SELF LEARNING MATERIAL
UNIT-7
Cubes

7.0 OBJECTIVES:

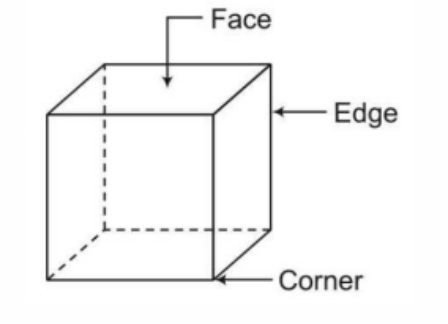
After reading this unit, you will be able to:

- Cutting of a Cube or Cuboid
- Counting of Blocks
- A Larger Cube/Cuboid is Painted and Cut

7.1 INTRODUCTION:

A Cube is a three-dimensional solid object bounded by six square faces or surfaces, it can also be called as 'regular hexahedron

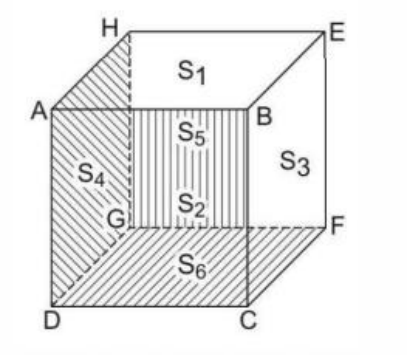
Cube is a three dimensional body having
- 6 surface/faces
- 8 corners
- 12 edges



In a cube, Length = Breadth = Height (L = B = H)

Surface/Faces

Every cube has six faces out of which a maximum of 3 faces are visible at a time.



In the given figure, ABCD, BCFE, ABEH, ADGH, EFGH and CDGF are the six faces of the figure

ABEH => 51
ABCD => 5_2
BCFE => S_3
} Visible faces

ADGH => S_4
EFGH => S_5
CDGF => S_6
} Invisible faces

ABCD => Front face => S_2
EFGH => Back face => S_5
} Faces opposite to each other

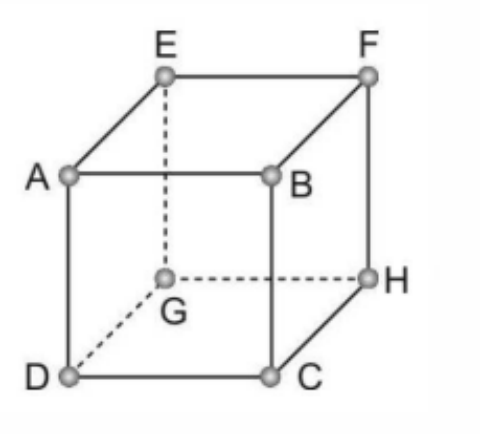
ABEH => Upper face => Si
CDGF => Lower face => S_6
} Faces opposite to each other

BCEF => Right face => S_3
ADGH => Left face => S 4
} Faces opposite to each other

From the above facts it is clear that
(i) Every face has four adjacent faces.
(ii) Adjacent faces are connected to each other.
(iii) Two adjacent faces meet at a single side and three face adjacent to each other meet at a single vertex.

Edges and Corners

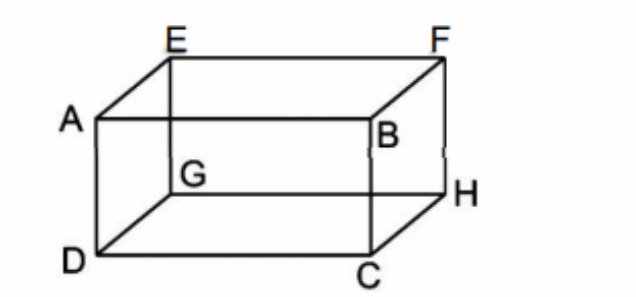
A cube consists of 12 edges and 8 corners. Maximum 9 edges and 7 corners are visible at a time and rest edges and corners are not visible.



