

Probability and Statistics for Economics Cheatsheet

Random experiments

The outcome in a random experiment is **unpredictable**:

- outcome is too complicated or poorly understood
- outcome is designed to be unpredictable
- coincidences, or independent chains of events

Random card shuffle experiment: take top card from a deck and insert randomly, to complete the shuffle of n cards, we need $T = n + \frac{n}{2} + \dots + \frac{n}{n-1} + 1 = n \log n$ shuffles.

Random number generator: $x_{n+1} = \frac{ax_n+b}{c} - \left\lfloor \frac{ax_n+b}{c} \right\rfloor$, the remainder after dividing by c , hence $x_{n+1} \in [0, c-1]$, let $u_{n+1} = \frac{x_{n+1}}{c}$, x_0, a, b, c all be integers. For very large a and good choice of b, c , the sequence u_1, u_2, \dots is like a sequence of numbers randomly picked from $[0, 1]$

Probabilities

Ways of assigning probabilities:

- symmetry: assume all outcomes are equally likely
- experimental method: relative frequency in repeated random experiment
- subjective method: assign probabilities using knowledge of random experiment
- market method

Elements of **probability space**:

- outcome space Ω and outcomes $\omega \in \Omega$
- event $E, E \subset \Omega$
- probability function/measure $P: \mathcal{A} \rightarrow [0, 1]$: a function from a **collection** \mathcal{A} of subsets of Ω to the interval $[0, 1]$.

Classes of events

Events E_1, E_2, \dots are just sets. They also follow the algebras of sets, such as:

$$\begin{aligned} A \cup (B \cap C) &= (A \cup B) \cap (A \cup C) \\ A \cap (B \cup C) &= (A \cap B) \cup (A \cap C) \\ A \cup A^C &= U \\ (\bigcup_{i=1}^{\infty} A_i)^C &= \bigcap_{i=1}^{\infty} A_i^C & A \setminus B &= A \cap B^C \\ A \cup B &= U, A \cap B = \emptyset \Leftrightarrow B = A^C & (\bigcap_{i=1}^{\infty} A_i)^C &= \bigcup_{i=1}^{\infty} A_i^C \\ A \subseteq B &\Leftrightarrow A \cap B = A \Leftrightarrow A \cup B = B \Leftrightarrow A \setminus B = \emptyset \Leftrightarrow B^C \subseteq A^C & (A^C)^C &= A \end{aligned}$$

Two special relations:

- **disjoint:** $E_1 \cap E_2 = \emptyset$
- **partition:** $\bigcup_{i=1}^{\infty} E_i = \Omega$

σ -field and Borel σ -field

Definition of σ -field: \mathcal{A} (a collection of subsets of Ω) is a σ -field if

- $\emptyset \in \mathcal{A}$
- $E \in \mathcal{A} \Rightarrow E^C \in \mathcal{A}$
- $E_1, E_2, \dots \in \mathcal{A} \Rightarrow \bigcup_{i=1}^{\infty} E_i \in \mathcal{A}$ ($\bigcap E_i^C \in \mathcal{A}$, $\bigcup E_i^C \in \mathcal{A}$ as well).

Some examples:

- Trivial σ -field: $\mathcal{A} = \{\emptyset, \Omega\}$

Variation of Parameters

$$F(x) = y'' + y'$$

$$y_h = b_1 y_1(x) + b_2 y_2(x), y_1 y_2 \text{ are L.I.}$$

$$y_p = u_1(x) y_1(x) + u_2(x) y_2(x)$$

$$u_1 = \int^t -\frac{y_2 F(t) dt}{w[y_1, y_2](t)}$$

$$u_2 = \int^t \frac{y_1 F(t) dt}{w[y_1, y_2](t)}$$

$$y = y_h + y_p$$

Systems

$$\vec{x}' = A\vec{x}$$

A is diagonalizable

$$\vec{x}(t) = a_1 e^{\lambda_1 t} \vec{v}_1 + \dots + a_n e^{\lambda_n t} \vec{v}_n$$

A is not diagonalizable

$$\vec{x}(t) = a_1 e^{\lambda_1 t} \vec{v}_1 + a_2 e^{\lambda t} (\vec{w} + t\vec{v})$$

where $(A - \lambda I)\vec{w} = \vec{v}$
 \vec{v} is an Eigenvector w/ value λ

i.e. \vec{w} is a generalized Eigenvector

$$\vec{x}' = A\vec{x} + \vec{B}$$

Solve y_h

$$\vec{x}_1 = e^{\lambda_1 t} \vec{v}_1, \vec{x}_2 = e^{\lambda_2 t} \vec{v}_2$$

$$\vec{X} = [\vec{x}_1, \vec{x}_2]$$

$$\vec{X}\vec{u}' = \vec{B}$$

$$y_p = \vec{X}\vec{u}$$

$$y = y_h + y_p$$

Matrix Exponentiation

$$A^n = S D^n S^{-1}$$

D is the diagonalization of A

Laplace Transforms

$$L[f](s) = \int_0^{\infty} e^{-sx} f(x) dx$$

$$f(t) = t^n, n \geq 0$$

$$F(s) = \frac{n!}{s^{n+1}}, s > 0$$

$$f(t) = e^{at}, a \text{ constant}$$

$$F(s) = \frac{1}{s-a}, s > a$$

$$f(t) = \sin bt, b \text{ constant}$$

$$F(s) = \frac{b}{s^2 + b^2}, s > 0$$

$$f(t) = \cos bt, b \text{ constant}$$

$$F(s) = \frac{s}{s^2 + b^2}, s > 0$$

$$f(t) = t^{-1/2}$$

$$F(s) = \frac{\pi}{s^{1/2}}, s > 0$$

$$f(t) = \delta(t-a)$$

$$F(s) = e^{-as}$$

$$f'$$

$$L[f'] = sL[f] - f(0)$$

$$f''$$

$$L[f''] = s^2 L[f] - sf(0) -$$

Gaussian Integral

$$\int_{-\infty}^{+\infty} e^{-1/2(\vec{x}^T A \vec{x})} = \frac{\sqrt{2\pi}^n}{\sqrt{\det A}}$$

Complex Numbers

Systems of equations If $\vec{w}_1 = u(\vec{t}) + iv(\vec{t})$ is a solution, $\vec{x}_1 = u(\vec{t}), \vec{x}_2 = v(\vec{t})$ are solutions

i.e. $\vec{x}_h = c_1 \vec{x}_1 + c_2 \vec{x}_2$

Euler's Identity

$$e^{ix} = \cos x + i \sin x$$

Vector Spaces

$$v_1, v_2 \in V$$

1. $v_1 + v_2 \in V$
2. $k \in \mathbb{F}, kv_1 \in V$
3. $v_1 + v_2 = v_2 + v_1$
4. $(v_1 + v_2) + v_3 = v_1 + (v_2 + v_3)$
5. $\forall v \in V, 0 \in V \mid 0 + v_1 = v_1 + 0 = v_1$
6. $\forall v \in V, \exists -v \in V \mid v + (-v) = (-v) + v = 0$
7. $\forall v \in V, 1 \in \mathbb{F} \mid 1 * v = v$
8. $\forall v \in V, k, l \in \mathbb{F}, (kl)v = k(lv)$
9. $\forall k \in \mathbb{F}, k(v_1 + v_2) = kv_1 + kv_2$
10. $\forall v \in V, k, l \in \mathbb{F}, (k+l)v = kv + lv$