Microeconomic Model Cheat Sheet

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Modelling essential: what we need to decide

- **Primitive assumptions**:
  - Who the **agents** are, what are their **preferences** and objective functions
  - What **technology** agents can access
  - What **endowment** agents have
- **Decision problems**: resource allocation problem (among agents, over time, etc.).
- **Information sets**: what do agents know, how will their knowledge change, what is their **expectation**.
- **Allocation mechanism**: how agents interact and achieve equilibrium. 2 main mechanisms are:
  - **price system** in competitive equilibrium
  - benevolent **central planner** maximizes a social welfare function.

Infinitely Lived Agent Model

Features

- **discrete** time, indexed by t
- economy lives **infinitely**, $t = 0, 1, 2, \cdots$
- single commodity exogenously produced, indexed by t, pure **exchange/endowment** economy.
- no firms/government, only **two types of households**.
- each type of households is continuum of **identical** households of that type, they are **price takers**, can be represented by a **representative** household

Agents' preferences

Utility of type i household is

$$U(c^i) = \sum_{t=0}^{\infty} \beta_i^t u(c_t^i)$$

where $\left(c^i\right) = \left\{c_t^i\right\}_{t=0}^{\infty}$, $\beta_i \in (0,1)$,

The utility function $u(c_t^i)$ is assumed to be:

- **continuously differentiable** of the second order
- **monotonically increasing, strictly concave**: $u'(c_t^i) > 0, u''(c_t^i) < 0$
- **satisfies Inada conditions** (never 0 or infinity consumption): $\lim_{c_t^i \to \infty} u'(c_t^i) = 0, \lim_{c_t^i \to 0} u'(c_t^i) = \infty$
- **time additivity**: $u(c_t^i)$ is independent of c_{t+j}^i, c_{t-j}^i.
- **impatient discounting** $\beta_i < 1$: households value today's consumption more than future's.
- Constant relative risk aversion (**CRRA**): $u(c_t^i) = \frac{c^{1-\sigma}-1}{1-\sigma}$ $\left(\lim_{\sigma \to 1} \frac{c^{1-\sigma}-1}{1-\sigma} = \lim_{\sigma \to 1} \frac{e^{(1-\sigma)\ln(c)}-1}{1-\sigma} = \ln(c)\right)$. **The RRA coefficient** $\sigma(c) = \frac{-u''(c_t^i)c}{u'(c_t^i)} = \frac{-\left(-\sigma c^{-(1+\sigma)} c\right)}{c^{-\sigma}} = \sigma$. Higher RRA means higher risk aversion.
- Constant intertemporal elasticity of substitution (**IES**):

$$IES = -\frac{\mathrm{d}\ln(c_{t+1}/c_t)}{\mathrm{d}\ln(u'(c_{t+1}/u'(c_t))} = \frac{1}{\sigma}$$

  hence higher RRA (more risk-averse), lower IES (consumption variation over time).

Agents' endowment

A deterministic endowment stream of the consumption good for type i household is

$$w^i = \left(w_0^i, w_1^i, \cdots\right) = \left\{w_t^i\right\}_{t=0}^{\infty}$$