Cuboid

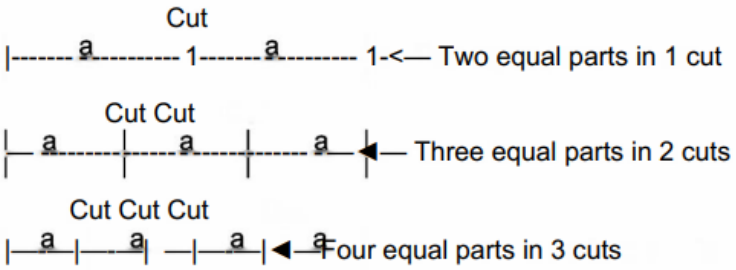
Like cube it is three-dimensional body which possesses all the properties of a cube except equality of dimensions.
It means a cuboid has

- 6 faces (3 are visible and 3 are hidden at a time)
- 8 corners (only 7 are visible at a time)
- 12 edges (only 9 are visible at a time)
- Length breadth and height not all same



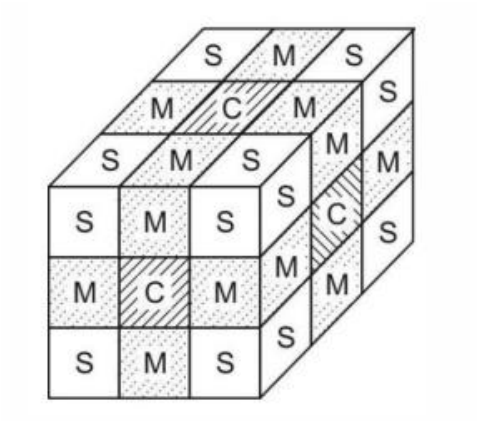
7.2 Cutting of a Cube or Cuboid

- When we cut a rod in two equal parts, then we cut it only once.
- Similarly, if a rod is divided into three equal parts, then 2 cuts are made.
- For 4 equal parts 3 cuts are made and so on.



For n equal parts, number of cuts = (n - 1)
Like a rod/stick, a cube can also be cut as per the rule given below
If each edge of a cube = 8 cm and it has to be cut into smaller cubes having each edge of 2 cm, then
Parts of each edge (n) = g/2 = 4 and to divide into four parts, the cube will have to be cut into (n - 1) = (4 - 1) = 3 times from three sides.

Identification of smaller cubes when a larger cube is cut into smaller cubes



After cutting a cube, following type of smaller cubes are obtained
• Corner cubes = S (exist at each corner)
• Middle cubes = M (exist at the middle of each edge)
• Central cubes = C (exist at the middle of each face)
• Nuclear cube/Inner central cube = N (hidden and exist at the centre of the larger cube)

Finding number of cubes when a larger cube is cut into smaller cubes

If a larger cube is cut into smaller cubes of equal volume so that each edge is divided into n parts, then

Number of smaller cubes so obtained = $(n)^3$

where, $n = \dfrac{\text{Edge of larger cube}}{\text{Edge of smaller cube}}$

- Number of inner central/nucleus cubes (N)= $(n - 2)^3$
- Number of central cubes (C) = $6\,(n - 2)^2$
- Number of middle cubes (M) = $12\,(n - 2)$
- Number of corner cubes (S) = 8

Example 7.1:

A cube has each side 6 cm. In how many smaller cubes of each side 1 cm can it be divided?

(a) 36 (b) 12 (c) 18 (d) 216

Solution:

Each side of bigger cube will be divided into 6 parts

Required number of smaller cubes = (n)^3 = 6^3=216

CHECK YOUR PROGRESS 7.1:

If a cube of 12 cm side is divided into smaller cubes of 3 cm side, then

(i)find the total number of smaller cubes.
(a) 16 (b) 64 (c) 128 (d) 32

(ii)find the total number of corner (vertex) cubes.
(a) 16 (b) 12 (c) 8 (d) 4

(iii) what is the total number of middle cubes?
(a) 8 (b) 16 (c) 24 (d) 32

(iv)what is the total number of central cubes?
(a) 45 (b) 9 (c) 15 (d) 24

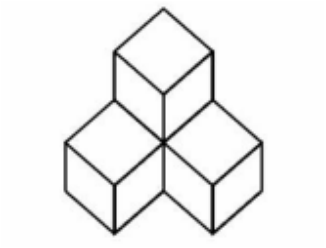
(v)find the total number of inner central cubes?
(a) 18 (b) 9 (c) 8 (d) 81

 # # Answers are given at the end of the document.

7.3 Counting of Blocks

When the number of cubes (or blocks/cuboids) in a figure needs to be counted, the procedure is described with the help of the following illustrations.

