Noise-Induced Randomization in Regression Discontinuity Designs

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Outline

Estimation

Estimation of Weighted Treatment Effects

Proposition 1

Let $\gamma_+(\cdot), \gamma_-(\cdot)$ be measurable functions of Z, then under A1-A3:

$$\mathbb{E}\left[\gamma_+(Z)\,Y\right] = \mathbb{E}\left[\alpha_{(1)}(U)\,h(U,\gamma_+)\right], \qquad \mathbb{E}\left[\gamma_-(Z)\,Y\right] = \mathbb{E}\left[\alpha_{(0)}(U)\,h(U,\gamma_-)\right]$$

where $h(u,\gamma) := \int \gamma(z)\,p(z \mid u)\,\mathrm{d}\lambda(z)$, $\alpha_{(w)}(u) = \mathbb{E}\left[Y_i(w) \mid U_i = u\right]$

ratio-form estimators:

$$\hat{\mu}_{\gamma,+} = \frac{\sum_i \gamma_+(Z_i)\,Y_i}{\sum_i \underbrace{\gamma_+(Z_i)}}, \qquad \hat{\mu}_{\gamma,-} = \frac{\sum_i \gamma_-(Z_i)\,Y_i}{\sum_i \underbrace{\gamma_-(Z_i)}}$$

Estimation of Weighted Treatment Effects

Proposition 1

Let $\gamma_+(\cdot), \gamma_-(\cdot)$ be measurable functions of Z, then under A1-A3:

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where $h(u,\gamma) := \int \gamma(z)\,p(z \mid u)\,\mathrm{d}\lambda(z)$, $\alpha_{(w)}(u) = \mathbb{E}\left[Y_i(w) \mid U_i = u\right]$

ratio-form estimators:

$$\hat{\mu}_{\gamma,+} = \frac{\sum_i \gamma_+(Z_i)\,Y_i}{\sum_i \underbrace{\gamma_+(Z_i)}_{\gamma_+(z)=0,z<c}} \qquad \hat{\mu}_{\gamma,-} = \frac{\sum_i \gamma_-(Z_i)\,Y_i}{\sum_i \underbrace{\gamma_-(Z_i)}_{\gamma_-(z)=0,z<\geq c}}$$

Estimation of Weighted Treatment Effects

Ratio-form estimators:

$$\frac{\mathbb{E}\left[\gamma_{+}(Z)\,Y\right]}{\mathbb{E}\left[\gamma_{+}(Z)\right]} \Rightarrow \quad \hat{\mu}_{\gamma,+} = \frac{\sum_i \gamma_{+}(Z_i)\,Y_i}{\sum_i \gamma_{+}(Z_i)}$$

$$\frac{\mathbb{E}\left[\gamma_{-}(Z)\,Y\right]}{\mathbb{E}\left[\gamma_{-}(Z)\right]} \Rightarrow \quad \hat{\mu}_{\gamma,-} = \frac{\sum_i \gamma_{-}(Z_i)\,Y_i}{\sum_i \gamma_{-}(Z_i)}$$

Estimation of Weighted Treatment Effects

Ratio-form estimators:

$$\left.\begin{aligned}\frac{\mathbb{E}\left[\gamma_{+}\left(Z\right)Y\right]}{\mathbb{E}\left[\gamma_{+}\left(Z\right)\right]} \Rightarrow \quad \hat{\mu}_{\gamma,+} &= \frac{\sum_i \gamma_{+}\left(Z_i\right)Y_i}{\sum_i \gamma_{+}\left(Z_i\right)} \\ \frac{\mathbb{E}\left[\gamma_{-}\left(Z\right)Y\right]}{\mathbb{E}\left[\gamma_{-}\left(Z\right)\right]} \Rightarrow \quad \hat{\mu}_{\gamma,-} &= \frac{\sum_i \gamma_{-}\left(Z_i\right)Y_i}{\sum_i \gamma_{-}\left(Z_i\right)}\end{aligned}\right. \Longrightarrow \hat{\tau}_{\gamma} = \hat{\mu}_{\gamma,+} - \hat{\mu}_{\gamma,-}$$

