Probability and Statistics for Economics Cheat Sheet

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Random experiments

The outcome in a random experiment is **unpredictable**:

- outcome is too complicates or poorly understood
- outcome is designed to be unpredictable
- coincidences, or independent chains of events

Two examples

Random card shuffle experiment: take top card from a deck and insert randomly, to complete the shuffle of n cards, we need

$$T = n + \frac{n}{2} + \cdots + \frac{n}{n-1} + 1 = n \log n$$

shuffles.

Random number generator:

$$x_{n+1} = \frac{ax_n + b}{c} - \left[\frac{ax_n + b}{c}\right]$$

the remainder after dividing by c, hence $x_{n+1} \in [0, c-1]$, let $u_{n+1} = \frac{x_{n+1}}{c}$, x_0, a, b, c all be integers. For very large a and good choice of b, c, the sequence $u_1, u_2, \cdots$ is like a sequence of numbers randomly picked from $[0, 1]$

Probabilities

Probability is a number in $[0, 1]$ that measures the likelihood of an outcome or a set of outcomes.

Ways of assigning probabilities:

- **symmetry**: assume all outcomes are equally likely
- **experimental method**: relative frequency in repeated random experiment
- **subjective method**: assign probabilities using knowledge of random experiment
- **market method**

Elements of probability space

- **outcome space** Ω and outcomes $\omega \in \Omega$
- **event** E, $E \subset \Omega$
- **probability function/measure** $P : \mathcal{A} \to [0, 1]$: a function from **a collection** $\mathcal{A}$ of subsets of Ω to the interval $[\mathbf{0}, \mathbf{1}]$.

Classes of events

Events $E_1, E_2, \cdots$ are just sets. They also follow the algebras of sets.

Some set algebras

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup A^C = U$, $A \setminus B = A \cap B^C$
$(\bigcup_{i=1}^{\infty} A_i)^C = \bigcap_{i=1}^{\infty} A_i^C$
$(\bigcap_{i=1}^{\infty} A_i)^C = \bigcup_{i=1}^{\infty} A_i^C$
$A \cup B = U, A \cap B = \varnothing \Leftrightarrow B = A^C$
$(A^C)^C = A$
$A \subseteq B \Leftrightarrow A \cap B = A \Leftrightarrow A \cup B = B \Leftrightarrow A \setminus B = \varnothing \Leftrightarrow B^C \subseteq A^C$

Two special relations:

- **disjoint**: $E_1 \cap E_2 = \varnothing$
- **partition**: $\bigcup_{i=1}^{\infty} E_i = \Omega$, $\{E_i\}$ are pairwise disjoint

σ−field and Borel σ−field

Definition of σ−field

$\mathcal{A}$ (a collection of subsets of Ω) is a σ−field if:

1. $\varnothing \in \mathcal{A}$
2. $E \in \mathcal{A} \Rightarrow E^C \in \mathcal{A}$
3. $E_1, E_2, \cdots \in \mathcal{A} \Rightarrow \bigcup_{i=1}^{\infty} E_i \in \mathcal{A}$

It is easy to see that $\bigcap E_i^C \in \mathcal{A}$, $\bigcup E_i^C \in \mathcal{A}$, $\bigcap E_i \in \mathcal{A}$ as well

Two important σ−field:

- Trivial σ−field: $\mathcal{A} = \{\varnothing, \Omega\}$
- Largest σ−field: **powerset** of Ω, $\mathcal{P}(\Omega)$