Example 7.2: Count the number of cubes in the given figure.



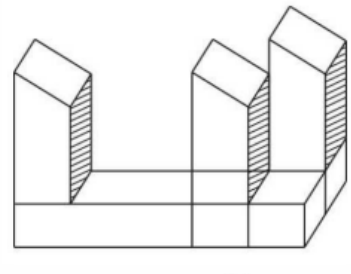
(a) 3 (b) 4 (c) 5 (d) 6

Solution: It is clear from the figure, 1 column contains 2 cubes and 2 columns contain 1 cube each.

Total number of cubes = (1 x 2) + (2 x 1) = 2 + 2 = 4

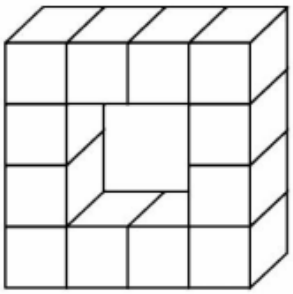
CHECK YOUR PROGRESS 7.2:

1. Count the number of blocks in the given figure



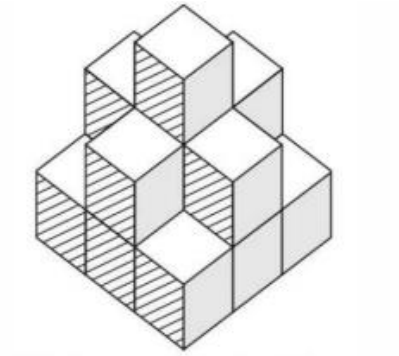
(a) 6 (b) 7 (c) 8 (d) 9

2. Count the number of blocks in the given figure



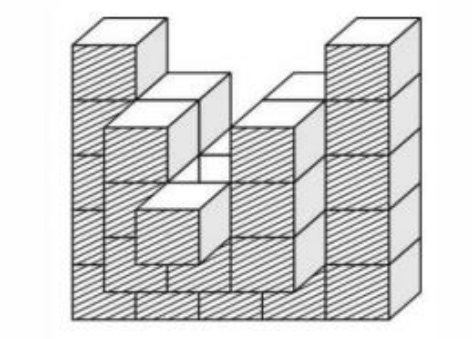
(a) 16 (b) 12 (c) 10 (d) 8

3. Count the number of blocks in the given figure



(a) 8 (b) 9 (c) 12 (d) 15

4. Count the number of blocks in the given figure



(a) 25 (b) 30 (c) 35 (d) 40

 # # Answers are given at the end of the document.

7.4 A Larger Cube/Cuboid is Painted and Cut

In different exams, questions on painted cubes/cuboid are asked. In such questions, a larger cube/cuboid is painted with one, two, three, four, five or maximum six different colours. This larger cube/cuboid is then cut into smaller cubes of same or different dimensions and it is asked to determine the number of smaller cubes with one or more surfaces painted (with same or different colours).

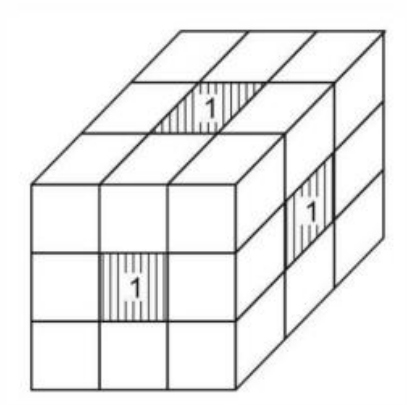
1. Larger Cube Painted with a Single Colour

In this type, a larger cube is painted with single colour and then cuts are made to form smaller cubes.

I. Smaller cubes with one painted face

Such cubes are central cubes and they are neither attached with any edges nor corners.

