

Matrix Project On Coordinate Geometry

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Index

Problem Statement

Steps to solve the problem

Solution

Figures

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P and **Q** are two distinct points on the parabola

$$\mathbf{x}^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x} - \begin{bmatrix} 4 \\ 0 \end{bmatrix} \mathbf{x} = 0$$

with parameters t and t_1 respectively. If the normal at **P** passes through **Q**, then find the minimum value of t_1^2 .

Steps to solve the problem

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- Write eqn of Normal using slope and parametric Point
- Minimise as a function of parameter of point P

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- $a = 1$

Solution Contd..

- Parametric form of points of parabola:

$$\begin{bmatrix} t^2 \\ 2t \end{bmatrix}$$

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- Differentiate the equation and substitute $\frac{dy}{dx} = m$ (the slope of the tangent)
- $\frac{dx}{dx} = \begin{bmatrix} 1 & m \end{bmatrix}^T$

Solution Contd..

-

$$\begin{bmatrix} 1 \\ m \end{bmatrix}^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x} + \mathbf{x}^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ m \end{bmatrix} - \begin{bmatrix} 4 \\ 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ m \end{bmatrix} = 0$$

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- Now, use the parametric form of \mathbf{x} (i.e.)

$$\mathbf{x} = \begin{bmatrix} t^2 \\ 2t \end{bmatrix}$$

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$$\begin{bmatrix} 1 & m \end{bmatrix} \begin{bmatrix} 0 \\ 2t \end{bmatrix} + \begin{bmatrix} t^2 & 2t \end{bmatrix} \begin{bmatrix} 0 \\ m \end{bmatrix} - 4 = 0$$

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- $m = \frac{1}{t}$

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- Slope of normal at point **p**: $m_N = -t$
- We calculate the equation of the line through P with slope = m_N

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$$t_1 = -t - \frac{2}{t}$$

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- $t_1^2 \geq 4 + 4 = 8$

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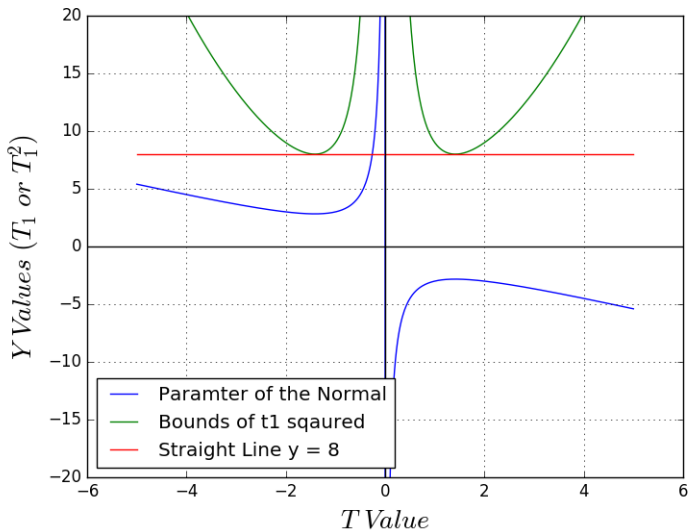
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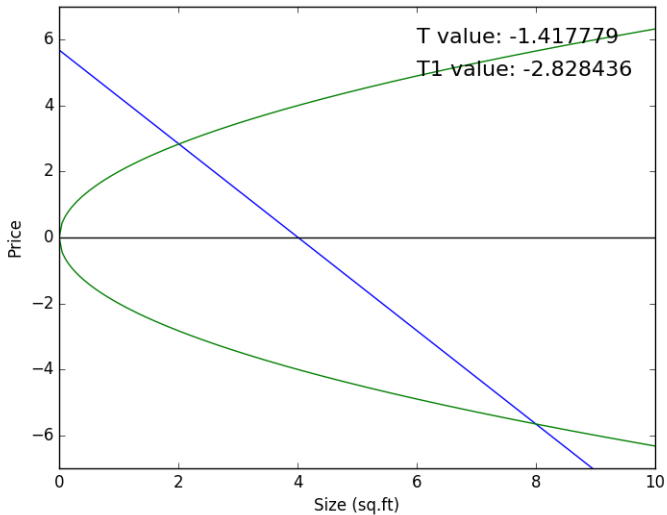
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- $t_1^2 \geq 4 + 4 = 8$
- \therefore The Minimum possible value of t_1^2 is 8.

Figures

Bounds of t_1 and t_1^2



Normal at minimum t_1^2



The End