

Roll No.: _____

Amrita Vishwa Vidyapeetham
Amrita School of Computing, Coimbatore
B.Tech First Assessment Examinations – October 2022
Fifth Semester
Computer Science and Engineering
19CSE302 Design and Analysis of Algorithms

Duration: Two hours

Maximum: 50 Marks

CO	Course Outcomes
CO01	Evaluate the correctness and analyze complexity of algorithms.
CO02	Understand and implement various algorithmic design techniques and solve classical problems.
CO03	Design solutions for real world problems by identifying, applying and implementing appropriate design techniques..
CO04	Design solutions for real world problem by mapping to classical problems.
CO05	Analyze the impact of various implementation choices on the algorithm complexity .

Answer all questions

- 1) Answer the following [6] [CO01][BTL 2]
- A. Many centuries ago, in a quiz competition of 20 questions, the reward announced was as follows: just a single bag of wheat for the first correct answer, two for the second, four for the third, eight for the fourth, and so on until all 20 questions are answered. Assume that Sagara answered all the questions correctly. How many bags did Sagara ultimately receive? Explain your answer.
- B. Write down a sum for the number of times the statement S is executed in the following nested loop. Obtain a tight bound on the asymptotic growth rate of the nested loop, assuming that the time for S is $O(1)$.

```
1.  for (i = 0; i < n; i++)
2.      for (j = 0; j < i; j++)
3.          for (k = 0; k < j; k++)
4.              S;
```

- 2) Given a sequence of numbers in a list, give an efficient algorithm (both in terms of time and space) to find the mode of the sequence. The mode is the value that appears the most number of times in this sequence. What are the time and space complexity of your algorithm?

[4] [CO02][BTL 3]

- 3) You are given n integers in the range $[1 \dots n^2]$. Show how to sort these integers in $O(n)$ time. Assume you can divide two integers and compute the remainder and quotient in constant time.

[5] [CO01][BTL 2]

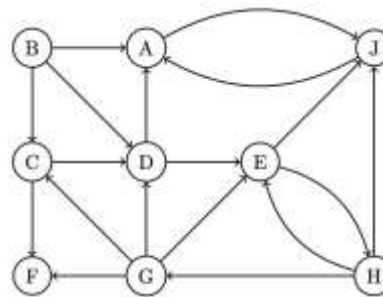
- 4) True or False and justify your answer

[4] [CO02][BTL 2]

- A. Heapsort can be used as a sorting sub-routine in radix sort because it operates in place.
- B. If the DFS finishing time $f[u] > f[v]$ for two vertices 'u' and 'v' in a directed graph G , and 'u' and 'v' are in the same DFS tree in the DFS forest, then 'u' is an ancestor of 'v' in the depth-first tree
- C. If $f(n) = O(g(n))$ and $g(n) = O(h(n))$, then $h(n) = \Omega(f(n))$
- D. Is $n(n + 1)/2 \in \Omega(n)$

- 5) For the directed graph below, find all the strongly connected components and draw the DAG of strongly connected components. Label each strongly connected component with all the nodes it contains.

[5][CO03][BTL 3]



- 6) Answer the following questions related to Merge-sort.

[CO01][BTL 2]

- A. Prove the correctness of the Merge function of Merge-sort using loop invariants.

[6]

```

Merge(L, R):
    m = length(L) + length(R)
    S = empty array of size m
    i = 1; j = 1
    for k = 1 to m:
        if L[i] <= R[j]:
            S[k] = L[i]
            i = i + 1
        else: (L[i] > R[j])
            S[k] = R[j]
            j = j + 1
    return S

```

- B. A sorting algorithm is called stable if the order of equal numbers is preserved in the output. Thus if a_i, a_j are equal and a_i appears before a_j in the input, then in the output, too a_i should

appear before a_j . With the help of an example, prove or disprove is mergesort stable or unstable. [4]

- 7) Suppose that rather than identifying a minimum spanning tree wrt. the sum of the edge costs, we want to minimise the maximum cost of any edge in the tree, i.e. we want to find a spanning tree with the most expensive edge being as cheap as possible compared to all spanning trees in the graph. The problem is called the MiniMax Spanning Tree problem. Prove that every MST is also an MMST. Does the converse hold? [8] [CO03][BTL 3]

- 8) Answer the following with respect to finding bi-connected components in a graph [CO04][BTL 3]
- Give the upper bound on the number of edges in the auxiliary graph constructed and explain you're bound. [2 Marks]
 - What is the ideal representation for the original graph to efficiently create the auxiliary graph? Justify your answer. [3 Marks]
 - What graph representation would you use for the auxiliary graph and why? [3 Marks]

Course Outcome /Bloom's Taxonomy Level (BTL) Mark Distribution Table

CO	Marks	BTL	Marks
CO01	21	BTL 1	
CO02	8	BTL 2	25
CO03	13	BTL 3	25
CO04	8	BTL 4	