Let us see the given picture in which digit 1 represents such cubes. As digit 1 exits only once in each surface and there are 6 surfaces in a cube, hence number of smaller cubes having one surface painted must be 6. If a cube is made up of n smaller cubes, then

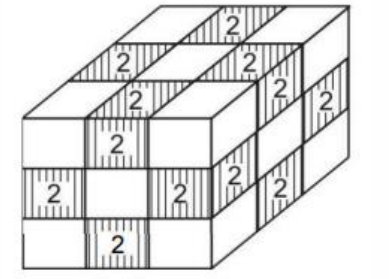


Number of smaller cubes having one face painted on each surface layer of cube = (n - 2)^2

As a cube has 6 surfaces, therefore number of such smaller cubes = 6(n - 2)^2

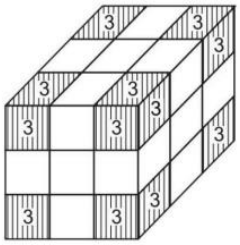
II. Smaller cubes with two painted faces

Such cubes are middle cubes and they are attached with edges (sides) as shown in the given figure by digit 2. As a cube has 12 edges, hence number of smaller cubes having 2 surfaces painted in a larger cube = 12(n - 2)



III. Smaller cubes with three painted surfaces

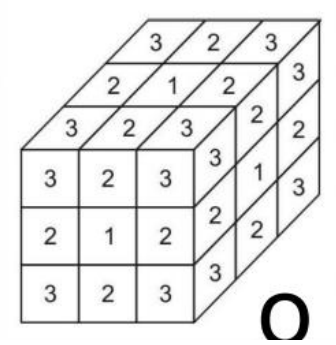
Such cubes are corner cubes which have been represented by digit 3 in the given figure. As a cube has 8 corners, hence number of smaller cubes having 3 faces painted is always 8.



Number of smaller cubes having three surface painted = 8

IV. Smaller cubes with no painted surfaces

Such cubes are inner central or nucleus cubes and they are invisible as shown in the figure. As we know, number of inner central or nucleus cubes = (n - 2)^3



Example 7.3:

A cube of side 4 cm is painted black on all of its surfaces and then divided into various smaller cubes of side 1 cm each. The smaller cubes so obtained are separated.

Solution:

Total number of cubes so obtained = 64

(i) Number of smaller cubes with three surfaces painted = 8

(ii) Number of smaller cubes with two surfaces painted = (n - 2) x 12

= (4 - 2) x 12 = 24

(iii) Number of smaller cubes with one surface painted = (n - 2)^2 x 6

= (4 - 2)^2 x 6

= 4 x 6 = 24

(iv) Number of smaller cubes with no surface painted = (n - 2)^3

= (4 - 2)^3 = 8

CHECK YOUR PROGRESS 7.3:

All the surfaces of a cube of 15 cm side are painted with red colour and then it is cut into smaller cubes of 3 cm side. Then,

(i) How many smaller cubes are there having only one surface painted with red colour?

(a) 18 (b) 24 (c) 36 (d) 54

(ii) How many smaller cubes are there having two surface painted with red colour?

(a) 8 (b) 24 (c) 36 (d) 54

(iii)How many smaller cubes are there having only three surface painted with red colour?

(a) 8 (b) 24 (c) 36 (d) 54

(iv) How many smaller cubes are there having 4 or more faces painted with red colour?

(a) 0 (b) 8 (c) 36 (d) 81

(v) How many smaller cubes are there having no surfaces painted with red colour?

(a) 3 (b) 9 (c) 27 (d) 81

 # # Answers are given at the end of the document.

2. When a Larger Cube is Painted with More Than One Colour

In this type, a larger cube is painted with more than one colour and then cuts are made to form smaller cubes.

I. Smaller cubes with one painted face

When it is asked to determine the number of smaller cubes with one face painted with a particular colour, then first and foremost task is to find the number of faces of larger cube on which that particular colour is used. If this particular colour is L, then Number of smaller cubes with one surface painted = (n - 2)^2 x Number of faces painted with colour L

II. Smaller cubes with two painted faces

Such cubes are related with the edges. If larger cube is painted with more than one colour, then such cubes are obtained as below.

