ECE-GY 6143 / Summer 2020

Homework 1

Please scan and upload your assignments on or before June 19, 2020.

- You are encouraged to discuss ideas with each other; but
- you **must acknowledge** your collaborator, and
- you **must compose your own** writeup and/or code independently.
- We **strongly** encourage answers to theory questions in LaTeX, and answers to coding questions in Python (Jupyter notebooks)

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1. **(5 points)** Let $\{x_1, x_2, \ldots, x_n\}$ be a set of points in d-dimensional space. Suppose we wish to produce a single point estimate $\mu \in \mathbb{R}^d$ that minimizes the mean squared-error:

$$\frac{1}{n}\left(\|x_1 - \mu\|_2^2 + \|x_2 - \mu\|_2^2 + \ldots + \|x_n - \mu\|_2^2\right)$$

   Find a closed form expression for μ and prove that your answer is correct.

2. **(10 points)** Not all norms behave the same; for instance, the ℓ_1-norm of a vector can be dramatically different from the ℓ_2-norm, especially in high dimensions. Prove the following norm inequalities for d-dimensional vectors, starting from the definitions provided in class and lecture notes. (Use any algebraic technique/result you like, as long as you cite it.)

   a. $\|x\|_\infty \leq \|x\|_2 \leq \sqrt{d}\|x\|_\infty$

   b. $\|x\|_\infty \leq \|x\|_1 \leq d\|x\|_\infty$

3. **(10 points)** In this problem, you will practice using Python for exploratory data analysis.

   - Open the Colab notebook from the following URL: click here
   - Use File > Save a Copy in Drive to create an editable copy in your own Google Drive
   - As you work through the notebook, look for the text and code cells marked with a **TODO** at the top. In these cells, answer the questions or fill in code as indicated.
   - When you are finished, run the entire notebook from beginning to end, using Runtime > Run All. Check the output and make sure everything looks OK.
   - Then, you can save your notebook as a PDF (to upload along with the rest of your submission) using File > Print.