# DOC-3

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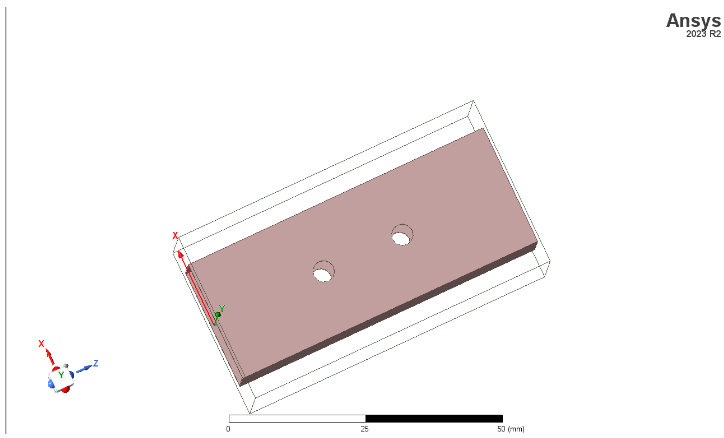


*WEEK3 fig1*

a=22.86mm,b=10.16mm,t=0.2mm,L=60mm,r=2mm,h= 10.56mm,$L_1 = 20mm, L_2 = 12mm$

## Conversion from S-parameters to ABCD-parameters

We begin with a 2-port network characterized by its scattering matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix},$$

with characteristic impedance $Z_0$.

### ABCD from S-parameters

For each frequency point, the ABCD parameters are obtained via the well-known relations:

$$A = \frac{(1+S_{11})(1-S_{22}) + S_{12}S_{21}}{2S_{21}},$$

$$B = Z_0 \frac{(1+S_{11})(1+S_{22}) - S_{12}S_{21}}{2S_{21}},$$

$$C = \frac{1}{Z_0} \frac{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}{2S_{21}},$$

$$D = \frac{(1-S_{11})(1+S_{22}) + S_{12}S_{21}}{2S_{21}}.$$

Thus, at each frequency, we obtain the transmission (ABCD) matrix

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}.$$

### Propagation Constant Extraction

For a transmission line of length $l$, the propagation constant $\gamma$ can be extracted from

$$A = \cosh(\gamma l).$$

Thus, given $A$ at the longer structure of length $l_{\text{long}}$, we compute

$$\gamma = \frac{1}{l_{\text{long}}} \operatorname{arccosh}(A).$$

## Characteristic Impedance

The characteristic impedance is then obtained from the $B$-parameter of the long line:

$$Z_c = \frac{B}{\sinh(\gamma l_{\text{long}})}.$$

## ABCD of a Shorter Section

For a line of shorter length $l_{\text{short}}$, the ABCD parameters can be reconstructed using

$$\begin{aligned}
A_{\text{short}} &= \cosh(\gamma l_{\text{short}}), \\
D_{\text{short}} &= A_{\text{short}}, \\
B_{\text{short}} &= Z_c \sinh(\gamma l_{\text{short}}), \\
C_{\text{short}} &= \frac{1}{Z_c} \sinh(\gamma l_{\text{short}}).
\end{aligned}$$

Let T $= \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, $\mathrm{T}_S = \begin{bmatrix} A_{short} & B_{short} \\ C_{short} & D_{short} \end{bmatrix}$,

If $A$ and $B$ are invertible (and we can take a square root of $B$):

Multiply on the left by $T^{-1}$ and on the right by $T^{-1}$:

$$XT_sX = N \quad \text{where } N := T^{-1}MT^{-1}.$$

Now set

$$Z := T_s^{1/2} X T_s^{1/2} \quad \text{so that} \quad X = T_s^{-1/2} Z T_s^{-1/2}.$$

Then

$$XT_sX = T_s^{-1/2} Z^2 T_s^{-1/2}.$$

Thus

$$Z^2 = T_s^{1/2} N T_s^{1/2}.$$

Pick any matrix square root

$$Z = \left(T_s^{1/2} N T_s^{1/2}\right)^{1/2},$$

and recover

$$X = T_s^{-1/2} \left(T_s^{1/2} T^{-1} M T^{-1} T_s^{1/2}\right)^{1/2} T_s^{-1/2}.$$

Notes:

- This gives one solution; in general the equation is quadratic in $X$, so multiple (or no) solutions may exist depending on $T, T_s, M$.
- If principal square roots exist (e.g. $T \succ 0$, $T_s \succ 0$, $M \succeq 0$), then the principal $X$ above is the symmetric PSD solution.
- Here $M$ = Matrix of total designed waveguide, $T$ = Matrix of first half of known length of the waveguide, $T_s$= Matrix of middle part of the waveguide, $X$ = Matrix of defect part

