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Topic 11: Lasso

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**Key points:** 

Disclaimer:

Lasso (Least absolute Shrinkage and Selection Operator), proposed by Tibshirani (1996), aims to minimize the SSR (sum of residual squares) subject to the L1-norm (sum of the absolute value) of the coefficients being less than a constant.

## 11.1 Set up

For data  $(\mathbf{x}_i, y_i)_{i=1}^n$ , where

- $y_i$  is the outcome for individual i
- $\mathbf{x}_i = (x_{i1}, \dots, x_{ip})'$  is the  $p \times 1$  vector of predictors

Then the Lasso estimator  $(\hat{\alpha}, \hat{\beta})$  is defined as

$$\left(\hat{\alpha}, \hat{\beta}\right) = \arg\min_{\alpha, \beta} \left\{ \sum_{i=1}^{n} \left( y_i - \alpha - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 \right\}$$
 s.t. 
$$\sum_{j=1}^{p} |\beta_j| \le t$$

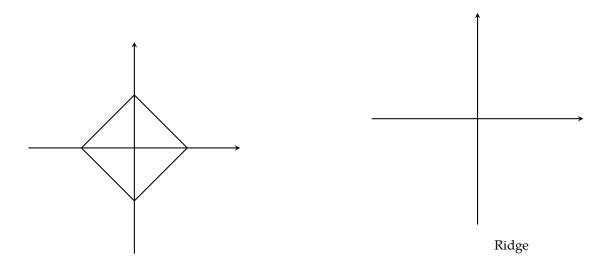
for the  $n \times 1$  response vector  $\mathbf{y} = (y_1, \dots, y_n)'$ , the  $n \times p$  design matrix  $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_n)'$  where  $\mathbf{x}_i = (x_{i1}, cdots, x_{ip})'$  is a  $p \times 1$  vector.

## 11.2 Penalized Least Square Estimation

Lasso is one special class of Penalized Least Square (PLS) Estimation. For the linear regression model  $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ , if  $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_n)$ , we have PLS as

$$\min_{\boldsymbol{\beta} \in \mathbb{R}^p} \left\{ \frac{1}{2n} \|\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\|_2^2 + \sum_{j=1}^p p_{\lambda} (|\beta_j|) \right\}$$

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## References

Robert Tibshirani. Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society: Series B (Methodological)*, 58(1):267–288, 1996.