

## Topic 11: Lasso

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**Key points:****Disclaimer:**

Lasso (Least absolute Shrinkage and Selection Operator), proposed by Tibshirani (1996), aims to minimize the **SSR (sum of residual squares)** subject to the **L1-norm (sum of the absolute value)** of the coefficients being less than a constant.

## 11.1 Set up

For data  $(\mathbf{x}_i, y_i)_{i=1}^n$ , where

- $y_i$  is the outcome for individual  $i$
- $\mathbf{x}_i = (x_{i1}, \dots, x_{ip})'$  is the  $p \times 1$  vector of predictors

Then the Lasso estimator  $(\hat{\alpha}, \hat{\beta})$  is defined as

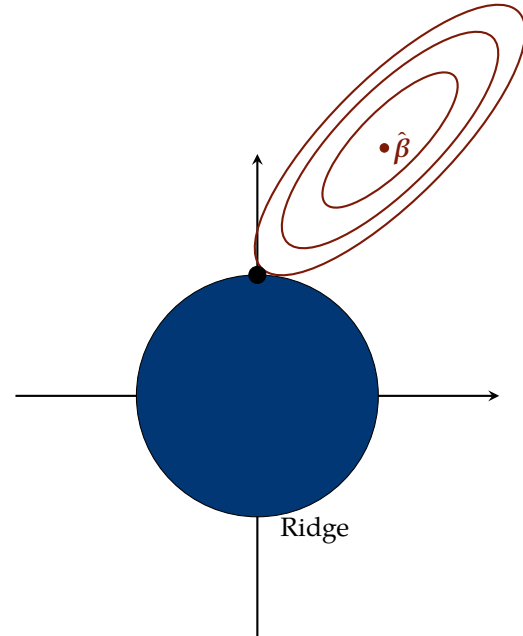
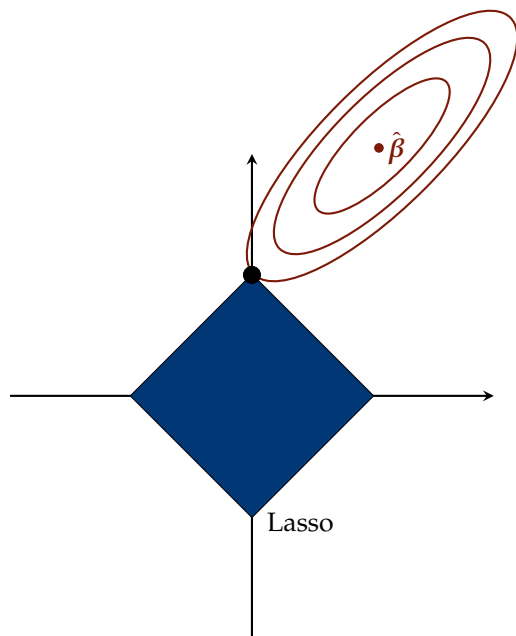
$$(\hat{\alpha}, \hat{\beta}) = \arg \min_{\alpha, \beta} \left\{ \sum_{i=1}^n \left( y_i - \alpha - \sum_{j=1}^p \beta_j x_{ij} \right)^2 \right\} \quad \text{s.t.} \quad \sum_{j=1}^p |\beta_j| \leq t$$

for the  $n \times 1$  response vector  $\mathbf{y} = (y_1, \dots, y_n)'$ , the  $n \times p$  design matrix  $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_n)'$  where  $\mathbf{x}_i = (x_{i1}, \dots, x_{ip})'$  is a  $p \times 1$  vector.

## 11.2 Penalized Least Square Estimation

Lasso is one special class of Penalized Least Square (PLS) Estimation. For the linear regression model  $\mathbf{y} = \mathbf{X}\beta + \epsilon$ , if  $\epsilon \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_n)$ , we have PLS as

$$\min_{\beta \in \mathbb{R}^p} \left\{ \frac{1}{2n} \|\mathbf{y} - \mathbf{X}\beta\|_2^2 + \sum_{j=1}^p p_\lambda(|\beta_j|) \right\}$$



## References

Robert Tibshirani. Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society: Series B (Methodological)*, 58(1):267–288, 1996.