Noise-Induced Randomization in Regression Discontinuity Designs

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Outline

Applications

Design Estimators

The goal: Make the confidence intervals **shorter**

$$\hat{\tau}_\gamma \pm l_\alpha, \qquad l_\alpha = \min\left\{l : \mathbf{P}\left[\left|b + n^{-\frac{1}{2}} \hat{V}_\gamma^{\frac{1}{2}} \tilde{Z}\right| \le l\right] \ge 1 - \alpha, \forall\, |b| \le \hat{B}_{\gamma,M}\right\}$$

by minimizing the worst-case MSE of

$$\hat{\tau} = \hat{\mu}_{\gamma,+} - \hat{\mu}_{\gamma,-} = \frac{\sum_i \gamma_+(Z_i) Y_i}{\sum_i \gamma_+(Z_i)} - \frac{\sum_i \gamma_-(Z_i) Y_i}{\sum_i \gamma_-(Z_i)}$$

Design Estimators: Quadratic Programming

Solve

$$\min_{\gamma_\pm(\cdot)} \frac{1}{n}\left(\int \gamma_-^2(z)\,\mathrm{d}\bar{F}(z) + \int \gamma_+^2(z)\,\mathrm{d}\bar{F}(z)\right) + (t_1+t_2)^2$$

s.t.

$$\begin{aligned}
\left|h(u,\gamma_+) - h(u,\gamma_-)\right| &\le t_1, && \forall u \\
M\left|h(u,\gamma_\diamond) - \bar{w}(u)\right| &\le t_2, && \forall u, \diamond \in \{\pm\}
\end{aligned}$$

$$\int \gamma_+(z)\,\mathrm{d}\bar{F}(z) = \int \gamma_-(z)\,\mathrm{d}\bar{F}(z) = 1$$

$$\begin{aligned}
\gamma_-(z) &= 0, && z \ge c \\
\gamma_+(z) &= 0, && z < c \\
\left|\gamma_\diamond(z)\right| &\le Cn^\beta, && \forall z, \diamond \in \{\pm\}
\end{aligned}$$

Design Estimators: Quadratic Programming

Solve

$$\min_{\gamma_{\pm}(\cdot)} \frac{1}{n}\left(\int \gamma_{-}^{2}(z)\, \mathrm{d}\bar{F}(z) + \int \gamma_{+}^{2}(z)\, \mathrm{d}\bar{F}(z)\right) + (t_1 + t_2)^2$$

s.t.

$$\begin{aligned}
|h(u,\gamma_{+}) - h(u,\gamma_{-})| &\leq t_1, & \forall u \\
M\,|h(u,\gamma_{\diamond}) - \bar{w}(u)| &\leq t_2, & \forall u, \diamond \in \{\pm\}
\end{aligned}$$

$$\int \gamma_{+}(z)\, \mathrm{d}\bar{F}(z) = \int \gamma_{-}(z)\, \mathrm{d}\bar{F}(z) = 1$$

$$\begin{aligned}
\gamma_{-}(z) &= 0, & z \geq c \\
\gamma_{+}(z) &= 0, & z < c \\
|\gamma_{\diamond}(z)| &\leq Cn^{\beta}, & \forall z, \diamond \in \{\pm\}
\end{aligned}$$

Design Estimators: Quadratic Programming

Solve

$$\min_{\gamma_\pm(\cdot)} \frac{1}{n}\left(\int \gamma_-^2(z)\,\mathrm{d}\bar{F}(z) + \int \gamma_+^2(z)\,\mathrm{d}\bar{F}(z)\right) + (t_1+t_2)^2$$

s.t.

$$\begin{aligned}
|h(u,\gamma_+) - h(u,\gamma_-)| &\le t_1, && \forall u && \text{confounding bias}\\
M\,|h(u,\gamma_\diamond) - \bar{w}(u)| &\le t_2, && \forall u, \diamond \in \{\pm\} && \text{CATE-hetrogeneity bias}
\end{aligned}$$

$$\int \gamma_+(z)\,\mathrm{d}\bar{F}(z) = \int \gamma_-(z)\,\mathrm{d}\bar{F}(z) = 1$$

$$\begin{aligned}
\gamma_-(z) &= 0, && z \ge c\\
\gamma_+(z) &= 0, && z < c\\
|\gamma_\diamond(z)| &\le Cn^\beta, && \forall z, \diamond \in \{\pm\}
\end{aligned}$$