Estimation of Weighted Treatment Effects

Ratio-form estimators:

$$\hat{\tau}_\gamma = \hat{\mu}_{\gamma,+} - \hat{\mu}_{\gamma,-} = \frac{\sum_i \gamma_+(Z_i) Y_i}{\sum_i \gamma_+(Z_i)} - \frac{\sum_i \gamma_-(Z_i) Y_i}{\sum_i \gamma_-(Z_i)}$$

What's the weighted treatment effects to be estimated?

Estimation of Weighted Treatment Effects

Ratio-form estimators:

$$\hat{\tau}_\gamma = \hat{\mu}_{\gamma,+} - \hat{\mu}_{\gamma,-} = \frac{\sum_i \gamma_+ (Z_i) Y_i}{\sum_i \gamma_+ (Z_i)} - \frac{\sum_i \gamma_- (Z_i) Y_i}{\sum_i \gamma_- (Z_i)}$$

What's the weighted treatment effects to be estimated?

$$\tau_w = \int \frac{w(u)}{\mathbb{E}_G [w(U)]} \tau(u) \mathrm{d}G(u), w(\cdot) \geq 0$$

Estimation of Weighted Treatment Effects

Ratio-form estimators:

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What's the weighted treatment effects to be estimated?

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where $\tau(u)$ **(Conditional Average Treatment Effects)** is

$$\tau(u) = \mathbb{E}[Y_i(1) - Y_i(0) \mid U_i = u] = \alpha_{(1)}(u) - \alpha_{(0)}(u)$$

Weighted Treatment Effects: Example 1

$$\tau_w = \int \frac{w(u)}{\mathbb{E}_G[w(U)]} \tau(u) \mathrm{d}G(u), w(\cdot) \geq 0$$

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Weighted Treatment Effects: Example 1

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- RD paramater:

$$\tau_c = \mathbb{E}[Y_i(1) - Y_i(0) \mid Z_i = c] = \int \frac{p(c \mid u)}{\int p(c \mid u) \, \mathrm{d}G(u)} \tau(u) \, \mathrm{d}G(u)$$

Weighted Treatment Effects: Example 2

$$\tau_w = \int \frac{w(u)}{\mathbb{E}_G[w(U)]} \tau(u) \mathrm{d}G(u), w(\cdot) \geq 0$$

where $\tau(u)$ **(CATE)** is $\tau(u) = \mathbb{E}[Y_i(1) - Y_i(0) \mid U_i = u] = \alpha_{(1)}(u) - \alpha_{(0)}(u)$

- Changing the cutoff from c to $c' < c$:

$$\tau_\pi = \mathbb{E}\left[Y_i(1) - Y_i(0) \mid c' \leq Z_i < c\right] = \int \tau(u) \frac{\int_{[c',c)} p(z \mid u) \, \mathrm{d}\lambda(z)}{\int_{[c',c)} \mathrm{d}F(z)} \mathrm{d}G(u)$$

Weighted Treatment Effects: Example 3

$$\tau_w = \int \frac{w(u)}{\mathbb{E}_G\left[w(U)\right]} \tau(u) \mathrm{d}G(u), w(\cdot) \geq 0$$

where $\tau(u)$ **(CATE)** is $\tau(u) = \mathbb{E}\left[Y_i(1) - Y_i(0) \mid U_i = u\right] = \alpha_{(1)}(u) - \alpha_{(0)}(u)$

- Measurement error reduced from $Z_i \mid U_i \sim \mathcal{N}(U_i, \nu^2)$ to $Z_i' \mid U_i \sim \mathcal{N}(U_i, \nu'^2)$:

$$\begin{aligned} \tau_\pi &= \mathbb{E}\left[Y_i(1) - Y_i(0) \mid W_i' > W_i\right] \\ &= \int \tau(u) \frac{(1 - \Phi_{\nu'}(c-u))\, \Phi_\nu(c-u)}{\int (1 - \Phi_{\nu'}(c-u))\, \Phi_\nu(c-u)\, \mathrm{d}G(u)} \mathrm{d}G(u) \end{aligned}$$

