Probability in Computing

Assignment - 4

- Sai Harsha K
CS17BTECH11036

1) The probability distribution vector after 4 time steps is

$[0.1938, 0.1772, 0.629]$

2) T_{ij} (expected time after i to j) $= j^2 - i^2$
$(i \le j)$

3) expected number of steps to reach is 10.

4) Stationary distribution for the given random walk is:
$\left[\frac{2}{7}, \frac{1}{7}, \frac{3}{14}, \frac{1}{7}, \frac{1}{7}, \frac{1}{14}\right]$

5a) We use constructive proof to show that there exists a colouring with two colours of graph s.t. no triangle is monochromatic.
Given a graph with 3-colouring using red, blue and green.
Replace all nodes' colour having blue with green. Now, in the graph, every ~~triangle~~ triangle previously had 3-coloured nodes but since one of them is changed, every triangle must have 2 colours (~~one~~ red which is unchanged and green)
There cant be a triangle with all green as that would imply red wasn't there previously and hence would violate 3 colouring.

Hence, proved.

5b) We can use Schöning's algorithm (variant).

1) Pick a random assignment $(x_1, x_2, \dots x_n) = (a_1, a_2, \dots a_n)$.
// x_i are nodes and $a_i \in \{R, B\}$.

2) Repeat $3n$ times, terminating if a satisfying assignment is found
- If there exists a monochromatic triangle, do:
- Pick a random vertex (uniformly) from the triangle, and flip the colour of node
- Re-evaluate the graph (check if ~~2-colourable~~ 2 colouring satisfies).

3) If assignment doesn't satisfy (there exists a monochromatic triangle), GOTO step 1 (restart)

Let $X_t = \binom{\text{num of}}{\text{nodes}}$ matching the current assignment, $(A_1, A_2, \dots A_n)$
where t is number of time steps.

Consider a random walk on $\{0, 1, \dots, n\}$ with X_t.
Need to calculate ~~expected~~ Expected time for $X_t = n$.

Consider a triangle with vertices (v_1, v_2, v_3).
Without loss of generality, let the colouring be $\{R, R, R\}$.
~~Let A_i be the correct assignment from the solution.~~
The current assignment can be of the form:
$\underbrace{(B, R, R), (R, B, R), (R, R, B)}$ or $\underbrace{(R, B, B), (B, R, B), (B, B, R)}$.

↳ If so with Prob $= \frac{1}{3}$, we get a monochromatic Δ & then flip a node to increase X_t value by 1

Similar to previous case but with prob $= \frac{2}{3}$ can increase X_t value by 1.

Hence, with prob = atleast $\frac{1}{3}$, X_t increases by 1 every time. This is similar to the case of Schönings algo.
Thus, using that algorithm ~~ex~~ result, the expected run time for $X_t = n$ is $O(1.33^n)$