

Noise-Induced Randomization in Regression Discontinuity Designs

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Outline

1 Introduction

Introduction

RD Identification

Z_i
running variable

RD Identification

Z_i
running variable

W_i
treatment

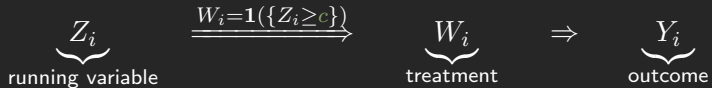
RD Identification

$$\underbrace{Z_i}_{\text{running variable}} \xrightarrow{W_i = \mathbf{1}(\{Z_i \geq c\})} \underbrace{W_i}_{\text{treatment}}$$

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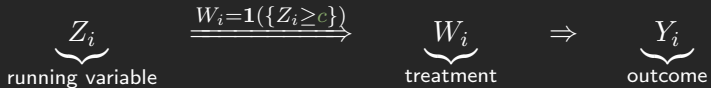
RD Identification



RD Identification



RD Identification



test scores
test results

admission
medication

outcomes
outcomes

RD Identification: Continuity Argument

For potential outcomes $\{Y_i(0), Y_i(1)\}$: $Y_i = Y_i(W_i)$, a weighted **causal effect** can be identified as

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- the conditional response functions $\mu_w(z) = \mathbb{E}[Y(w) \mid Z = z]$ are continuous
- $\mu_w(z)$ to have a uniformly bounded 2nd derivative for CIs (Armstrong and Kolesár, 2018, 2020)

RD Identification: Problems of Continuity Argument

Assumption: continuous $\mu_w(z) = \mathbb{E}[Y(w) \mid Z = z]$

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Where does this continuity come from?

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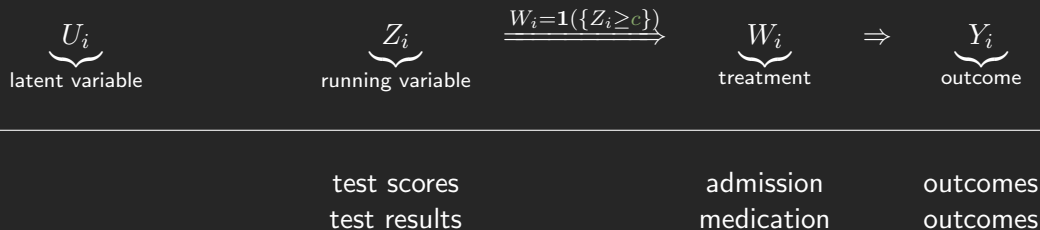
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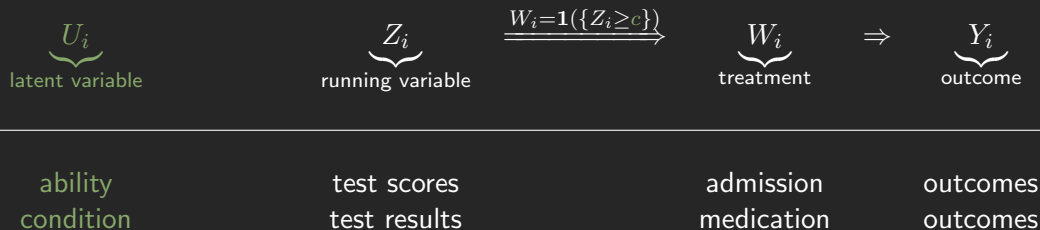
Where does this continuity come from?

Lee (2008): continuous measurement error in the running variable by units

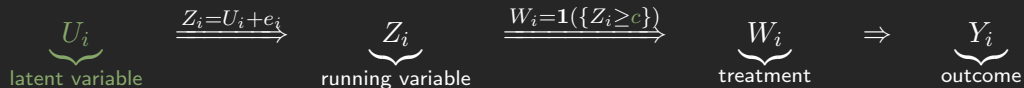
RD Identification: Measurement Error



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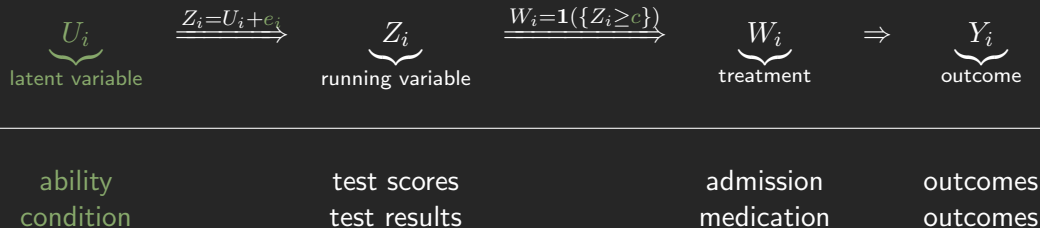
ability
condition

test scores
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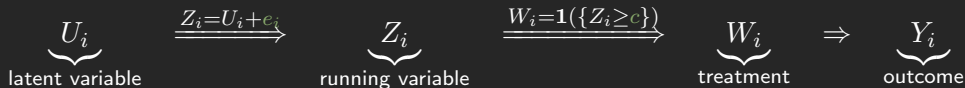
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RD Identification: Measurement Error



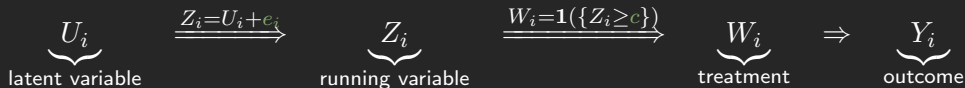
Why don't we take advantage of the measurement error itself for inference?

This Paper



Weighted treatment effects can be estimated if the measurement error in Z_i

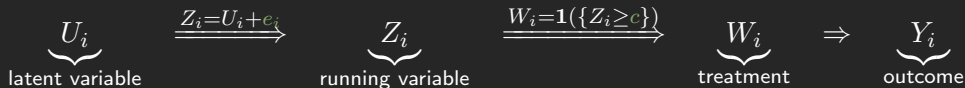
This Paper



Weighted treatment effects can be estimated if the measurement error in Z_i

- has a known distribution

This Paper



Weighted treatment effects can be estimated if the measurement error in Z_i

- has a known distribution
- is conditionally (on U_i) independent of potential outcomes

References I

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Thank you!