Econometrics **May 18, 2023**

Topic 16: Graphical Network Inference

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Key points:

Disclaimer: *The note is built on Prof. Jinchi Lv's lectures of the course at USC, DSO 607, High-Dimensional Statistics and Big Data Problems.*

16.1 Motivation

Consider a classic question: Suppose we have N observations of dimension p follow $\mathcal{N}(\mu, \Sigma)$. let $\Theta = \Sigma^{-1}$, and $\mathbf{S}$ be the empirical covariance matrix. How can we capture the statistical relationships between the variables of interest? Write this question in matrix form:

Example 16.1.1: Multivariate Gaussian Distribution

$\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$ with the probability density

$$f(\mathbf{x}) = \frac{1}{(2\pi)^{p/2}\det(\mathbf{\Sigma})^{1/2}} \exp\left\{-\frac{1}{2}\mathbf{x}'\mathbf{\Sigma}^{-1}\mathbf{x}\right\} \propto \det(\mathbf{\Theta})^{1/2} \exp\left\{-\frac{1}{2}\mathbf{x}'\mathbf{\Theta}\mathbf{x}\right\}$$

where $\mathbf{\Sigma} = \mathbb{E}[\mathbf{x}\mathbf{x}'] > \mathbf{0}$ is the covariance matrix, and $\mathbf{\Theta} = \mathbf{\Sigma}^{-1}$ is the **inverse covariance matrix** or **precision matrix**