Matrix Project On Coordinate Geometry

Sai Harsha Kottapalli and Abhishek Agarwal

February 14, 2019

Indian Institute of Technology Hyderabad

Index

Problem Statement

Steps to solve the problem

Solution

Problem Statement

Problem Statement

P and **Q** are two distinct points on the parabola

$$\mathbf{x}^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x} - \begin{bmatrix} 4 \\ 0 \end{bmatrix} \mathbf{x} = 0$$

with parameters t and t_1 respectively. If the normal at **P** passes through **Q**, then find the minimum value of t_1^2.

Steps to solve the problem

Solution

Solution

- Given equation:

$$\mathbf{x}^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x} - \begin{bmatrix} 4 \\ 0 \end{bmatrix} \mathbf{x} = 0$$

- Since, 3 entries of the 2x2 matrix is 0, the parabola is of the standard form (i.e.)

$$y^2 = 4ax$$

- Coefficient of x = 4a
- a = 1

Solution Contd..

- Parametric form of points of parabola:

$$\begin{bmatrix} t^2 \\ 2t \end{bmatrix}$$

- Differentiate the equation and substitute $\frac{dy}{dx} = \mathrm{m}$ (the slope of the tangent)
- $\frac{d\mathbf{x}}{dx} = \begin{bmatrix} 1 & m \end{bmatrix}^T$

Solution Contd..

- $$\begin{bmatrix} 1 \\ m \end{bmatrix}^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x} + \mathbf{x}^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ m \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ m \end{bmatrix} = 0$$
- Now, use the parametric form of $\mathbf{x}$ (i.e.)

$$\mathbf{x} = \begin{bmatrix} t^2 \\ 2t \end{bmatrix}$$

Solution Contd..

- $$\begin{bmatrix} 1 & m \end{bmatrix} \begin{bmatrix} 0 \\ 2t \end{bmatrix} + \begin{bmatrix} t^2 & 2t \end{bmatrix} \begin{bmatrix} 0 \\ m \end{bmatrix} + 4 = 0$$
- 2mt + 2mt + 4 = 0
- 4mt + 4 = 0
- m = -$\frac{1}{t}$

Solution Contd..

- We know that, Two lines are perpendicular if the product of their slopes is -1.
- Slope of normal at point **p**: $m_N =$ t
- We calculate the equation of the line through P with slope $= m_N$

Solution Contd..

- Note: $\mathbf{p} = (x_1, y_1) = (t^2, 2t)$
- $$\begin{bmatrix} -m & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -m & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$
- $$\begin{bmatrix} -t & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -t & 1 \end{bmatrix} \begin{bmatrix} t^2 \\ 2t \end{bmatrix}$$
- $$\begin{bmatrix} -t & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = -t^3 + 2t$$

Solution Contd..

- Now, Let Q: $(t_1^2, 2t_1)$
- Then, We have,

$$\begin{bmatrix} -t & 1 \end{bmatrix} \begin{bmatrix} t_1^2 \\ 2t_1 \end{bmatrix} = -t^3 + 2t$$

- $$2t_1 + tt_1^2 = t^3 + 2t$$

- $$-2(t - t_1) = t(t_1 + t)(t - t_1)$$

- $$t_1 = -t - \frac{2}{t}$$

Solution Contd..

- $$t_1^2 = t^2 + \frac{4}{t^2} + 4$$
- To solve above equation we use the property: Arithmetic Mean $\geq$ Geometric Mean on $t^2 + \frac{4}{t^2}$
- A.M $\geq$ G.M. $\implies t^2 + \frac{4}{t^2} \geq 2\sqrt{4} = 4$
- $t_1^2 \geq 4 + 4 = 8$
- $\therefore$ The Minimum possible value of t_1^2 is 8.

The End