UNIT- IV 9L

Replacement Models: Introduction, replacement of items that deteriorate with time - value of money unchanging and changing, simple probabilistic model for replacement of items that fail completely.

Game Theory: Introduction, game with pure strategies, game with mixed strategies, dominance principle, graphical method for 2xn and mx2 games, linear programming approach for game theory.

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Replacement Models:

In any system the efficacy (efficiency) of an item deteriorates with time. In such cases, either the old item should be replaced by a new item, or some kind of restorative action (maintenance) is necessary to restore the efficiency of the whole system.

The cost of maintenance depends upon a number of factors, and a stage comes at which the maintenance cost is so large that it is more profitable to replace the old item. Thus, there is a need to formulate the most effective replacement policy.

Replacement models are concerned with the problem of replacement of machines, individuals, capital assets, etc. due to their deteriorating efficiency, failure, or breakdown.

It is evident that the study of replacement is a field of application rather than a method of analysis. Actually, it is concerned with methods of comparing alternative replacement policies.

Types of Replacement Problems:

i) Replacement policy for items, efficiency of which declines gradually with time without change in money value.

ii) Replacement policy for items, efficiency of which declines gradually with time but with change in money value.

iii) Replacement policy of items breaking down suddenly

- a) Individual replacement policy
- b) Group replacement policy

iv) Staff replacement

Reasons for Replacement of Equipment's:

Equipment are generally considered for replacement for the following reasons:

(i) Deterioration:

It is the decline in performance due to wear and tear or misalignment indicated by:

(i) Increase in maintenance costs.

(ii) Reduction in product quality and rate of production.

(iii) Increase in labour costs, and

(iv) Loss of operating time due to breakdowns.

(ii) Obsolescence:

Technology is progressing fast; newer and better equipment are being developed and produced every year.

The equipment gets obsolete due to advancement in technology and the unwarranted manufacturing costs arising from such obsolete equipment will:

(i) Reduce profits.

(ii) Impair competition.

(iii) Cause loss in value of machinery.

(iii) Inadequacy:

When the existing equipment becomes inadequate to meet the demand or it is not able to increase the production rate to desired level, the question of replacement arises.

(iv) Working Conditions:

It may be thought of replacing the old equipment and machinery which creates unpleasantness i.e. give rise to unsafe conditions for workers and leads to accidents, making the environment noisy and smoky etc.

(v) Economy:

The existing units/equipment have outlived their effective life and it is not economical to continue with them.

Assumptions – Replacement Decisions:

Following assumptions are essentially required for replacement decisions:

i) The quality of the output remains constant.
ii) Replacement and maintenance costs remain constant.
iii) The operational efficiency of the equipment remains constant.
iv) There is no change in technology of the asset under consideration.

OR Methodology of Solving Replacement Problem:

OR provides a methodology for tackling replacement problem which is discussed below:

i) Identify the items to be replaced and also their failure mechanism.
ii) Collect the data relating to the depreciation cost and the maintenance cost for the items which follow gradual failure mechanism. In case of sudden failure of items, collect the data for replacement cost of the failed items.
iii) Select a suitable replacement model and proceed.

Model-I: Replacement policy for items whose running cost increases with time and value of money remains constant during a period.

Example-I: A firm is considering the replacement of a machine, whose price is Rs 12,200 /- and its scrap value is Rs 200. From experience the running (maintenance and operating) costs are found to be as follows:

Year	1	2	3	4	5	6	7	8
Running Cost	200	500	800	1,200	1,800	2,500	3,200	4,000

When should the machine be replaced?