Asymptotic Bias

Theorem 2: Asymptotic Limit of $\hat{\tau}_\gamma$

$$\hat{\tau}_\gamma = \hat{\mu}_{\gamma,+} - \hat{\mu}_{\gamma,-} = \frac{\sum_i \gamma_+(Z_i) Y_i}{\sum_i \gamma_+(Z_i)} - \frac{\sum_i \gamma_-(Z_i) Y_i}{\sum_i \gamma_-(Z_i)}$$

Asymptotic Bias

Theorem 2: Asymptotic Limit of $\hat{\tau}_\gamma$

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$$\xrightarrow{p} \frac{\mathbb{E}\left[\alpha_{(1)}(U)\, h(U, \gamma_+)\right]}{\mathbb{E}\left[h(U, \gamma_+)\right]} - \frac{\mathbb{E}\left[\alpha_{(0)}(U)\, h(U, \gamma_-)\right]}{\mathbb{E}\left[h(U, \gamma_-)\right]} = \mu_{\gamma,+} - \mu_{\gamma,-} \equiv \theta_\gamma$$

Asymptotic Bias

Theorem 2: Asymptotic Limit of $\hat{\tau}_\gamma$

$$\hat{\tau}_\gamma = \hat{\mu}_{\gamma,+} - \hat{\mu}_{\gamma,-} = \frac{\sum_i \gamma_+ (Z_i) Y_i}{\sum_i \gamma_+ (Z_i)} - \frac{\sum_i \gamma_- (Z_i) Y_i}{\sum_i \gamma_- (Z_i)}$$
$$\xrightarrow{p} \frac{\mathbb{E}\left[\alpha_{(1)} (U) h (U, \gamma_+)\right]}{\mathbb{E}\left[h (U, \gamma_+)\right]} - \frac{\mathbb{E}\left[\alpha_{(0)} (U) h (U, \gamma_-)\right]}{\mathbb{E}\left[h (U, \gamma_-)\right]} = \mu_{\gamma,+} - \mu_{\gamma,-} \equiv \theta_\gamma$$

How biased is this asymptotic limit? Comparing to

$$\tau_w = \int \frac{w(u)}{\mathbb{E}_G\left[w(U)\right]} \tau(u) \mathrm{d}G(u), w(\cdot) \geq 0$$

Asymptotic Bias: Decomposition

$$a\text{Bias}\left[\gamma_{\pm}, \tau_w; \alpha_{(0)}(\cdot), \tau(\cdot), G\right] = \theta_{\gamma} - \tau_w$$

Asymptotic Bias: Decomposition

$$
\begin{aligned}
& a\text{Bias}\left[\gamma_{\pm}, \tau_w; \alpha_{(0)}(\cdot), \tau(\cdot), G\right] = \theta_\gamma - \tau_w \\
= & \frac{\mathbb{E}\left[\alpha_{(1)}(U)\, h(U, \gamma_+)\right]}{\mathbb{E}\left[h(U, \gamma_+)\right]} - \frac{\mathbb{E}\left[\alpha_{(0)}(U)\, h(U, \gamma_-)\right]}{\mathbb{E}\left[h(U, \gamma_-)\right]} - \int \frac{w(u)}{\mathbb{E}_G\left[w(U)\right]} \tau(u)\, \mathrm{d}G(u)
\end{aligned}
$$

