Noise-Induced Randomization in Regression Discontinuity Designs

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Presented by: Sai Zhang

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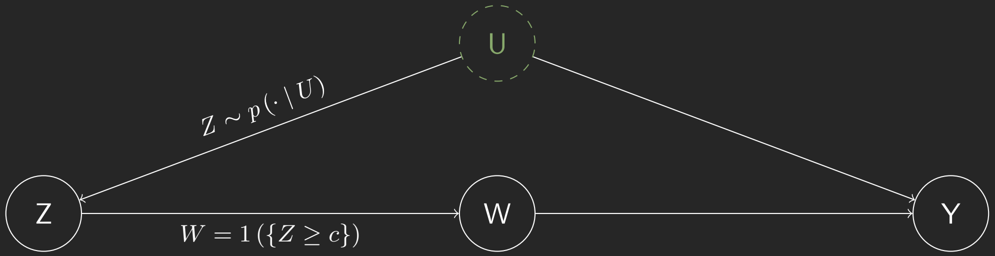
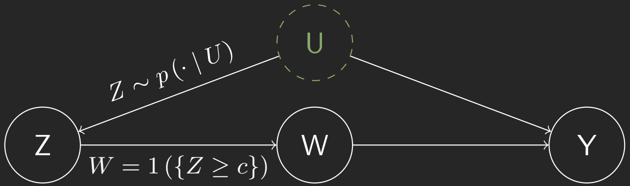
Outline

Key Argument

Sharp RD Design with A Noisy Running Variable



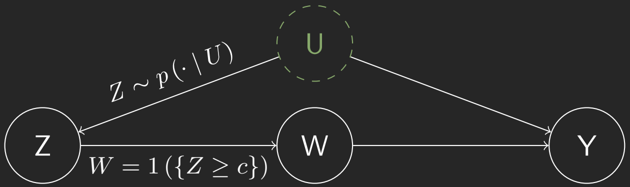
Sharp RD Design with A Noisy Running Variable



Assumption 1: Sharp RD design

- **I.I.D.** samples $\{Y_i(0), Y_i(1), Z_i\} \in \mathbb{R}^3, i = 1, \cdots, n$
- treatment assignment: $W_i = 1\left(\{Z_i \geq c\}\right)$, where $c \in \mathbb{R}$ is the **cutoff**
- observation: $\{Y_i, Z_i\}$ where $Y_i = Y_i(W_i)$

Sharp RD Design with A Noisy Running Variable



Assumption 2: Noisy running variable

$$Z_i \mid U_i \sim p(\cdot \mid U_i)$$

where $p(\cdot \mid \cdot)$ is a **known** conditional density w.r.t. to a measure λ, the latent variable U_i has an **unknown** distribution G

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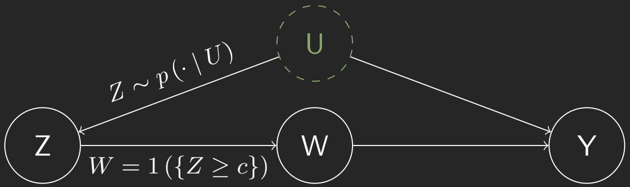

Assumption 2: Noisy running variable

$$Z_i \mid U_i \sim \mathcal{N}(U_i, \nu^2), \nu > 0$$

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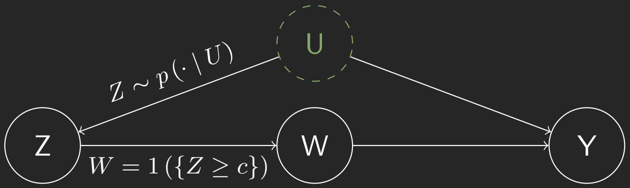

Assumption 2: Noisy running variable

$$Z_i \mid U_i \sim \text{Binomial}(K, U_i), K \in \mathbb{N}$$

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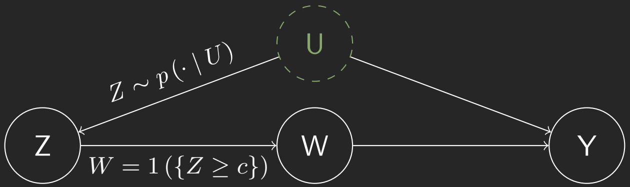

Assumption 3: Exogeneity

$$[\{Y_i(0), Y_i(1)\} \perp Z_i] \mid U_i$$

which implies $\mathbb{E}[Y_i \mid U_i, Z_i] = \alpha_{(W_i)}(u)$

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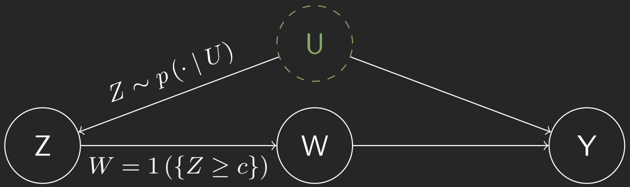

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which implies $\mathbb{E}[Y_i \mid U_i, Z_i] = \alpha_{(W_i)}(u)$, where $\alpha_{(w)}(u) = \mathbb{E}[Y_i(w) \mid U_i = u]$ is the **response functions** for the potential oucomes conditional on the latent variable u

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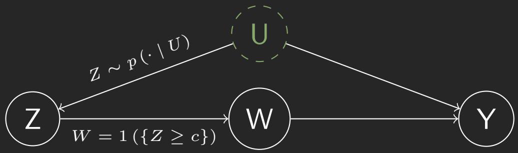

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A2 **Noisy** Z_i: $Z_i \mid U_i \sim p(\cdot \mid U_i)$

A3 **Exogeneity**:
$[\{Y_i(0), Y_i(1)\} \perp Z_i] \mid U_i$

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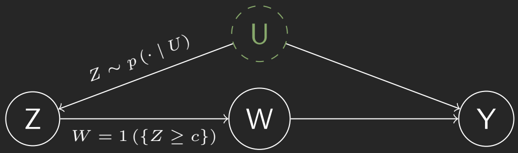

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Proposition 1

Let $\gamma_+(\cdot), \gamma_-(\cdot)$ be measurable functions of Z, then under A1-A3:

$$\mathbb{E}[\gamma_+(Z) Y] = \mathbb{E}\left[\alpha_{(1)}(U) h(U, \gamma_+)\right], \qquad \mathbb{E}[\gamma_-(Z) Y] = \mathbb{E}\left[\alpha_{(0)}(U) h(U, \gamma_-)\right]$$

where $h(u, \gamma) := \int \gamma(z) p(z \mid u) \, \mathrm{d}\lambda(z)$

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To achieve balance in the **latent variable**: $h(\cdot, \gamma_+) \approx h(\cdot, \gamma_-)$

References I

Eckles, D., Ignatiadis, N., Wager, S., & Wu, H. (2020). Noise-induced randomization in regression discontinuity designs. *arXiv preprint arXiv:2004.09458.*

Thank you!