  $\epsilon_{t-1}$ ) might provide incremental information about the value of Ford, and therefore might be associated with an increase in Ford's quotes.

The main difference between **ETOB** and **SRFI** is that **ETOB** is an own-stock effect, while **SRFI** is a cross-stock effect. And **Froot and Perold** show that **SRFI** appears to be correct, which means that bid-ask bounce cannot explain the empirical observations.

-  $\frac{Cov(m_{t-1}, \epsilon_t)}{Var(l_t)}$  measures the covariance between past increases/decreases in the midquote index and current buys/sells. This statistic is negative and decreasing over time. 2 possible explanations were considered as well:

- (1) **see-'em-coming (SEC)**: specialists expect the upcoming order flow and raise/reduce prices as buy/sell orders arrive (bid-ask prices rise/fall as buy/sell orders peak locally, therefore  $m_{t-1}$  and  $\epsilon_t$  are negatively correlated).
- (2) **slow-response-to-price-information (SRPI)**: some stocks' quoted prices respond slowly to information (sticky), when the quickly-responding stocks trade at higher/lower quote prices, the "sticky" stocks' quoted prices remain the same. Smart investors will buy/sell these sticky stocks to profit.

Once again, SEC is an own-stock effect while SRPI is a cross-stock effect. Empirically, the authors proved that SRPI appears correct.

In summary, **Froot and Perold** proved that bid-ask effects only account for 1/3 of the decline of AR. of this 1/3, nearly 1/2 might be attributed to slow-response-to-information hypotheses (SRFI and SRPI).

- **Nontrading**: They examined two factors of the non-trading effect: trade frequency and degree of synchronisousness. They calculated a measure of *staleness*: the difference between last-trade midquote  $m_t$  and the *current* midquote  $cm_t$ :  $m_t = cm_t + s_t = cm_t + (m_t - cm_t)$ , where  $cm_t$  is the return on the current midquote index, free of staleness and bid-ask bounce. Empirically, the decline of AR of  $cm_t$  is greater than that of  $m_t$ , which means that nontrading staleness does NOT explain the reduction of index's AR.

Define equally-weighted current midquote index as  $eq_t$  (versus value-weighted index  $cm_t$ ) and  $z_t = cm_t - eq_t$ , then decompose the autocovariance of  $cm_t$ , get:  $\frac{Cov(cm_t, cm_{t-1})}{Var(l_t)} = \frac{Cov(eq_t, eq_{t-1})}{Var(l_t)} + \frac{Cov(eq_t, z_{t-1})}{Var(l_t)} + \frac{Cov(z_t, eq_{t-1})}{Var(l_t)} + \frac{Cov(z_t, z_{t-1})}{Var(l_t)}$ , loosely speaking,

the four items on the right-hand side can be seen as a predictability matrix:

		Predictor	
		small stocks	large stocks
Predictor	small stocks	$\frac{Cov(eq_t, eq_{t-1})}{Var(l_t)}$	$\frac{Cov(z_t, eq_{t-1})}{Var(l_t)}$
	large stocks	$\frac{Cov(eq_t, z_{t-1})}{Var(l_t)}$	$\frac{Cov(z_t, z_{t-1})}{Var(l_t)}$

If stocks respond symmetrically to market-wide information, the decline in the AR of  $cm_t$  would be distributed equally across the four components. However, the authors showed that the decline in AR of  $cm_t$  was driven by  $\frac{Cov(eq_t, eq_{t-1})}{Var(l_t)}$  and  $\frac{Cov(z_t, eq_{t-1})}{Var(l_t)}$ , in other words, it has become more difficult to predict small stocks' returns, no matter with other small stocks' returns or large stocks' returns.

To incorporate information dissemination, consider  $\frac{Cov(m_t, m_{t-1})}{Var(l_t)} - \frac{Cov(cm_t, cm_{t-1})}{Var(l_t)} = \frac{Cov(s_t, s_{t-1})}{Var(l_t)} + \frac{Cov(s_t, cm_{t-1})}{Var(l_t)} + \frac{Cov(cm_t, s_{t-1})}{Var(l_t)}$ , where  $s_t$  is the new information in quotes beyond that reflected in the trade. They found that the changes of both  $\frac{Cov(s_t, s_{t-1})}{Var(l_t)}$  and  $\frac{Cov(s_t, cm_{t-1})}{Var(l_t)}$  are negligible, while  $\frac{Cov(cm_t, s_{t-1})}{Var(l_t)}$  was negative and rising over time, which means the responsiveness of current quotes ( $cm_t$ ) to information between  $t - 1$  and the last trade as of time  $t - 1$  ( $s_{t-1}$ ) declines, the evidence of more rapid dissemination of market-wide information.