(i) Smaller cubes having same colour on both the faces

Such cubes are obtained when two faces of the larger cube having the same colour have an edge in common. Hence, when the number of cubes with two faces painted with a particular colour is asked, then first and foremost task is to find the number of edges to which two faces (painted with a particular colour) of the larger cube are attached with.

Number of smaller cubes having both the faces painted with a particular colour (x) = (n - 2) x Number of edges to which colour x is attached.

(ii) Smaller cubes having different colours on both the faces

Number of smaller cubes having two faces painted with a colour (P) on one surface and another colour (Q) on another surface = (n - 2) x Number of edges to which colours P and Q are attached together.

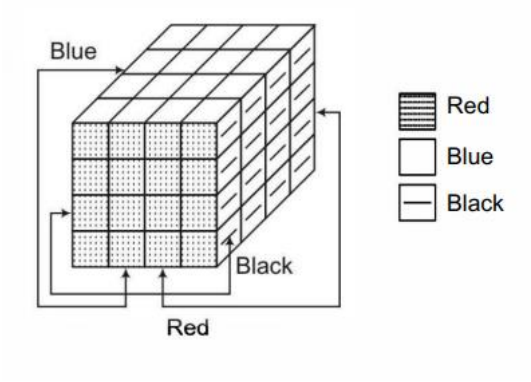
(III) Smaller cubes with three painted faces

Such smaller cubes are always 8 in number. In this case three situations arise

(i) If all the three surfaces have same colour.
(ii) If all the three surfaces have different colours.
(III)If one surface has one colour and two other surfaces have another colour.

Example 7.4: A cube of side 4 cm is painted black on one pair of opposite surfaces, blue on another pair of opposite surfaces and red on the remaining pair of opposite surfaces. The cube is now divided into smaller cubes of equal side of 1 cm each.

Solution:



(i) Number of smaller cubes with three surfaces painted = 8 (these smaller cubes will have all three surfaces painted with different colours—blue, black and red).

(ii) Number of smaller cubes with two surfaces painted = 24

(a) Number of cubes with two surfaces painted with black and blue colours = 8
(b) Number of cubes with two surfaces painted with blue and red colours = 8
(c) Number of cubes with two surfaces painted with black and red colours = 8

(iii) Number of smaller cubes with one surface painted = 24 and out of these,

(a) Number of cubes with one surface painted with black colour = 8
(b) Number of cubes with one surface painted with blue colour = 8
(c) Number of cubes with one surface painted with red colour = 8

CHECK YOUR PROGRESS 7.4:

1. A solid cube has been formed with 64 smaller cubes. How many smaller cubes are completely invisible?

(a) 2 (b) 4 (c) 6 (d) 8

2. A cube has to be painted in such a way that there is no

similar colour on adjacent faces. How many colours are needed for this?

(a) 3 (b) 4 (c) 6 (d) 2

3. All the faces of a cube are painted in blue and then it is divided into 27 smaller cubes. How many smaller cubes have only one face painted?

(a) 0 (b) 6 (c) 8 (d) 18

4. A bigger cube of 3 inch side is formed by keeping together smaller cubes of 1 inch side. All the faces of this bigger cube is painted red. When the bigger cube is divided into original smaller cubes, then how many smaller cubes have two faces painted?

(a) 4 (b) 8 (c) 12 (d) 0

5. All the faces of a cube having side 5 cm are painted green and then it is divided into smaller cubes of 1 cm side. How many smaller cubes have three faces painted with green?

(a) 4 (b) 8 (c) 12 (d) 24

SUMMARY:

- Cutting of a Cube or Cuboid is explained
- Counting of Blocks is explained
- A Larger Cube/Cuboid is Painted and Cut is explained

7.5 Glossary:

Cuboid : a solid which has six rectangular faces at right angles to each other

7.6 Suggested Readings:

- Quantitative Aptitude for Competitive Examinations by R.S.Agarwal. Published by S. CHAND
- Study material for CAT, SAT, GRE, GMAT by TIME, CareerLauncher and IMS etc.
- Quantitative Aptitude by Pearson Publications

7.7 Practice exercise:

1. A cube has six faces each of a different colour. The red face is opposite to black. The green face is in between red and black. The blue face is adjacent to white and brown face is adjacent to blue. The four colours adjacent to green are

(a)red, black, brown and white
(b) red, black, brown and blue
(c) red, black, blue and white
(d) red, brown, blue and white

2. Wooden little cubes each with an edge of one inch are put togther to form a solid cube with an edge of three inches. This big cube is then painted red all over the outside. When the big cube is broken-up into the original little ones, how many cubes will be without paint?

