Econometrics	**September 30, 2023**

Topic 20: Random Forest

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Key points: .

Disclaimer: *The note is built on Prof. Jinchi Lv's lectures of the course at USC, DSO 607, High-Dimensional Statistics and Big Data Problems.*

20.1 Motivation

Denote by $m(\mathbf{X})$ the measurable nonparametric regression function with p-dimensional random vector $\mathbf{X}$ taking values in $[0,1]^p$. The Random Forest algorithm aims to learn the regression function in a non-parametric way based on the observations $\mathbf{x}_i \in [0,1]^p, y_i \in \mathbb{R}, i = 1, \cdots, n$, from the model

$$y_i = m(\mathbf{x}_i) + \epsilon_i$$

where $\mathbf{X}, \mathbf{x}_i, \epsilon_i, i = 1, \cdots, n$ are independent, and $\{\mathbf{x}_i\}$ and $\{\epsilon_i\}$ are two sequences of identically distributed random variables. $\mathbf{x}_i$ is distributed identically as $\mathbf{X}$.

Why Random Forest (RF)? **RF** has gained significant popularity due to its

- **High accuracy**: **RF** consistently rank among the top performer, often surpassing more complex models
- **Robustness**: **RF** are less subject to overfitting due to the ensemble nature leveraging multiple decision trees
- **Interpretability**: **RF** provide rankings of feature importance

As illustrated in Figure 20.1, in a level-2 tree, each node (cell) defines the point where the current cell split and new cells are produced. The sets of features eligible for splitting cells at level $k-1$ are denoted as $\Theta_k := \left\{\Theta_{k,1}, \cdots, \Theta_{k,2^{k-1}}\right\}$, where $\Theta_{k,s} \subset \left\{1, \cdots, p\right\}$.

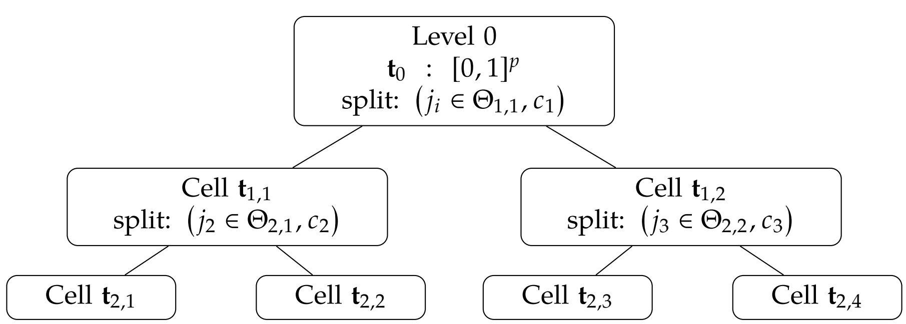

Figure 20.1: Level-2 Tree Example

Given any T (and the associated splitting criterion) and $\Theta_{1:k}$, the tree estimate denoted as $\hat{m}_{T(\Theta_{1:k})}$ for a test point $\mathbf{c} \in [0,1]^p$

Chi et al. (2022)

References

Chien-Ming Chi, Patrick Vossler, Yingying Fan, and Jinchi Lv. Asymptotic properties of high-dimensional random forests. *The Annals of Statistics*, 50(6):3415–3438, 2022.