

## Topic 5: Two-Way Cluster-Robust (TWCR) Standard Errors

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**Key points:** The validity of Two-Way Cluster-Robust (TWCR) standard errors

**Disclaimer:** *This note is compiled by Sai Zhang.*

### 5.1 One-Way Clustering

First, consider the case of one-way clustering. The linear model with one-way clustering

$$y_{ig} = \mathbf{x}_{ig}\boldsymbol{\beta} + u_{ig}$$

where  $i$  denotes the  $i$ th of the  $N$  individuals in the sample,  $j$  denotes the  $g$ th of the  $G$  clusters, assume that

- $\mathbb{E}[u_{ig} | \mathbf{x}_{ig}] = 0$
- error independence across clusters: for  $i \neq j$

$$\mathbb{E}[u_{ig}u_{jg'} | \mathbf{x}_{ig}, \mathbf{x}_{jg'}] = 0 \quad (5.1)$$

unless  $g = g'$ , that is, errors for individuals within the same cluster may be correlated.

Grouping observations by cluster, get

$$\mathbf{y}_g = \mathbf{X}_g\boldsymbol{\beta} + \mathbf{u}$$

where  $\mathbf{X}_g$  has dimension  $N_g \times K$  and  $\mathbf{y}_g$  has dimension  $N_g \times 1$ , with  $N_g$  observations in cluster  $g$ . Stacking over cluster, get the matrix form of the model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

with  $\mathbf{y}, \mathbf{u}$  being  $N \times 1$  vectors,  $\mathbf{X}$  being an  $N \times K$  matrix. OLS estimator gives

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{y} = \left( \sum_{g=1}^G \mathbf{X}_g' \mathbf{X}_g \right)^{-1} \sum_{g=1}^G \mathbf{X}_g' \mathbf{y}_g \quad (5.2)$$

Chiang and Sasaki (2023) (Menzel, 2021) Cameron et al. (2011)

## References

- A Colin Cameron, Jonah B Gelbach, and Douglas L Miller. Robust inference with multiway clustering. *Journal of Business & Economic Statistics*, 29(2):238–249, 2011.
- Harold D Chiang and Yuya Sasaki. On using the two-way cluster-robust standard errors. *arXiv preprint arXiv:2301.13775*, 2023.
- Konrad Menzel. Bootstrap with cluster-dependence in two or more dimensions. *Econometrica*, 89(5):2143–2188, 2021.