

Topic 5: Two-Way Cluster-Robust (TWCR) Standard Errors

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Key points: The validity of Two-Way Cluster-Robust (TWCR) standard errors

Disclaimer: *This note is compiled by Sai Zhang.*

5.1 One-Way Clustering

First, consider the case of one-way clustering. The linear model with one-way clustering

$$y_{ig} = \mathbf{x}_{ig}\boldsymbol{\beta} + u_{ig}$$

where i denotes the i th of the N individuals in the sample, j denotes the g th of the G clusters, assume that

- $\mathbb{E}[u_{ig} | \mathbf{x}_{ig}] = 0$
- error independence across clusters: for $i \neq j$

$$\mathbb{E}[u_{ig}u_{jg'} | \mathbf{x}_{ig}, \mathbf{x}_{jg'}] = 0 \quad (5.1)$$

unless $g = g'$, that is, errors for individuals within the same cluster may be correlated.

Grouping observations by cluster, get

$$\mathbf{y}_g = \mathbf{X}_g\boldsymbol{\beta} + \mathbf{u}$$

where \mathbf{X}_g has dimension $N_g \times K$ and \mathbf{y}_g has dimension $N_g \times 1$, with N_g observations in cluster g . Stacking over cluster, get the matrix form of the model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

with \mathbf{y}, \mathbf{u} being $N \times 1$ vectors, \mathbf{X} being an $N \times K$ matrix. OLS estimator gives

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{y} = \left(\sum_{g=1}^G \mathbf{X}'_g \mathbf{X}_g \right)^{-1} \sum_{g=1}^G \mathbf{X}'_g \mathbf{y}_g \quad (5.2)$$

then, by CLT, we have that $\sqrt{G}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \xrightarrow{d} \mathcal{N}(0, \boldsymbol{\Sigma})$ where the variance matrix of the limit normal distribution $\boldsymbol{\Sigma}$ is

$$\left(\lim_{G \rightarrow \infty} \frac{1}{G} \sum_{g=1}^G \mathbb{E}[\mathbf{X}'_g \mathbf{X}_g] \right)^{-1} \left(\lim_{G \rightarrow \infty} \frac{1}{G} \sum_{g=1}^G \mathbb{E}[\mathbf{X}'_g \mathbf{u}'_g \mathbf{u}_g \mathbf{X}_g] \right) \times \left(\lim_{G \rightarrow \infty} \frac{1}{G} \sum_{g=1}^G \mathbb{E}[\mathbf{X}'_g \mathbf{X}_g] \right) \quad (5.3)$$

Chiang and Sasaki (2023) (Menzel, 2021) Cameron et al. (2011)

References

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