Solution:

Given: the running cost, R(n), the scrap value S = Rs 200 and the cost of machine, C = Rs 12,200

The given running cost, R(n)

Year	1	2	3	4	5	6	7	8
Running Cost	200	500	800	1,200	1,800	2,500	3,200	4,000

In order to determine the optimal time n when the machine should be replaced, we first calculate the average cost per year during the life of the machine,

Depreciation Cost = Cost of machine – Scrap value = 12,200 – 200 = 12,000

Total cost = Depreciation Cost + Total maintenance cost in 'n' years

Year n (1)	Running Cost $R(n)$ (2)	Cumulative Running Cost $\Sigma R(n)$ (3)	Depreciation Cost $C\text{-}S$ (4)	Total Cost TC (5) = (3) + (4)	Average Total Cost $ATC\,n$ (6) = (5) / (1)
1	200	200	12,000	12,200	12,200
2	500	700	12,000	12,700	6,350
3	800	1,500	12,000	13,500	4,500
4	1,200	2,700	12,000	14,700	3,675
5	1,800	4,500	12,000	16,500	3,300
6	2,500	7,000	12,000	19,000	3,166.67
7	3,200	10,200	12,000	22,200	3,171.43
8	4,000	14,200	12,000	26,200	3,275

The calculations in table show that the average cost is lowest during the 6*th* year (Rs 3,166.67).

Hence, the machine should be replaced after every 6*th* year, otherwise the average cost per year for running the machine would start increasing.

Example -2: Solve the following problem, using Replacement Model. Cost Price = Rs 60,000 /-

Year	1	2	3	4	5
Resale value	42000	30000	20400	14400	9650
Cost of spares	4000	4270	4880	5700	6800
Cost of labour	14000	16000	18000	21000	25000

Solution:

The data collected in running a machine,

Year	1	2	3	4	5
Resale value	42000	30000	20400	14400	9650
Cost of spares	4000	4270	4880	5700	6800
Cost of labour	14000	16000	18000	21000	25000

The costs of spares and labour, together, determine the running cost

The running costs and resale price of the machine in successive years

Year	1	2	3	4	5
Resale value	42000	30000	20400	14400	9650
Running Cost	18,000	20,270	22,880	26,700	31,800

In order to determine the optimal time n when the machine should be replaced, we first calculate the average cost per year during the life of the machine,

Year n (1)	Running Cost $R(n)$ (2)	Cumulative Running Cost $\Sigma R(n)$ (3)	Resale Value S (4)	Depreciation Cost $C - S$ (5) = 60,000-(4)	Total Cost TC (6) = (3) +(5)	Average Total Cost $ATC\,n$ (7) = (5) / (1)
1	18,000	18,000	42,000	18,000	36,000	36,000
2	20,270	38,270	30,000	30,000	68,270	34,135
3	22,880	61,150	20,400	39,600	100,750	33,583.3
4	26,700	87,850	14,400	45,600	133,450	33,362.5
5	31,800	119,650	9,650	50,350	170,000	34,000

The calculations in table show that the average cost is lowest during the 4*th* year (Rs 33,362.5).
Hence, the machine should be replaced after every 4*th* years, otherwise the average cost per year for running the machine would start increasing.

Example -3: **Solve the following problem, using Replacement Model**

For Machine-1
Cost Price = 45000, Operating Cost = 1000, Increment By = 10000
For Machine-2
Cost Price = 50000, Operating Cost = 2000, Increment By = 4000

Solution:
In order to determine the optimal time n when the machine A should be replaced, we first calculate the average cost per year during the life of the machine A,

Year n (1)	Running Cost $R(n)$ (2)	Cumulative Running Cost $\Sigma R(n)$ (3)	Depreciation Cost $C\text{-}S$ (4)	Total Cost TC (5) = (3) + (4)	Average Total Cost $ATC\,n$ (6) = (5) / (1)
1	1,000	1,000	45,000	46,000	46,000
2	11,000	12,000	45,000	57,000	28,500
3	21,000	33,000	45,000	78,000	26,000
4	31,000	64,000	45,000	109,000	27,250
5	41,000	105,000	45,000	150,000	30,000
6	51,000	156,000	45,000	201,000	33,500

The calculations in table show that the average cost of machine A is lowest during the 3*rd* year (Rs 26,000).
Hence, the machine A should be replaced after every 3*rd* year.

