

Topic 14: Regularization Methods in Thresholded Parameter Space

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Key points: The connections and differences of all regularization methods and some interesting phase transition phenomena.

Disclaimer: The note is built on Prof. *Jinchi Lv*'s lectures of the course at USC, DSO 607, High-Dimensional Statistics and Big Data Problems.

14.1 Model Setup

Now, consider a generalized linear model (GLM) linking a p -dimensional predictor \mathbf{x} to a scalar response Y . With canonical link, the conditional distribution of Y given \mathbf{x} has density

$$f(y; \theta, \phi) = \exp [y\theta - b(\theta) + c(y, \phi)]$$

where $\theta = \mathbf{x}'\boldsymbol{\beta}$ with $\boldsymbol{\beta}$ a p -dimensional regression coefficient vector, $b(\cdot)$ and $c(\cdot, \cdot)$ are known functions and ϕ is dispersion parameter. Again, $\boldsymbol{\beta} = (\beta_{0,1}, \dots, \beta_{0,p})'$ is sparse with many zero components, and $\log p = O(n^a)$ for some $0 < a < 1$.