Asymptotic Bias: Decomposition

$$
\begin{aligned}
&a\text{Bias}\left[\gamma_{\pm},\tau_w;\alpha_{(0)}\left(\cdot\right),\tau\left(\cdot\right),G\right]=\theta_\gamma-\tau_w\\
=&\frac{\mathbb{E}\left[\alpha_{(1)}\left(U\right)h\left(U,\gamma_+\right)\right]}{\mathbb{E}\left[h\left(U,\gamma_+\right)\right]}-\frac{\mathbb{E}\left[\alpha_{(0)}\left(U\right)h\left(U,\gamma_-\right)\right]}{\mathbb{E}\left[h\left(U,\gamma_-\right)\right]}-\int\frac{w\left(u\right)}{\mathbb{E}_G\left[w\left(U\right)\right]}\tau\left(u\right)\mathrm{d}G\left(u\right)\\
=&\int\left(\frac{h\left(u,\gamma_+\right)}{\mathbb{E}_G\left[h\left(U,\gamma_+\right)\right]}\right)\alpha_{(1)}\left(u\right)\mathrm{d}G\left(u\right)-\int\left(\frac{h\left(u,\gamma_-\right)}{\mathbb{E}_G\left[h\left(U,\gamma_-\right)\right]}\right)\alpha_{(0)}\left(u\right)\mathrm{d}G\left(u\right)\\
&-\int\frac{w\left(u\right)}{\mathbb{E}_G\left[w\left(U\right)\right]}\tau\left(u\right)\mathrm{d}G\left(u\right)
\end{aligned}
$$

Asymptotic Bias: Decomposition

$$
\begin{aligned}
& a\mathrm{Bias}\left[\gamma_{\pm}, \tau_w; \alpha_{(0)}(\cdot), \tau(\cdot), G\right] = \theta_\gamma - \tau_w \\
=& \frac{\mathbb{E}\left[\alpha_{(1)}(U)\, h(U, \gamma_+)\right]}{\mathbb{E}\left[h(U, \gamma_+)\right]} - \frac{\mathbb{E}\left[\alpha_{(0)}(U)\, h(U, \gamma_-)\right]}{\mathbb{E}\left[h(U, \gamma_-)\right]} - \int \frac{w(u)}{\mathbb{E}_G\left[w(U)\right]} \tau(u)\, \mathrm{d}G(u) \\
=& \int \left(\frac{h(u, \gamma_+)}{\mathbb{E}_G\left[h(U, \gamma_+)\right]} \right) \alpha_{(1)}(u)\, \mathrm{d}G(u) - \int \left(\frac{h(u, \gamma_-)}{\mathbb{E}_G\left[h(U, \gamma_-)\right]} \right) \alpha_{(0)}(u)\, \mathrm{d}G(u) \\
& - \int \frac{w(u)}{\mathbb{E}_G\left[w(U)\right]} \tau(u)\, \mathrm{d}G(u)
\end{aligned}
$$

Remember? $\tau(u)$ **(Conditional Average Treatment Effects)** is

$$
\tau(u) = \mathbb{E}\left[Y_i(1) - Y_i(0) \mid U_i = u\right] = \alpha_{(1)}(u) - \alpha_{(0)}(u) \Rightarrow \boxed{\alpha_{(1)}(u) = \tau(u) + \alpha_{(0)}(u)}
$$

