# Noise-Induced Randomization in Regression Discontinuity Designs

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# Outline

# CIs

# Asymptotic Normality

$$\hat{\tau} = \frac{\sum_i \gamma_+(Z_i) Y_i}{\sum_i \gamma_+(Z_i)} - \frac{\sum_i \gamma_-(Z_i) Y_i}{\sum_i \gamma_-(Z_i)}$$

$$\hat{\tau}_\gamma \xrightarrow{p} \theta_\gamma = \frac{\mathbb{E}\left[\alpha_{(1)}(U) h(U, \gamma_+)\right]}{\mathbb{E}\left[h(U, \gamma_+)\right]} - \frac{\mathbb{E}\left[\alpha_{(0)}(U) h(U, \gamma_-)\right]}{\mathbb{E}\left[h(U, \gamma_-)\right]}$$

$$a\text{Bias} = \theta_\gamma - \tau_w = \underbrace{\int \left( \frac{h(u, \gamma_+)}{\mathbb{E}_G\left[h(U, \gamma_+)\right]} - \frac{h(u, \gamma_-)}{\mathbb{E}_G\left[h(U, \gamma_-)\right]} \right) \alpha_{(0)}(u) \, \mathrm{d}G(u)}_{\text{Confounding bias}}$$

$$+ \underbrace{\int \left( \frac{h(u, \gamma_+)}{\mathbb{E}_G\left[h(U, \gamma_+)\right]} - \frac{w(u)}{\mathbb{E}_G\left[w(U)\right]} \right) \tau(u) \, \mathrm{d}G(u)}_{\text{CATE heterogeneity bias}}$$

# Asymptotic Normality

**Theorem: Asymptotic Normality of $\hat{\tau}$**

Suppose the sequence of weighting kernels $\gamma_{+}^{(n)}$ and $\gamma_{-}^{(n)}$ is deterministic, and $\exists \beta \in \left(0, \frac{1}{2}\right), C, C' > 0$ s.t. $\forall n$ large enough:

$$\sup_{z} \left| \gamma_{\diamond}^{(n)}(z) \right| < C n^{\beta} \left[ \gamma_{\diamond}^{(n)}(Z_i) \right] \qquad \sup_{u} \left| h\left(u, \gamma_{\diamond}^{(n)}\right) \right| < C' \mathbb{E}\left[ \gamma_{\diamond}^{(n)}(Z_i) \right], \qquad \diamond = \{+, -\}$$

where $h(u, \gamma) := \int \gamma(z)\, p(z \mid u)\, \mathrm{d}\lambda(z)$, $\alpha_{(w)}(u) = \mathbb{E}\left[Y_i(w) \mid U_i = u\right]$

# References I

Eckles, D., Ignatiadis, N., Wager, S., & Wu, H. (2020). Noise-induced randomization in regression discontinuity designs. *arXiv preprint arXiv:2004.09458.*

Thank you!