

## Assignment - 4.

### Task 1:-

Given joint probability distribution for a domain of two variables

	Color = Red	Color = Green	Color = Blue
Vehicle = Car	0.1184	0.1280	0.0736
Vehicle = van	0.0444	0.0480	0.0276
Vehicle = Truck	0.1554	0.1680	0.0966
Vehicle = SUV	0.0518	0.0560	0.0322

Sol:-  $P(A \text{ and } B) = P(A) * P(B)$

Where  $P(A)$  = colour is green and  
 $P(B)$  = Vehicle is truck.

from Bayes's theorem,

$$P(A) = 0.1280 + 0.0480 + 0.1680 + 0.0560 = 0.4$$

add all green color values, and then  
 $P(A) = 1 - P(A) = 1 - 0.4 = 0.6$

$$P(B) = 1/4 = 0.25$$

For calculating given condition

$$P(\text{Color is not green} \wedge \text{Vehicle is Truck}) = P(\text{Vehicle is Truck})$$

$$\Rightarrow \frac{0.1554 + 0.0966}{0.1554 + 0.1680 + 0.0966}$$

$$\Rightarrow \frac{0.252}{0.42} = 0.6$$



Part b:

Check if color are totally independent from each other

$$P(\text{color is green}) = 1 - P(\text{color is not green})$$

$$P(\text{color is green}) = 0.4$$

$$P(\text{color is not green}) = 1 - 0.4 = 0.6$$

$$P(\text{color is not green} | \text{given/vehicle is truck}) = P(\text{color is not green})$$

if the color is not green and  $P(\text{color is not given/vehicle is truck})$  then they are totally independent of each other.

Task 2:-

given In, a certain probability problem we have 11 variables A,  $B_1, B_2, \dots, B_{10}$ . Variables has 7 Rules and Each of Variable  $B_1, \dots, B_{10}$  have 8 possible values. Given that each  $B_i$  is conditionally independent of all other  $B_j$  variables (with  $j \neq i$ ) given A.

Part a:-

Given 11 variables :  $A, B, B_1, \dots, B_{10}$

$A$  has 7 values

$B_1$  to  $B_{10}$  has 8 possible values, Each  $B_i$  is conditionally independent possible.  $A = 7$   
possible value of  $B = 8^{10}$

$7 \times 8^{10}$  is the total numbers to be stored in joint distribution table =  $7 \times 8^{10}$ .

Part b:-

The most space-efficient way of representation for that joint probability distribution of these 11 are

$$P(B/A) = 7 \times 8 = 56 \text{ values or}$$

$$7 \times 7 = 49 \text{ values}$$

$$\text{for the 10 variables: } 49 \times 10 = 490$$

$$\text{we need to calculate } P(A) = (7-1) = 6$$

$$\text{Total Space} = 490 + 6 = 496.$$

Part c:-

Yes, this scenario follows the naive-Bayes model.



Task-4

Given table

Class	A	B	C
X	1	2	1
X	2	1	2
X	3	2	2
X	1	3	3
X	1	2	1
Y	2	1	2
Y	3	1	1
Y	2	2	2
Y	3	3	1
Y	2	1	1

The information

Entropy before split  $X=5, Y=5$ 

and splitting with A

for  $A=1$ 

$$X=3, Y=0$$

$$H_A = -\frac{3}{3} \log_2 \frac{3}{3} - \frac{0}{3} \log_2 \frac{0}{3} = 0$$

$$\text{for } A=2 = X=1, Y=3 \quad H_B = 0.8113$$

$$H_C = A=3 = 0.9183 \quad \therefore I_A = \underline{0.4}$$

Splitting it with B.

$$\text{for } B=1 \quad H_A = 1/4 \log_2 1/4 + 3/4 \log_2 3/4$$

$$= 0.8113$$

$$\text{for } B=2$$

$$X=3, Y=1$$

$$\text{for } B=3, H_A$$

$$H_A = -1/2 \log_2 1/2 - 1/2 \log_2 1/2 = 1$$

$$I_B = H - 0.4/10 H_A - 4/10 H_C - \frac{2}{10} H_T$$

$$0.1511$$

Splitting with C.

$$\text{for } C=1, X=2, Y=3$$

$$H_B = -2/5 \log_2 2/5 - 3/5 \log_2 3/5$$

$$= 0.921$$

$$\text{for } C=2, X=2, Y=2$$

$$H_C = -1/1 \log_2 1/1 - 0/1 \log_2 0/1 = 0$$

$$I_C = H - 5/10 H_B - 4/10 H_C - 1/10 H_T$$

$$= 0.1145 \approx 0.115$$

Therefore, A is the best attribute.



Task-5

Class	A	B	C
X	25	24	31
X	22	14	24
X	28	22	25
X	24	13	30
X	26	20	24
Y	20	31	17
Y	18	32	14
Y	21	25	20
Y	13	32	15
Y	12	27	18

gain = Parent node, where  
 The gain works as  $-1/2 \log 1/2$   
 $-1/2 \log 1/2$

for the threshold is 15.  $\Rightarrow 1$   
 $-0/2 \log 0/2 - 2/2 \log 2/2 = 0$

if threshold is 20  $\Rightarrow 0$

if threshold is 25  $\Rightarrow 0.286$

Gain at A  $= 1 - (0.286 + 0 + 0)$   
 $0.713$

Case B:-

if threshold is 15  $\Rightarrow -2/2 \log 2/2 - 0/2 \log 0/2$   
 $= 0$

If threshold is 1.0  $\Rightarrow \frac{3}{3} \log \frac{3}{3} - \frac{0}{3} \log \frac{0}{3}$   
 $= 0$

If threshold is 1.5  $\Rightarrow -\frac{5}{6} \log \frac{5}{6} - \frac{1}{6} \log \frac{1}{6}$   
 $= 0.065 + 0.124$

$\approx 0.199$   
 $\therefore B = 1 - (0 + 0 + 0.295)$   
 $= 0.805$

Case (c) i.e. at 'c'

If threshold is 1.5  $\Rightarrow -\frac{0}{2} \log \frac{0}{2} - \frac{2}{2} \log \frac{2}{2}$

If threshold is 1.8  $\Rightarrow -\frac{0}{5} \log \frac{0}{5} - \frac{5}{5} \log \frac{5}{5}$

If threshold is 1.5  $\Rightarrow -\frac{3}{8} \log \frac{3}{8} - \frac{5}{8} \log \frac{5}{8}$   
~~gain at c = 0.286~~  $= 0.286$

gain at c =  $1 - (0 + 0 + 0.286)$   
 $= 0.713$

Attribute "B" achieves the highest information gain at the root.