(a) 0 (b) 1 (c) 3 (d) 4

3. In a solid cube made up of 27 small cubes, two opposite sides are painted red, two opposite sides yellow and two other sides while. How many small cubes have the colours yellow and white along in them?

(a) 4 (b) 8 (c) 12 (d) 16

4. A solid cube of 4 inches has been painted red, green, and black on pair of opposite faces. It has been cut into one inch cubes. How many cubes have only one face painted that too only red?

(a) 4 (b) 8 (c) 18 (d) 24

5. A solid red coloured cube is painted yellow on all sides.The cube is cut into 125 equal cubes. How many cubes will have 3 sides yellow?

(a) 10 (b) 4 (c) 8 (d) 12

Directions (Q. Nos. 6-10) Read the following information carefully and answer the questions given below. A cube of 4 cm has been painted on its surfaces in such a way that two opposite surfaces have been painted blue and two adjacent surfaces have been painted red. Two remaining surfaces have been left unpainted. Now, the cube is cut into smaller cubes of side 1 cm each.

6. How many cubes will have no side painted?

(a) 18 (b) 16 (c) 22 (d) 8

7. How many cubes will have atleast red colour on its surfaces?

(a) 20 (b) 22 (c) 28 (d) 32

8. How many cubes will have atleast blue colour on its surfaces?

(a) 20 (b) 8 (c) 24 (d) 32

9. How many cubes will have only two surfaces painted with red and blue colours, respectively?

(a) 8 (b) 12 (c) 24 (d) 30

10. How many cubes have three surface coloured?

(a) 3 (b) 4 (c) 2 (d) 16

Directions (Q. Nos. 11-20) Read the following information carefully and answer the questions given below. Two adjacent faces of a solid cube have been painted red and the faces just opposite to the red painted faces have been

painted black while the remaining faces have been painted green. After painting this cube has been divided into 64 smaller cubes.

11. How many cubes have only one face painted?

(a) 12 (b) 16 (c) 20 (d) 24

12. How many cubes have only two faces painted?

(a) 20 (b) 24 (c) 32 (d) 48

13. How many cubes have three surface painted?

(a) 4 (b) 6 (c) 8 (d) 12

14. How many cubes have four faces painted?

(a) 0 (b) 2 (c) 8 (d) 12

15. How many cubes have no faces painted?

(a) 4 (b) 6 (c) 8 (d) 12

16. How many cubes have atleast one face painted in red?

(a) 20 (b) 16 (c) 28 (d) 32

17. How many cubes have one or two faces painted but they have no three faces painted?

(a) 16 (b) 32 (c) 48 (d) 6

18. How many cubes have one face painted in green and have black or red colour on the face just adjacent to green face?

(a) 12 (b) 16 (c) 20 (d) 24

19. How many cubes have two adjacent faces painted in red or black?

(a) 2 (b) 4 (c) 6 (d) 8

20. How many cubes have black colour at the face which is just opposite to the face having red colour?

(a) 0 (b) 8 (c) 12 (d) 24

ANSWERS:

CHECK YOUR PROGRESS 7.1

(i) b
(ii) c
(iii) c
(iv) d
(v) c

CHECK YOUR PROGRESS 7.2

1. b
2. b
3. d
4. b

CHECK YOUR PROGRESS 7.3

(i) d
(ii) c
(iii) a
(iv) a
(v) c

CHECK YOUR PROGRESS 7.4

1. d
2. a

3. b
4. c
5. b

Practice exercise 7.7

1.a
2.b
3. a
4. b
5. c
6. a
7. c
8. d
9. b
10.c
11.c
12.b
13.c
14.a
15.c
16.c
17.c
18. b
19.b
20.a