With market excess return data from [Kenneth French's website](#), I replicate the AR(1)s of daily value-weighted market index returns from 1926 to 2021<sup>6</sup>. The auto-correlations are estimated in a rolling window of 300 trading days. The time series of AR(1) coefficients is shown in Figure 1.3. The daily ARs of the market returns were positive till late 1990s, but the decline started from 1970s. Negative daily ARs are observed in recent years (since mid-2000s).

## 1.3 Excess volatility puzzle

[insert text]

## 1.4 Decomposing prices

[insert text]

### 1.4.1 Campbell-Schiller decomposition

[insert text]

<sup>6</sup>As documented on Kenneth French's website, the market return consists of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ with a CRSP share code of 10/11



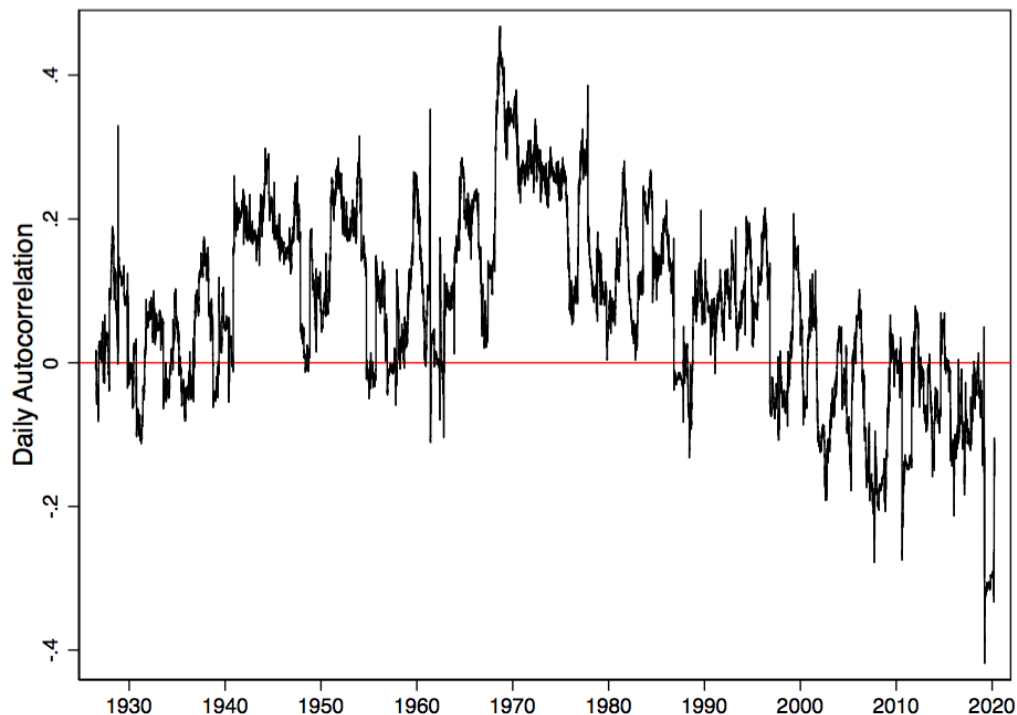


Figure 1.3: Daily Autocorrelation of the Excess Market Return, 1926-2021

### 1.4.2 Lettau-Ludvigson decomposition

[insert text]

## 1.5 Prediction zoo

[insert text]

## 1.6 Issues and extensions

[insert text]

### 1.6.1 Persistency of most regressors

[insert text]

### 1.6.2 Aggregate predictors without ex-ante choice

[insert text]

### **1.6.3 Instability in the prediction relation**

[insert text]

### **1.6.4 Measurement**

[insert text]

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# CHAPTER 2

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## CROSS-SECTION PREDICTABILITY

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## TERM STRUCTURE OF RETURNS

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In this chapter, I summarize the stylized facts and models of interest rates, and, combining with the time-series and cross-sectional properties of equities, discuss how the term structure of equity can be incorporated into the asset pricing dynamic. Instead of assuming the risk-free rate to be one period, as classic asset pricing models implying in the Euler equations and SDFs, one would expect that an ideal asset pricing model could not only explain the dynamic of equity, but reconcile the property of the term structure of interest rates as well.

The first part of this chapter summarizes studies of risk free bonds and the term structure of this asset class.

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In this chapter, I summarize the learning in empirical finance. This is one of the most cutting-edge research area now.

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