Asymptotic Bias: Decomposition

$$
\begin{aligned}
& a\text{Bias}\left[\gamma_{\pm}, \tau_w; \alpha_{(0)}(\cdot), \tau(\cdot), G\right] = \theta_\gamma - \tau_w \\
&= \int \left(\frac{h(u, \gamma_+)}{\mathbb{E}_G\left[h(U, \gamma_+)\right]} \right) \underbrace{\alpha_{(1)}(u)}_{=\tau(u)+\alpha_{(0)}(u)} \mathrm{d}G(u) \\
& \quad - \int \left(\frac{h(u, \gamma_-)}{\mathbb{E}_G\left[h(U, \gamma_-)\right]} \right) \alpha_{(0)}(u) \,\mathrm{d}G(u) - \int \frac{w(u)}{\mathbb{E}_G\left[w(U)\right]} \tau(u) \,\mathrm{d}G(u)
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= & \int \left(\frac{h(u, \gamma_+)}{\mathbb{E}_G\left[h(U, \gamma_+)\right]} - \frac{h(u, \gamma_-)}{\mathbb{E}_G\left[h(U, \gamma_-)\right]} \right) \alpha_{(0)}(u)\, \mathrm{d}G(u)
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&\quad - \int \left(\frac{h(u, \gamma_-)}{\mathbb{E}_G[h(U, \gamma_-)]} \right) \alpha_{(0)}(u)\,\mathrm{d}G(u) - \int \frac{w(u)}{\mathbb{E}_G[w(U)]} \tau(u)\,\mathrm{d}G(u) \\
&= \int \left(\frac{h(u, \gamma_+)}{\mathbb{E}_G[h(U, \gamma_+)]} - \frac{h(u, \gamma_-)}{\mathbb{E}_G[h(U, \gamma_-)]} \right) \alpha_{(0)}(u)\,\mathrm{d}G(u) \\
&\quad + \int \left(\frac{h(u, \gamma_+)}{\mathbb{E}_G[h(U, \gamma_+)]} - \frac{w(u)}{\mathbb{E}_G[w(U)]} \right) \tau(u)\,\mathrm{d}G(u)
\end{aligned}
$$

Asymptotic Bias: Decomposition

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& - \int \left(\frac{h(u, \gamma_-)}{\mathbb{E}_G\left[h(U, \gamma_-)\right]} \right) \alpha_{(0)}(u) \, \mathrm{d}G(u) - \int \frac{w(u)}{\mathbb{E}_G\left[w(U)\right]} \tau(u) \, \mathrm{d}G(u) \\
= & \int \left(\frac{h(u, \gamma_+)}{\mathbb{E}_G\left[h(U, \gamma_+)\right]} - \frac{h(u, \gamma_-)}{\mathbb{E}_G\left[h(U, \gamma_-)\right]} \right) \alpha_{(0)}(u) \, \mathrm{d}G(u) \\
& + \int \left(\frac{h(u, \gamma_+)}{\mathbb{E}_G\left[h(U, \gamma_+)\right]} - \frac{w(u)}{\mathbb{E}_G\left[w(U)\right]} \right) \tau(u) \, \mathrm{d}G(u)
\end{aligned}
$$

Confounding bias

CATE heterogeneity bias

Asymptotic Bias: Decomposition

$$\int \left(\frac{h\left(u, \gamma_+\right)}{\mathbb{E}_G\left[h\left(U, \gamma_+\right)\right]} - \frac{h\left(u, \gamma_-\right)}{\mathbb{E}_G\left[h\left(U, \gamma_-\right)\right]} \right) \alpha_{(0)}\left(u\right) \mathrm{d}G\left(u\right)$$

Confounding bias

$$\int \left(\frac{h\left(u, \gamma_+\right)}{\mathbb{E}_G\left[h\left(U, \gamma_+\right)\right]} - \frac{w\left(u\right)}{\mathbb{E}_G\left[w\left(U\right)\right]} \right) \tau\left(u\right) \mathrm{d}G\left(u\right)$$

CATE heterogeneity bias

How to minimize them?

Asymptotic Bias: Decomposition

$$\int \left(\frac{h\left(u, \gamma_{+}\right)}{\mathbb{E}_G\left[h\left(U, \gamma_{+}\right)\right]} - \frac{h\left(u, \gamma_{-}\right)}{\mathbb{E}_G\left[h\left(U, \gamma_{-}\right)\right]} \right) \alpha_{(0)}\left(u\right) \mathrm{d}G\left(u\right)$$

Confounding bias

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CATE heterogeneity bias

How to minimize them?