In order to determine the optimal time n when the machine B should be replaced, we first calculate the average cost per year during the life of the machine B,

Year n (1)	Running Cost $R(n)$ (2)	Cumulative Running Cost $\Sigma R(n)$ (3)	Depreciation Cost $C\text{-}S$ (4)	Total Cost TC (5) = (3) + (4)	Average Total Cost $ATC\,n$ (6) = (5) / (1)
1	2,000	2,000	50,000	52,000	52,000
2	6,000	8,000	50,000	58,000	29,000
3	10,000	18,000	50,000	68,000	22,666.67
4	14,000	32,000	50,000	82,000	20,500

5	18,000	50,000	50,000	100,000	20,000
6	22,000	72,000	50,000	122,000	20,333.33

The calculations in table show that the average cost of machine B is lowest during the 5*th* year (Rs 20,000). Hence, the machine B should be replaced after every 5*th* year.

The lowest average running cost of (Rs 20,000) per year for machine B is less than the lowest average running cost of (Rs 26,000) per year for machine A.
Hence machine A should be replaced by machine B.

Example 4: An engineering company is offered a material handling equipment A. It is priced at Rs 60,000 including cost of installation. The costs for operation and maintenance are estimated to be Rs 10,000 for each of the first five years, increasing every year by Rs 3,000 in the sixth and subsequent years. The company expects a return of 10% on all its investment. What is the optimal replacement period?

Year	1	2	3	4	5	6	7	8	9	10
Running Cost	10,000	10,000	10,000	10,000	10,000	13,000	16,000	19,000	22,000	25,000

Solution:
Since money is worth 10 per cent per year, the discounted factor over a period of one year is given by:

$$d = \frac{1}{1 + \frac{10}{100}} = 0.9091$$

It is also given that C = Rs 60,000

The optimum replacement age must satisfy the condition $R_n < W(n) < R_{n+1}$
R_n, R_{n+1} : Running cost in year n and year n+1
W(n): Weighted Average Cost

Year of Service n (1)	Running Cost Rn (2)	Discounted factor $d^{(n-1)}$ (3) $=0.9091^{(n-1)}$	Discounted Cost $R_n \cdot d^{(n-1)}$ (4) = (2) × (3)	Summation of Discounted Cost $\Sigma R_i . d^{(n-1)}$ (5) = Σ (4)	$C + \Sigma R_i . d^{(n-1)}$ (6) = 60,000 + (5)	Summation of Discount $\Sigma d^{(n-1)}$ (7) = Σ (3)	Weighted Average Cost W(n) (8) = (6)/ (7)
1	10,000	1	10,000	10,000	70,000	1	70,000
2	10,000	0.9091	9,090.91	19,090.91	79,090.91	1.9091	41,428.57
3	10,000	0.8264	8,264.46	27,355.37	87,355.37	2.7355	31,933.53
4	10,000	0.7513	7,513.15	34,868.52	94,868.52	3.4869	27,207.5
5	10,000	0.683	6,830.13	41,698.65	101,698.65	4.1699	24,388.95
6	13,000	0.6209	8,071.98	49,770.63	109,770.63	4.7908	22,912.86
7	16,000	0.5645	9,031.58	58,802.21	118,802.21	5.3553	22,184.21
8	19,000	0.5132	9,750	68,552.22	128,552.22	5.8684	21,905.77
9	22,000	0.4665	10,263.16	78,815.38	138,815.38	6.3349	21,912.71
10	25,000	0.4241	10,602.44	89,417.82	149,417.82	6.759	22,106.42

The calculations in table show that the average cost is lowest during the 8*th* year (Rs 21,905.77).
Hence, the machine should be replaced after every 8*th* years, otherwise the average cost per year for running the machine would start increasing.

Example 5: A company is buying minicomputers. It costs Rs 5 lakh, and its running and maintenance costs are Rs 60,000/- for each of the first five years, increasing by Rs 20,000 per year in the sixth and subsequent years. If the money is worth 12 percent per year, what is the optimal replacement period?

