Noise-Induced Randomization in Regression Discontinuity Designs

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Outline

Introduction

RD Identification

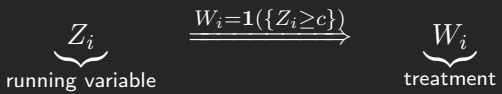
$$\underbrace{Z_i}_{\text{running variable}}$$

RD Identification

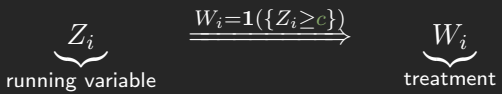
$\underbrace{Z_i}$
running variable

$\underbrace{W_i}$
treatment

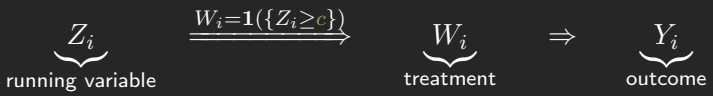
RD Identification



RD Identification



RD Identification



RD Identification

test scores	admission	outcomes

RD Identification

$$\underbrace{Z_i}_{\text{running variable}} \xrightarrow{W_i=\mathbf{1}(\{Z_i \geq c\})} \underbrace{W_i}_{\text{treatment}} \Rightarrow \underbrace{Y_i}_{\text{outcome}}$$

test scores	admission	outcomes
test results	medication	outcomes

RD Identification: Continuity Argument

For potential outcomes $\{Y_i(0), Y_i(1)\}$: $Y_i = Y_i(W_i)$, a weighted **causal effect** can be identified as

$$\tau_c = \mathbb{E}\left[Y_i(1) - Y_i(0) \mid Z_i = c\right]$$

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assuming the conditional response functions $\mu_w(z) = \mathbb{E}\left[Y(w) \mid Z = z\right]$ are **continuous**

RD Identification: Problems of Continuity Argument

Assumption: **continuous** $\mu_w(z) = \mathbb{E}[Y(w) \mid Z = z]$

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Where does this continuity come from?

Lee (2008): **continuous measurement error** in the running variable by units

References I

Eckles, D., Ignatiadis, N., Wager, S., & Wu, H. (2020). Noise-induced randomization in regression discontinuity designs. *arXiv preprint arXiv:2004.09458*.

Lee, D. S. (2008). Randomized experiments from non-random selection in us house elections. *Journal of Econometrics*, *142*(2), 675–697.

Thank you!