

## Topic 18: Eigenvalue and Spike Models

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**Key points:** .

**Disclaimer:** The note is built on Prof. *Jinchi Lv*'s lectures of the course at USC, DSO 607, High-Dimensional Statistics and Big Data Problems.

### 18.1 Motivation

Consider  $n$  independent observations  $\mathbf{X}_i \in \mathbb{R}^p$  drawn from a  $\mathcal{N}(\mathbf{0}, \Sigma)$ , then the covariance can be decomposed into 2 parts, white noise and low rank

$$\Sigma = \text{Cov}(\mathbf{X}_i) = \mathbf{I} + \sum_{k=1}^M \theta_k \mathbf{v}_k \mathbf{v}_k' = \Sigma_0 + \Phi$$

where  $M$  denotes the **number of spikes** in the distribution of eigenvalues. The idea is: spikes deviate from a reference model along a **small fixed number** of unknown directions. If  $\Phi = \mathbf{0}$ , then none of the sample eigenvalues is separated from the bulk.

**Why a spike model is interesting?** A spike model can help determine the latent dimension of the data, some examples being

- Principal component analysis (PCA): spikes are related to the directions of the most variations of the data, i.e., the principal components
- Clustering model:  $M$  spikes is equivalent to  $M + 1$  clusters
- Economic significance:  $M$  is related to the number of factor loadings

Then the question is threefold:

- How to determine  $M$
- How to estimate  $\mathbf{v}_k$
- How to test  $\theta_k$

Under rank one alternative, we would like to test the hypothesis

$$the H_1 : \Sigma = \mathbf{I}_p + \theta \mathbf{v} \mathbf{v}', \theta > 0$$

against the null

$$H_0 : \Sigma = \mathbf{I}_p$$

## References