# Extraction of 12 mm ABCD from the 20 mm section (computed at 7 GHz)

Given the ABCD parameters of a non-uniform (long) section at frequency $f = 7$ GHz, we denote the long-section ABCD entries by $A_{\text{long}}, B_{\text{long}}, C_{\text{long}}, D_{\text{long}}$. For the 20 mm section (denoted with subscript 20) the values at 7 GHz are:

$$A_{20} = -3.4557829029938545 + 0.867472783988988\,\mathrm{j},$$

$$B_{20} = -1263.5135908447594 + 1041.6935147605195\,\mathrm{j},$$

$$C_{20} = -0.007155426556438465 - 0.0011446833803803918\,\mathrm{j},$$

$$D_{20} = -3.4684231482364623 + 0.8677268886920003\,\mathrm{j}.$$

We treat the long section as (locally) a transmission-line segment characterized by a complex propagation constant $\gamma$ and characteristic impedance $Z_c$. For a uniform line of length $l$ the ABCD entries satisfy

$$A = \cosh(\gamma l), \qquad B = Z_c \sinh(\gamma l), \qquad C = \frac{1}{Z_c}\sinh(\gamma l), \qquad D = \cosh(\gamma l).$$

## Step 1: extract $\gamma$

From $A_{20} = \cosh(\gamma\, l_{\text{long}})$ we obtain

$$\gamma \;=\; \frac{1}{l_{\text{long}}}\operatorname{arccosh}\big(A_{20}\big).$$

Using $l_{\text{long}} = 20$ mm and the numeric $A_{20}$ above yields

$$\gamma \;=\; 0.09730334030136867 + 0.14429210548883942\,\mathrm{j} \quad \text{(per mm)}.$$

## Step 2: compute characteristic impedance $Z_c$

Compute $\sinh(\gamma l_{\text{long}})$ and then

$$Z_c \;=\; \frac{B_{20}}{\sinh(\gamma l_{\text{long}})}.$$

Numerically this gives

$$Z_c \;=\; 434.1651291723434 - 195.73803396441718\,\mathrm{j} \quad \Omega.$$

## Step 3: synthesize 12 mm ABCD

For the short length $l_{\text{short}} = 12$ mm we use

$$A_{12} = \cosh(\gamma\, l_{\text{short}}),$$

$$B_{12} = Z_c\, \sinh(\gamma\, l_{\text{short}}),$$

$$C_{12} = \frac{1}{Z_c}\, \sinh(\gamma\, l_{\text{short}}),$$

$$D_{12} = A_{12}.$$

Substituting the computed $\gamma$ and $Z_c$ yields (numerically):

$$A_{12} = -0.2820716758413925 + 1.4329430430868093\,\mathrm{j},$$

$$B_{12} = 239.73870101614557 + 800.9302863615875\,\mathrm{j},$$

$$C_{12} = -0.001946289084536531 + 0.003130310540358743\,\mathrm{j},$$

$$D_{12} = -0.2820716758413925 + 1.4329430430868093\,\mathrm{j}.$$

## Remarks

- The computed $A_{12}, B_{12}, C_{12}, D_{12}$ match previously obtained values for the 12 mm section at 7 GHz, showing that the short-section ABCD can be synthesized from the long-section ABCD by extracting $\gamma$ and $Z_c$ and re-evaluating the transmission-line expressions.
- All complex arithmetic above is performed in SI units with lengths in mm; the propagation constant is given per mm. If you prefer per meter, multiply $\gamma$ by 1000.

At frequency $f = 7.000$ GHz, the following matrices are given:

$$A_{12} = \begin{bmatrix} -0.2820717 + 1.4329430j & 239.7387010 + 800.9302864j \\ -0.0019463 + 0.0031303j & -0.2820717 + 1.4329430j \end{bmatrix}$$

$$A_{20} = \begin{bmatrix} -3.4557829 + 0.8674728j & -1263.5135908 + 1041.6935148j \\ -0.0071554 - 0.0011447j & -3.4684231 + 0.8677269j \end{bmatrix}$$

The defect matrix obtained is

$$D = \begin{bmatrix} -6.4383558 + 4.0286366j & 1872.553551 - 2990.336240j \\ 0.0160460 - 0.0019935j & -6.4119568 + 4.0203491j \end{bmatrix}$$

We want to compute:

$$M_{\text{total}} = A_{20} \cdot D \cdot A_{12} \cdot D \cdot A_{20}$$

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## Step 1: Multiply $A_{20}$ and $D$

$$M_1 = A_{20} \cdot D$$

## Step 2: Multiply by $A_{12}$

$$M_2 = M_1 \cdot A_{12}$$

## Step 3: Multiply by $D$

$$M_3 = M_2 \cdot D$$

## Step 4: Multiply by $A_{20}$

$$M_{\text{total}} = M_3 \cdot A_{20}$$

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## Final Result

After carrying out the matrix multiplications numerically (complex arithmetic), we obtain:

$$M_{\text{total}} = \begin{bmatrix} -0.687351971 + 0.024208933\,\mathrm{i} & -3.09231628 + 123.99621598\,\mathrm{i} \\ -0.0001614926 + 0.0042705521\,\mathrm{i} & -0.686042058 + 0.024183463\,\mathrm{i} \end{bmatrix}$$