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Topic 3: *Moving the Goalposts* Approach

by Sai Zhang

Key points:

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Disclaimer: *These notes are written by Sai Zhang (email me or check my Github page). The main reference for this topic is Armstrong, Kolesár, and Kwon (2020), I thank Prof. Armstrong for his valuable advice.*

3.1 Finite Sample Bias-Variance Tradeoffs

3.1.1 Setup

Consider the fixed design regression model

$$y_i = w_i\beta\left(z_i\right) + h\left(z_i\right) + \epsilon_i \tag{3.1}$$

where

- w_i, z_i are treated as **fixed**
- ϵ_i is **independent**, with $\mathbb{E}\left[\epsilon_i\right] = 0, \mathbb{E}\left[\epsilon_i^2\right] = \sigma_i^2$
- observation: $\left\{\left(y_i, w_i, z_i'\right)'\right\}_{i=1}^n$

one example is the case where w_i is **binary**, then

$$\beta(z) = f(1,z) - f(0,z)$$

which is just the ATE conditional on z under the unconfoundedness assumption. This includes the RD design, where z_i is the running variable and w_i is the treatment assignment.

Now, consider for the weighted average treatment effect

$$L_\mu\left[\beta(\cdot)\right] = \int \beta(z)\mathrm{d}\mu(z)$$

where $\int \mu(z) = 1$ is a **signed** measure (weight, allowing **negative** weights), construct a linear estimator

$$\hat{L}_a = \sum_{i=1}^n a_i y_i$$

where the estimation weights a_i can depend on $\left\{z_i, w_i, \sigma_i^2\right\}_{i=1}^n$, but **not** on y_i. Together, the bias of $\hat{L}_a$ for $L_\mu\left[\beta(\cdot)\right]$, given the regression function $\beta(\cdot), h(\cdot)$, is

$$\mathbb{E}_{\beta(\cdot),h(\cdot)}\left[\hat{L}_a\right] - L_\mu\left[\beta(\cdot)\right] = \sum_{i=1}^n a_i\left[w_i\beta(z_i) + h(z_i)\right] - \int \beta(z)\mathrm{d}\mu(z)$$

and its variance, given the regression function $\beta(\cdot), h(\cdot)$, is just

$$\mathrm{Var}_{\beta(\cdot),h(\cdot)}\left[\hat{L}_a\right] = \sum_{i=1}^{n} a_i^2 \sigma_i^2$$

To bound the bias, assume $h(\cdot)$ is known to belong in a class of functions $\mathcal{H}$

References

Timothy B Armstrong, Michal Kolesár, and Soonwoo Kwon. Bias-aware inference in regularized regression models. *arXiv preprint arXiv:2012.14823*, 2020.