Year	1	2	3	4	5	6	7	8	9	10
Running Cost	60,000	60,000	60,000	60,000	60,000	80,000	1,00,000	1,20,000	1,40,000	1,60,000

Solution:

Since money is worth 12 % per year, the discounted factor over a period of one year is given by:

$$d = \frac{1}{1 + \frac{12}{100}} = 0.8929$$

It is also given that C = Rs 500,000

The optimum replacement age must satisfy the condition $R_n < W(n) < R_{n+1}$

R_n, R_{n+1} : Running cost in year n and year n+1

W(n): Weighted Average Cost

Year of Service n (1)	Running Cost Rn (2)	Discounted factor $d^{(n-1)}$ (3) $=0.8929^{(n-1)}$	Discounted Cost $R_n \cdot d^{(n-1)}$ (4) = (2) × (3)	Summation of Discounted Cost $\Sigma R_i . d^{(n-1)}$ (5) =Σ (4)	$C + \Sigma R_i . d^{(n-1)}$ (6) = 60,000 + (5)	Summation of Discount $\Sigma d^{(n-1)}$ (7) = Σ (3)	Weighted Average Cost W(n) (8) = (6)/ (7)
1	60,000	1	60,000	60,000	560,000	1	560,000
2	60,000	0.8929	53,571.43	113,571.43	613,571.43	1.8929	324,150.94
3	60,000	0.7972	47,831.63	161,403.06	661,403.06	2.6901	245,870.08
4	60,000	0.7118	42,706.81	204,109.88	704,109.88	3.4018	206,979.66
5	60,000	0.6355	38,131.08	242,240.96	742,240.96	4.0373	183,843.63
6	80,000	0.5674	45,394.15	287,635.11	787,635.11	4.6048	171,047.42
7	100,000	0.5066	50,663.11	338,298.22	838,298.22	5.1114	164,005.36
8	120,000	0.4523	54,281.91	392,580.13	892,580.13	5.5638	160,427.6
9	140,000	0.4039	56,543.65	449,123.78	949,123.78	5.9676	**159,045.09**
10	160,000	0.3606	57,697.6	506,821.38	1,006,821.38	6.3282	159,099.5

The calculations in table show that the average cost is lowest during the 9*th* year (Rs 159,045.09).

Hence, the machine should be replaced after every 9*th* years, otherwise the average cost per year for running the machine would start increasing.

Example 6: **Solve the following problem, using Replacement Model**

Year	1	2	3	4	5	6	7	8	9	10
Running Cost	120000	120000	120000	120000	120000	140000	160000	180000	200000	220000

Machine Cost = 250000 and Discounted Rate = 14%

Solution:

Since money is worth 14 per cent per year, the discounted factor over a period of one year is given by:

$$d = \frac{1}{1 + \frac{14}{100}} = 0.8772$$

It is also given that C = Rs 250,000

The optimum replacement age must satisfy the condition $R_n < W(n) < R_{n+1}$

R_n, R_{n+1} : Running cost in year n and year n+1

W(n): Weighted Average Cost

Year of Service n (1)	Running Cost Rn (2)	Discounted factor $d^{(n-1)}$ (3) =$0.89291^{(n-1)}$	Discounted Cost $Rn \cdot d^{(n-1)}$ (4) = (2) × (3)	Summation of Discounted Cost $\Sigma Ri.\ d^{(n-1)}$ (5) =Σ (4)	$C+ \Sigma Ri.\ d^{(n-1)}$ (6) = 60,000 + (5)	Summation of Discount $\Sigma\ d^{(n-1)}$ (7) = Σ (3)	Weighted Average Cost W(n) (8) = (6)/ (7)
1	120,000	1	120,000	120,000	370,000	1	370,000
2	120,000	0.8772	105,263.16	225,263.16	475,263.16	1.8772	253,177.57
3	120,000	0.7695	92,336.1	317,599.26	567,599.26	2.6467	214,458.66
4	120,000	0.675	80,996.58	398,595.84	648,595.84	3.3216	195,264.21
5	120,000	0.5921	71,049.63	469,645.48	719,645.48	3.9137	183,877.97
6	140,000	0.5194	72,711.61	542,357.09	792,357.09	4.4331	178,737.34
7	160,000	0.4556	72,893.85	615,250.94	865,250.94	4.8887	176,991.16
8	180,000	0.3996	71,934.72	687,185.66	937,185.66	5.2883	177,218.54
9	200,000	0.3506	70,111.81	757,297.47	1,007,297.47	5.6389	178,634.83
10	220,000	0.3075	67,651.75	824,949.21	1,074,949.21	5.9464	180,773.96

The calculations in table show that the average cost is lowest during the $7th$ year (Rs 176,991.16).

