# Noise-Induced Randomization in Regression Discontinuity Designs

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# Outline

# Discussion

# Literature: Continuity-Based RD

Most popular: local linear regression (Hahn et al., 2001; G. W. Imbens and Lemieux, 2008)

$$\hat{\tau}_c = \arg\min_{\tau} \left\{ \sum_{i=1}^{n} \underbrace{K}_{\text{weighting}} \left( \frac{|Z_i - c|}{\underbrace{h_n}_{\text{bandwidth}}} \right) \left( Y_i - a - \tau W_i - \beta_- (Z_i - c)_- - \beta_+ (Z_i - c)_+ \right)^2 \right\}$$

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**Optimization-based RD**: the treatment effect $\tau_c$ can be estimated (minimax linear estimation) via **numerical convex optimization** (Armstrong and Kolesár, 2018; G. Imbens and Wager, 2019)

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Posit a non-trivial interval $\mathcal{I}$ with $c \in \mathcal{I}$ s.t.

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- Design-based approach (Rubin, 2008)
- Strong assumption
  No **data-driven way** of choosing $\mathcal{I}$
  If the interval $\mathcal{I}$ is known a-priori, the problem collapses to a **RCT**

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- **NOT** assuming prior knowledge of the noise distribution $p(\cdot \mid u)$
- A stronger assumption: observing at least 3 noisy measurements of the latent variable $U_i$, $\{Z_i, Z_i', Z_i''\}$
  - $(U_i, Z_i, Z_i', Z_i'')$ is **joint normal**
  - $\alpha_{(w)}(u) = \mathbb{E}\left[Y_i(w) \mid U_i = uu\right]$ is **linear** w.r.t. $u$

# RD with Ordinal Running Variables

Similarly, ordinal $Z_i$ (bond rating, custody security score, etc.) can be seen as a noisy measurement of a latent variable $U_i$.
Li et al. (2021) assume

$$U_i = \mathbf{X}_i \beta$$

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then use **inverse-propensity weighting** with estimated propensities $e(u) = \mathbb{P}\left[Z_i \geq c \mid U_i = u\right]$ for inference.
**Assuming**: $U_i$ can be observed, and well predicted by $\mathbf{X}_i$

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Thank you!