

## Assignment - 6

19/04/2022  
R. Sai Manojanya ①

Estimate the bicarbonates of well water based on its pH value using polynomial regression model.

Manual calculations: degree  $\Rightarrow 2$

Step 1: Initialise  $m_1 = 1, m_2 = 1, c = -1, epochs = 1, \eta = 0.1, n_s = 2$ .

Step 2: iter = 1

Step 3:  $i = 1$

$$\begin{aligned} \text{Step 4: } \frac{\partial E}{\partial m_1} &= -(y_i - m_2 x_i^2 - m_1 x_i - c) x_i \\ &= -(157 - (7.6)^2 - 7.6 + 1)(7.6) \\ &= -(92.64)(7.6) \\ &= -704.064 \end{aligned}$$

$$\begin{aligned} \frac{\partial E}{\partial m_2} &= -(y_i - m_2 x_i^2 - m_1 x_i - c) x_i^2 \\ &= -(157 - (7.6)^2 - 7.6 + 1)(7.6)^2 \\ &= -5350.8864 \end{aligned}$$

$$\begin{aligned} \frac{\partial E}{\partial c} &= -(y_i - m_2 x_i^2 - m_1 x_i - c) \\ &= -(157 - (7.6)^2 - 7.6 + 1) \\ &= -92.64 \end{aligned}$$

$$\begin{aligned} \text{Step 5: } \Delta m_1 &= -\eta \frac{\partial E}{\partial m_1} = -(0.1)(-704.064) \\ &= 70.4064 \end{aligned}$$

$$\begin{aligned} \Delta m_2 &= -\eta \frac{\partial E}{\partial m_2} = -(0.1)(-5350.8864) \\ &= 535.08864 \end{aligned}$$

$$\begin{aligned} \Delta c &= -\eta \frac{\partial E}{\partial c} = -(0.1)(-92.64) \\ &= 9.264 \end{aligned}$$

Step 6:  $m_1 = m_1 + \Delta m_1$   
 $= 1 + 70.4064$   
 $= 71.4064$

$m_2 = m_2 + \Delta m_2$   
 $= 1 + 535.08864$   
 $= 536.08864$

$c = c + \Delta c$   
 $= -1 + 9.264$   
 $= 8.264$

Step 7:  $i = i + 1 = 1 + 1 = 2$

Step 8: if  $(i \leq n)$   
 True  $\rightarrow$  step 4  
 False  $\rightarrow$  step (Next)

Step 4:  $\frac{\partial E}{\partial m_1} = -(y_i - m_2 x_i^2 - m_1 x_i - c) x_i$   
 $= -(174 - (536.08864)(7.1)^2 - (71.4064)(7.1) - 8.264)(7.1)$   
 $= 194294.88$

$\frac{\partial E}{\partial m_2} = -(y_i - m_2 x_i^2 - m_1 x_i - c) x_i^2$   
 $= -(174 - (536.08864)(7.1)^2 - (71.4064)(7.1) - 8.264)(7.1)^2$   
 $= 1379493.69$

$\frac{\partial E}{\partial c} = -(y_i - m_2 x_i^2 - m_1 x_i - c)$   
 $= -(174 - (536.08864)(7.1)^2 - (71.4064)(7.1) - 8.264)$   
 $= 27365.477$



$$\text{step 5: } \Delta m_1 = -\eta \frac{\partial \mathcal{E}}{\partial m_1} = -(0.1)(194294.88) \\ = -19429.48$$

$$\Delta m_2 = -\eta \frac{\partial \mathcal{E}}{\partial m_2} = -(0.1)(1379493.69) \\ = -137949.36$$

$$\Delta C = -\eta \frac{\partial \mathcal{E}}{\partial C} = -(0.1)(27365.47) \\ = -2736.547$$

$$\text{step 6: } m_1 = m_1 + \Delta m_1 \\ = 71.4064 - 19429.48 \\ = -19358.07$$

$$m_2 = m_2 + \Delta m_2 \\ = 536.0864 - 137949.36 \\ = -137413.27$$

$$C = C + \Delta C \\ = 8264 - 2736.547 \\ = -2728.283$$

$$\text{step 7: } i = i + 1 = 2 + 1 = 3$$

$$\text{step 8: } \text{if } (i \leq n_s) \\ \text{True} \rightarrow \text{step 4} \\ \text{false} \rightarrow \text{Nextstep}$$

$$\text{step 9: } \text{iter} = \text{iter} + 1 = 2$$

$$\text{step 10: } \text{if } (C_{\text{iter}} \leq \text{epochs}) \\ \text{True} \rightarrow \text{step 3} \\ \text{false} \rightarrow \text{nextstep}$$

$$\text{step 11: } \text{Read final model parameters} \\ m_1 = -19358.07 \\ m_2 = -137413.27 \\ C = -2728.283$$