Design Estimators: Quadratic Programming

Solve

$$\min_{\gamma_{\pm}(\cdot)} \frac{1}{n}\left(\int \gamma_{-}^{2}(z)\,\mathrm{d}\bar{F}(z) + \int \gamma_{+}^{2}(z)\,\mathrm{d}\bar{F}(z)\right) + (t_1 + t_2)^2$$

s.t.

$$\begin{aligned}
|h(u,\gamma_{+}) - h(u,\gamma_{-})| &\leq t_1, & \forall u & \quad \text{confounding bias} \\
M\,|h(u,\gamma_{\diamond}) - \bar{w}(u)| &\leq t_2, & \forall u, \diamond \in \{\pm\} & \quad \text{CATE-hetrogeneity bias} \\
\int \gamma_{+}(z)\,\mathrm{d}\bar{F}(z) = \int \gamma_{-}(z)\,\mathrm{d}\bar{F}(z) &= 1 & & \quad \text{normalization constraint} \\
\gamma_{-}(z) &= 0, & z \geq c & \quad \text{Sharp RD} \\
\gamma_{+}(z) &= 0, & z < c & \\
|\gamma_{\diamond}(z)| &\leq C n^{\beta}, & \forall z, \diamond \in \{\pm\} &
\end{aligned}$$

Design Estimators: Quadratic Programming

Solve

$$\min_{\gamma_\pm(\cdot)} \frac{1}{n}\left(\int \gamma_-^2(z)\,\mathrm{d}\bar{F}(z) + \int \gamma_+^2(z)\,\mathrm{d}\bar{F}(z)\right) + (t_1+t_2)^2$$

s.t.

$$
\begin{aligned}
|h(u,\gamma_+) - h(u,\gamma_-)| &\le t_1, & \forall u \quad & \text{confounding bias}\\
M\,|h(u,\gamma_\diamond) - \bar{w}(u)| &\le t_2, & \forall u, \diamond \in \{\pm\} \quad & \text{CATE-hetrogeneity bias}\\
\int \gamma_+(z)\,\mathrm{d}\bar{F}(z) = \int \gamma_-(z)\,\mathrm{d}\bar{F}(z) &= 1 & & \text{normalization constraint}\\
\gamma_-(z) &= 0, & z \ge c \quad & \text{Sharp RD}\\
\gamma_+(z) &= 0, & z < c \quad & \\
|\gamma_\diamond(z)| &\le Cn^\beta, & \forall z, \diamond \in \{\pm\} \quad & \text{no observation is given excessive influence}
\end{aligned}
$$

Design Estimators: Quadratic Programming

Solve

$$\min_{\gamma_{\pm}(\cdot)} \frac{1}{n} \left(\int \gamma_{-}^{2}(z)\, \mathrm{d}\bar{F}(z) + \int \gamma_{+}^{2}(z)\, \mathrm{d}\bar{F}(z) \right) + (t_1 + t_2)^2$$

s.t.

$$M \left| h(u, \gamma_{\diamond}) - \bar{w}(u) \right| \leq t_2, \qquad \forall u, \diamond \in \{\pm\} \qquad \text{CATE-hetrogeneity bias}$$

$$\int \gamma_{+}(z)\, \mathrm{d}\bar{F}(z) = \int \gamma_{-}(z)\, \mathrm{d}\bar{F}(z) = 1 \qquad \text{normalization constraint}$$

Design Estimators: Quadratic Programming

Solve

$$\min_{\gamma_\pm(\cdot)} \frac{1}{n}\left(\int \gamma_-^2(z)\,\mathrm{d}\bar{F}(z) + \int \gamma_+^2(z)\,\mathrm{d}\bar{F}(z)\right) + (t_1+t_2)^2$$

s.t.

$$M\left|h(u,\gamma_\diamond) - \bar{w}(u)\right| \le t_2, \qquad \forall u, \diamond \in \{\pm\} \qquad \text{CATE-hetrogeneity bias}$$

$$\int \gamma_+(z)\,\mathrm{d}\bar{F}(z) = \int \gamma_-(z)\,\mathrm{d}\bar{F}(z) = 1 \qquad \text{normalization constraint}$$

References I

Eckles, D., Ignatiadis, N., Wager, S., & Wu, H. (2020). Noise-induced randomization in regression discontinuity designs. *arXiv preprint arXiv:2004.09458.*

Thank you!