

## Topic 6: DID and TWFE

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**Key points:** This note is on the causal panel data, building upon [Arkhangelsky and Imbens \(2023\)](#).

**Disclaimer:** *This note is compiled by Sai Zhang.*

## 6.1 Panel Data Configurations

### 6.1.1 Data Types

#### 6.1.1.1 Panel Data

For observations on  $N$  units, indexed by  $i = 1, \dots, N$ , over  $T$  periods, indexed by  $t = 1, \dots, T$ , the outcome of interest is denoted by  $Y_{it}$ , the treatment  $W_{it}$ . These observations may themselves consist of averages over more basic units:

$$\mathbf{Y} = \begin{pmatrix} Y_{11} & \cdots & Y_{1T} \\ \vdots & \ddots & \vdots \\ Y_{N1} & \cdots & Y_{NT} \end{pmatrix} \quad \mathbf{W} = \begin{pmatrix} W_{11} & \cdots & W_{1T} \\ \vdots & \ddots & \vdots \\ W_{N1} & \cdots & W_{NT} \end{pmatrix}$$

we may also observe exogenous variables  $X_{it}$  or  $X_i$ . Typically, we focus on a balanced panel where for all units  $i = 1, \dots, N$  we observe outcomes for all  $t = 1, \dots, T$ .

#### 6.1.1.2 Grouped Repeated Cross-Section Data

In a GRCS data, we have observations on  $N$  units, each observed only once in period  $T_i$  for unit  $i$ . Different units may be observed at different points in time,  $T_i$  typically takes on only a few values, with many units sharing the same value for  $T_i$ . The outcome  $Y_i$  and treatment  $W_i$  are indexed by the unit index  $i$ . The set of units is **partitioned** into 2 or more groups, with the group that unit  $i$  belongs to denoted by  $G_i \in \mathcal{G} = \{1, 2, \dots, G\}$ .

Define the average outcomes for each group-time-period pair:

$$\bar{Y}_{gt} \equiv \frac{\sum_{i=1}^N \mathbf{1}_{G_i=g, T_i=t} Y_i}{\sum_{i=1}^N \mathbf{1}_{G_i=g, T_i=t}}$$

for treatment

$$\bar{W}_{gt} \equiv \frac{\sum_{i=1}^N \mathbf{1}_{G_i=g, T_i=t} W_i}{\sum_{i=1}^N \mathbf{1}_{G_i=g, T_i=t}}$$

then treat the  $G \times T$  group averages  $\bar{Y}_{gt}$  and  $\bar{W}_{gt}$  as the unit of observation, then the grouped data is just a panel. The major issue in practice is that the number of groups is very small comparing to proper panel data.

### 6.1.1.3 Row and Column Exchangeable Data

The data are doubly indexed by  $i = 1, \dots, N$  and  $j = 1, \dots, J$ , with outcomes  $Y_{ij}$ . They are different from panel data in that there is **no time ordering** for the second index. Many methods developed for panel data are also applicable here.

## 6.1.2 Shapes of Data Frames

Panel data can also be loosely classified by the shape:

- **Thin Frames** ( $N \gg T$ ), where the number of cross-section units is large relative to the number of time periods:
  - unit-specific parameters (individual FEs) **can not be estimated consistently** due to the short time series
  - REs might be more suitable since they place a stochastic structure on the individual components
- **Fat Frames** ( $N \ll T$ ), where the number of cross-section units is large relative to the number of time periods.
- **Square**  $N \approx T$ , where the number of units and time periods is comparable.

## 6.1.3 Assignment Mechanisms

### 6.1.3.1 The General Case

In the most general case, the treatment may vary both across units and over time, with units **switching** in and out of the treatment group:

$$\mathbf{W}^{\text{general}} = \begin{pmatrix} 1 & 1 & 0 & 0 & \dots & 1 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 1 & 0 & 1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 1 & 0 & \dots & 0 \end{pmatrix}$$

This is more relevant for the RCED configurations, and for panel data of products and promotions as treatments. The assumption on the absence/presence of **dynamic treatment** effects is very important.

### 6.1.3.2 Single Treated Period

One special case arises when a substantial number of units is treated, but these units are only treated **in the last period**

$$\mathbf{W}^{\text{last}} = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

If  $T$  is relatively small, this case is often analyzed as a cross-section problem, the lagged outcomes are used as exogenous covariates or pre-treatment variables to be adjusted. Here, dynamic effects are not testable,

nor do they matter since the shortness of the panel.

$$\mathbf{W}^{\text{last}} = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 1 & 1 & \cdots & 0 \end{pmatrix}$$

this setting is prominent in the original applications of the synthetic control literature, here  $T$  is usually small.

### 6.1.3.3 Single Treated Unit and Single Treated Period

An extreme case is where only a single unit is treated, and it is only treated in a single period (typically the last). Normally, we focus on the effect for the single treated/time-period pair and construct prediction intervals.

### 6.1.3.4 Block Assignment

The case of block assignment is where a subset of units is treated every period after a common starting date:

$$\mathbf{W}^{\text{block}} = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 1 & 1 & \cdots & 1 \end{pmatrix}$$

There is typically a sufficient number of treated unit/time-period pairs to allow for reasonable approximations. The presence of dynamic effects change the interpretation of the average effect of the treated: the average effect for the treated now is an average over short **and** medium term effects during different periods.

## References

Dmitry Arkhangelsky and Guido Imbens. Causal models for longitudinal and panel data: A survey. Technical report, National Bureau of Economic Research, 2023.