- **Confounding bias:** $h(\cdot, \gamma_{+}) \approx h(\cdot, \gamma_{-})$

Asymptotic Bias: Decomposition

$$\int \left(\frac{h\left(u,\gamma_{+}\right)}{\mathbb{E}_G\left[h\left(U,\gamma_{+}\right)\right]} - \frac{h\left(u,\gamma_{-}\right)}{\mathbb{E}_G\left[h\left(U,\gamma_{-}\right)\right]} \right) \alpha_{(0)}\left(u\right) \mathrm{d}G\left(u\right)$$

Confounding bias

$$\int \left(\frac{h\left(u,\gamma_{+}\right)}{\mathbb{E}_G\left[h\left(U,\gamma_{+}\right)\right]} - \frac{w\left(u\right)}{\mathbb{E}_G\left[w\left(U\right)\right]} \right) \tau\left(u\right) \mathrm{d}G\left(u\right)$$

CATE heterogeneity bias

How to minimize them?

- **Confounding bias:** $h(\cdot,\gamma_{+}) \approx h(\cdot,\gamma_{-})$ where $h\left(u,\gamma\right) := \int \gamma\left(z\right) p\left(z \mid u\right) \mathrm{d}\lambda\left(z\right)$
  How well the units are **balanced** via the latent variable u

Asymptotic Bias: Decomposition

$$\int \left(\frac{h\left(u, \gamma_{+}\right)}{\mathbb{E}_{G}\left[h\left(U, \gamma_{+}\right)\right]} - \frac{h\left(u, \gamma_{-}\right)}{\mathbb{E}_{G}\left[h\left(U, \gamma_{-}\right)\right]} \right) \alpha_{(0)}\left(u\right) \mathrm{d}G\left(u\right)$$

Confounding bias

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Asymptotic Bias: Decomposition

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$$\int \left(\frac{h\left(u,\gamma_{+}\right)}{\mathbb{E}_{G}\left[h\left(U,\gamma_{+}\right)\right]} - \frac{w\left(u\right)}{\mathbb{E}_{G}\left[w\left(U\right)\right]} \right) \tau\left(u\right) \mathrm{d}G\left(u\right) \qquad \textbf{CATE heterogeneity bias}$$

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Asymptotic Bias: Decomposition

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Confounding bias

$$\int \left(\frac{h\left(u,\gamma_+\right)}{\mathbb{E}_G\left[h\left(U,\gamma_+\right)\right]} - \frac{w\left(u\right)}{\mathbb{E}_G\left[w\left(U\right)\right]} \right) \tau\left(u\right) \mathrm{d}G\left(u\right)$$

CATE heterogeneity bias

How to minimize them?

- **Confounding bias:** $h(\cdot,\gamma_+) \approx h(\cdot,\gamma_-)$
  How well the units are **balanced** via the latent variable u
- **CATE heterogeneity bias:**
  - $\tau(u)$ being constant w.r.t. u, a constant conditional treatment effect

Asymptotic Bias: Decomposition

$$\int \left(\frac{h\left(u,\gamma_{+}\right)}{\mathbb{E}_{G}\left[h\left(U,\gamma_{+}\right)\right]} - \frac{h\left(u,\gamma_{-}\right)}{\mathbb{E}_{G}\left[h\left(U,\gamma_{-}\right)\right]} \right) \alpha_{(0)}\left(u\right) \mathrm{d}G\left(u\right) \quad \text{Confounding bias}$$

$$\int \left(\frac{h\left(u,\gamma_{+}\right)}{\mathbb{E}_{G}\left[h\left(U,\gamma_{+}\right)\right]} - \frac{w\left(u\right)}{\mathbb{E}_{G}\left[w\left(U\right)\right]} \right) \tau\left(u\right) \mathrm{d}G\left(u\right) \quad \text{CATE heterogeneity bias}$$

How to minimize them?

- **Confounding bias:** $h(\cdot,\gamma_{+}) \approx h(\cdot,\gamma_{-})$
  How well the units are **balanced** via the latent variable u
- **CATE heterogeneity bias:**
  - $\tau(u)$ being constant w.r.t. u, a constant conditional treatment effect
  - $h(u,\gamma_{+}) = w(u), \forall u$, an absolutely correct weighting function

References I

Eckles, D., Ignatiadis, N., Wager, S., & Wu, H. (2020). Noise-induced randomization in regression discontinuity designs. *arXiv preprint arXiv:2004.09458.*

Thank you!