Hence, the machine should be replaced after every $7th$ years, otherwise the average cost per year for running the machine would start increasing.

Simple probabilistic model for replacement of items that fail completely:

Example 6: A computer contains 10,000 resistors. When any resistor fails, it is replaced. The cost of replacing a resistor individually is Rs 1 only. If all the resistors are replaced at the same time, the cost per resistor would be reduced to 35 paise. The percentage of surviving resistors say S(t) at the end of month t and the probability of failure P(t) during the month t are as follows:

t	0	1	2	3	4	5	6
P(t)	0	0.03	0.07	0.20	0.40	0.15	0.15

What is the optimal replacement plan?

Solution:

The percentage of survivors resistors at the end of month t and the probability of failure are as follows,

t	0	1	2	3	4	5	6
P(t)	0	0.03	0.07	0.20	0.40	0.15	0.15

Let Ni be that number of resistors replaced at the end of the ith month.
The different value of Ni can then be calculated as follows
N0 = number of resistors in the beginning = 10000

N1 = number of resistors being replaced by the end of 1st month

= N0.P1 = 10000 x 0.03 = 300

N2 = number of resistors being replaced by the end of 2nd month
= N0.P2 + N1.P1

= (10000 x 0.07) + (300 x 0.03) = 709

N3 = number of resistors being replaced by the end of 3rd month
= N0.P3 + N1.P2 + N2.P1

= (10000 x 0.2) + (300 x 0.07) + (709 x 0.03) = 2042

$N4$ = number of resistors being replaced by the end of $4th$ month
$= N0.P4 + N1.P3 + N2.P2 + N3.P1$

$= (10000 \times 0.4) + (300 \times 0.2\ 0 + (709 \times 0.07) + (2042 \times 0.03) = 4171$

$N5$ = number of resistors being replaced by the end of $5th$ month

$= N0P5+N1P4+N2P3+N3P2+N4P1$

$= (10000 \times 0.15) + (300 \times 0.4) + (709 \times 0.2) + (2042 \times 0.07) + (4171 \times 0.03) = 2030$

$N6$ = number of resistors being replaced by the end of $6th$ month

$= N0P6+N1P5+N2P4+N3P3+N4P2+N5P1$

$= (10000 \times 0.15) + (300 \times 0.15) + (709 \times 0.4) + (2042 \times 0.2) + (4171 \times 0.07) + (2030 \times 0.03) = 2590$

The expected life of each resistor is given by

$= \sum X_i P(X_i) = 1 \cdot 0.03+2 \cdot 0.07+3 \cdot 0.2+4 \cdot 0.4+5 \cdot 0.15+6 \cdot 0.15 = 4.02$ months.

Average number of failures per month is given by

$N0/(\text{Mean age})=10000/4.02=2487.56 = 2488$ resistors (approx.)

Hence, the total cost of individual replacement at the cost of Rs 1 per resistor will be Rs (2488 x 1) = Rs 2488 /-

The cost replacement of all the resistors at the same time can be calculated as follows:

End of month	Total Cost of Group Replacement (Rs)	Average Cost per Month (Rs)
1	(300) x 1+ (10000 x 0.35) = 3800	3800
2	(300+709) x 1 + (10000 x 0.35) =4509	2254.5
3	(300+709+2042) x 1 + (10000 x 0.35) =6551	2183.67
4	(300+709+2042+4171) x 1 + (10000 x 0.35) =10722	2680.5
5	(300+709+2042+4171+2030) x 1 + (10000 x 0.35) =12752	2550.4
6	(300+709+2042+4171+2030+2590) x 1 + (10000 x 0.35) =15342	2557

Since the average cost per month of Rs 2,183.67 is obtained in the $3rd$ month, it is optimal to have a group replacement after every $3rd$ month

Example 7: The following mortality rates have been observed for a certain type of fuse:

t	0	1	2	3	4	5
P(t)	0	0.05	0.10	0.20	0.40	0.25

There are 1,000 fuses in use, and it costs Rs 5 to replace an individual fuse. If all fuses were replaced simultaneously, it would cost Rs 1.25 per fuse. It is proposed to replace all fuses at fixed intervals of time, whether or not they have burnt out, and to continue replacing burnt out fuses as they fail. At what time intervals should the group replacement be made? Also prove that this optimal policy is superior to the straightforward policy of replacing each fuse only when it fails.

