Noise-Induced Randomization in Regression Discontinuity Designs

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Outline

Discussion

Literature: Continuity-Based RD

Most popular: local linear regression (Hahn et al., 2001; G. W. Imbens and Lemieux, 2008)

$$\hat{\tau}_c = \arg\min_{\tau} \left\{ \sum_{i=1}^{n} \underbrace{K}_{\text{weighting}} \left(\frac{|Z_i - c|}{\underbrace{h_n}_{\text{bandwidth}}} \right) \left(Y_i - a - \tau W_i - \beta_- (Z_i - c)_- - \beta_+ (Z_i - c)_+ \right)^2 \right\}$$

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Robust CIs (Armstrong and Kolesár, 2020; Calonico et al., 2014; Kolesár and Rothe, 2018); Data-adaptive bandwidths (G. Imbens and Kalyanaraman, 2012)

Literature: Continuity-Based RD extended

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If further assume **convexity** of $\mu_{(w)}(z)$, e.g.:

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Optimization-based RD: the treatment effect τ_c can be estimated (minimax linear estimation) via **numerical convex optimization** (Armstrong and Kolesár, 2018; G. Imbens and Wager, 2019)

Literature: Randomization Inference RD

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Thank you!