Solution:

The percentage of survivers resistors at the end of month t and the probability of failure are as follows,

t	0	1	2	3	4	5
P(t)	0	0.05	0.10	0.20	0.40	0.25

Let Ni be that number of resistors replaced at the end of the ith month.

The different value of Ni can then be calculated as follows

$N0$ = number of resistors in the beginning = 1000

$N1$ = number of resistors being replaced by the end of $1st$ month

= $N0P1$ = = 1000 x 0.05 = 50

$N2$ = number of resistors being replaced by the end of $2nd$ month

= $N0P2 + N1P1$ = 1000 x 0.1 + 50 x 0.05 = 103

$N3$ = number of resistors being replaced by the end of $3rd$ month

= $N0P3+N1P2+N2P1$ = 1000 x 0.2 + 50 x 0.1 + 103 x 0.05 = 210

$N4$ = number of resistors being replaced by the end of $4th$ month

= $N0P4+N1P3+N2P2+N3P1$ = 1000 x 0.4 + 50 x 0.2 + 103 x 0.1 + 210 x 0.05 = 431

$N5$ = number of resistors being replaced by the end of $5th$ month

= $N0P5+N1P4+N2P3+N3P2+N4P1$ = 1000 x 0.25 + 50 x 0.4 + 103 x 0.2 + 210 x 0.1 + 431 x 0.05 = 333

The expected life of each resistor is given by $= \sum X_i P(X_i) =$ 1·0 x 05 + 2 x 0.1 +3 x 0.2 + 4 x 0.4 + 5 x 0.25
= 3.7 months.

Average number of failures per month is given by = $N0$/(Mean age) = 1000/3.7 = 270.27 =270 resistors (approx.)

Hence, the total cost of individual replacement at the cost of Rs 5 per resistor will be Rs (270 x 5) = Rs 1350 /-

The cost replacement of all the resistors at the same time can be calculated as follows:

End of month	Total Cost of Group Replacement (Rs)	Average Cost per Month (Rs)
1	(50) x 5 + 1000 x 1.25=1500	1500
2	(50+103) x 5 + 1000 x 1.25=2015	1007.5
3	(50+103+210) x 5 + 1000 x 1.25=3065	1021.67
4	(50+103+210+431) x 5 + 1000 x 1.25=5220	1305
5	(50+103+210+431+333) x 5 + 1000 x 1.25=6885	1377

Since the average cost per month of Rs 1,007.5 is obtained in the $2nd$ month, it is optimal to have a group replacement after every $2nd$ month.

Introduction to Game Theory

Game theory is a type of decision theory in which one's choice of action is determined after taking into account all possible alternatives available to an opponent playing the same game, rather than just by the possibilities of several outcome results. Game theory does not insist on how a game should be played but tells the procedure and principles by which action should be selected. Thus, it is a decision theory useful in competitive situations.

Game is defined as an activity between two or more persons according to a set of rules at the end of which each person receives some benefit or suffers loss. The set of rules defines the game. Going through the set of rules once by the participants defines a play.

Properties of a Game:

a) There are finite numbers of competitors called 'players'
b) Each player has a finite number of possible courses of action called 'strategies'
c) All the strategies and their effects are known to the players, but player does not know which strategy is to be chosen.
d) A game is played when each player chooses one of his strategies. The strategies are assumed to be made simultaneously with an outcome such that no player knows his opponent's strategy until he decides his own strategy.
e) The game is a combination of the strategies and in certain units which determines the gain or loss.
f) The figures shown as the outcomes of strategies in a matrix form are called 'pay-off matrix'.
g) The player playing the game always tries to choose the best course of action which results in optimal pay off called 'optimal strategy'.
h) The expected pay off when all the players of the game follow their optimal strategies is known as 'value of the game'. The main objective of a problem of a game is to find the value of the game.
i) The game is said to be 'fair' game if the value of the game is zero otherwise it's known as 'unfair'.

Characteristics of Game Theory:

a) **Competitive game:** A competitive situation is called a competitive game if it has the following four properties

- → There are finite number of competitors such that $n \geq 2$. In case $n = 2$, it is called a two-person game and in case $n > 2$, it is referred as n-person game.
- → Each player has a list of finite number of possible activities.
- → A play is said to occur when each player chooses one of his activities. The choices are assumed to be made simultaneously i.e., no player knows the choice of the other until he has decided on his own.
- → Every combination of activities determines an outcome which results in a gain of payments to each player, provided each player is playing uncompromisingly to get as much as possible. Negative gain implies the loss of same amount.

b) **Strategy:** The strategy of a player is the predetermined rule by which player decides his course of action from his own list during the game.

The two types of strategy are

→ Pure strategy
→ Mixed strategy

Pure Strategy: If a player knows exactly what the other player is going to do, a deterministic situation is obtained, and objective function is to maximize the gain. Therefore, the pure strategy is a decision rule always to select a particular course of action.

Mixed Strategy: If a player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained, and objective function is to maximize the expected gain. Thus, the mixed strategy is a selection among pure strategies with fixed probabilities.

c) **Number of persons:** A game is called 'n' person game if the number of persons playing is 'n'.

The person means an individual or a group aiming at a particular objective.

Two-person, zero-sum game: A game with only two players (player A and player B) is called a 'two-person, zero-sum game', if the losses of one player are equivalent to the gains of the other so that the sum of their net gains is zero.

Two-person, zero-sum games are also called rectangular games as these are usually represented by a payoff matrix in a rectangular form.

d) Number of activities: The activities may be finite or infinite.

e) Payoff: The quantitative measure of satisfaction a person gets at the end of each play is called a payoff

f) Payoff matrix: Suppose the player A has 'm' activities and the player B has 'n' activities.

Then a payoff matrix can be formed by adopting the following rules

- → Row designations for each matrix are the activities available to player A
- → Column designations for each matrix are the activities available to player B
- → Cell entry Vij is the payment to player A in A's payoff matrix when A chooses the activity i and B chooses the activity j.
- → With a zero-sum, two-person game, the cell entry in the player B's payoff matrix will be negative of the corresponding cell entry Vij in the player A's payoff matrix so that sum of payoff matrices for player A and player B is ultimately zero.

g) Value of the game: Value of the game is the maximum guaranteed game to player A (maximizing player) if both the players uses their best strategies. It is generally denoted by 'V' and it is unique.

Classification of Games

All games are classified into

→ Pure strategy games

→ Mixed strategy games

Strategy: It is the pre-determined rule by which each player decides his course of action from his list available to him. How one course of action is selected out of various courses available to him is

(i) Pure Strategy: It is the predetermined course of action to be employed by the player. The players knew it in advance. It is usually represented by a number with which the course of action is associated.

(ii) Mixed Strategy: In mixed strategy the player decides his course of action in accordance with some fixed probability distribution. Probability is associated with each course of action and the selection is done as per these probabilities.

In mixed strategy the opponent cannot be sure of the course of action to be taken on any particular occasion. Pure strategy games can be solved by saddle point method.

Decision of a Game. In Game theory, best strategy for each player is determined on the basis of some rule. Since both the players are expected to be rational in their approach this is known as the criteria of optimality. Each player lists the possible outcomes from his action and selects the best action to achieve his objectives. This criterion of optimality is expressed as Maximin for the maximising player and Minimax for the minimising player.

Maximin and Minimax strategy

1. Maximin criteria

This criterion is the decision to take the course of action which maximizes the minimum possible pay-off. Since this decision criterion locates the alternative strategy that has the least possible loss, it is also known as a pessimistic decision criterion. The working method is:

(i) Determine the lowest outcome for each alternative.

(ii) Choose the alternative associated with the maximum of these.

2. Minimax criteria

This criterion is the decision to take the course of action which minimizes the maximum possible pay-off. Since this decision criterion locates the alternative strategy that has the greatest possible gain. The working method is:

(i) Determine the highest outcome for each alternative.

(ii) Choose the alternative associated with the minimum of these.