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Assignment - 009.

6.2 given that Probability maxtrix

$$P = \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix}$$

(a) need to compute 2 step transition probability matrix
ie. P^2

$$P^2 = \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix} \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix}$$

$$\begin{bmatrix} (0.4\times0.4)+(0.6)(0.6) & (0.4\times0.6)+(0.6\times0.4) \\ (0.6\times0.4)+(0.4\times0.6) & (0.6\times0.6)+(0.4\times0.4) \end{bmatrix}$$

$$= \begin{bmatrix} 0.16+0.36 & 0.24+0.24 \\ 0.24+0.24 & 0.36+0.16 \end{bmatrix}$$

$$= \begin{bmatrix} 0.52 & 0.48 \\ 0.48 & 0.52 \end{bmatrix}$$

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b) Now, we need probability is model at 8:30pm
on the Sameday
ie, $P^3 = P^2 \times P$

$$\begin{bmatrix} 0.52 & 0.48 \\ 0.48 & 0.52 \end{bmatrix} \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix}$$

$$\begin{bmatrix} (0.52\times0.4)+(0.48\times0.6) & (0.48\times0.6)+(0.52\times0.4) \\ (0.48\times0.4)+(0.52\times0.6) & (0.48\times0.6)+(0.52\times0.4) \end{bmatrix}$$

$$= \begin{bmatrix} 0.496 & 0.504 \\ 0.504 & 0.496 \end{bmatrix}$$

So the probability of system is model I
at 8:30pm is P_{11}
So $P_{11} = 0.496$
Hence proved.

6.3 Given,
Probability of a block dog is Black = 0.6
is Brown is = 0.4.

⇒ Probability of a brown dog is black.
= 0.2
and is brown = 0.8.

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So,

(a) We need to write transition probability matrix
Let P_A be probability that child be brown
$P_A = (0.4)(0.8)$
(0.32)
P_B is Probability that child be black.
$P_B = (0.6)(0.2)$
$\Rightarrow 0.12$

Transition probability matrix
$\Rightarrow \begin{bmatrix} 0.32 & 0.12 \end{bmatrix}$

(b) Need to compute probability the Pep's grandchild is black

$P_{black} = \frac{0.8}{0.12} = \frac{2}{3} = 0.66.$

6.5 given,

Probability of Sunny day followed by another Sunny day $= 0.8$
Probability of rainy day followed by another rainy day $= 0.6$

So, we need to compute probability that April 1st next year is rainy

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So, we consider System of equation

$$\bar{\pi} P = \bar{\pi}$$

$$\bar{\pi}_1 + \bar{\pi}_2 = 1$$

using Study State distribution

$$0.8\bar{\pi}_1 + 0.4\bar{\pi}_2 = \bar{\pi}_1 \quad \text{—— (1)}$$

$$0.2\bar{\pi}_1 + 0.6\bar{\pi}_2 = \bar{\pi}_2 \quad \text{—— (2)}$$

$$\bar{\pi}_1 + \bar{\pi}_2 = 1 \quad \text{—— (3)}$$

Now

from (1)

$$0.8\bar{\pi}_1 - \bar{\pi}_1 = 0.4\bar{\pi}_2$$

$$0.2\bar{\pi}_1 = 0.4\bar{\pi}_2$$

$$\Rightarrow 0.2\bar{\pi}_1 = 0.4\bar{\pi}_2$$

$$\Rightarrow \bar{\pi}_1 = 2\bar{\pi}_2$$

from (2)

$$0.2\bar{\pi}_1 + 0.6\bar{\pi}_2 = \bar{\pi}_2$$

$$0.2\bar{\pi}_1 = \bar{\pi}_2 - 0.6\bar{\pi}_2$$

$$0.\cancel{2}\bar{\pi}_1 = \cancel{0.4}^{2}\bar{\pi}_2$$

$$\bar{\pi}_1 = 2\bar{\pi}_2$$

Now from (3)

$$\bar{\pi}_1 + \bar{\pi}_2 = 1$$

using from equation (1) & (2)

$$2\bar{\pi}_2 + \bar{\pi}_2 = 1$$

$$3\bar{\pi}_2 = 1$$

$$\bar{\pi}_2 = 1/3$$

then $\bar{\pi}_1$

$$\Rightarrow \bar{\pi}_1 + 1/3 = 1$$

$$\Rightarrow \bar{\pi}_1 + 1 - 1/3$$

$$\bar{\pi}_1 = 2/3$$

∴ Probability that April 1^{st} next year is rainly is $\bar{\pi}_2$

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$\Rightarrow \pi_2 = 1/3$ //

Hence proved //

(14.8) Given

$$P = \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$$

(a) Now, P^2

$$P \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (0.9 \times 0.9) + (0.1 \times 0.3) & (0.9 \times 0.1) + (0.1 \times 0.7) \\ (0.3 \times 0.9) + (0.7 \times 0.3) & (0.3 \times 0.1) + (0.7 \times 0.7) \end{bmatrix}$$

$$= \begin{bmatrix} 0.84 & 0.16 \\ 0.48 & 0.52 \end{bmatrix}$$

$$= \quad P^3 = P^2 \times P$$

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$$= \begin{bmatrix} 0.89 & 0.16 \\ 0.48 & 0.52 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$$

$$\begin{bmatrix} (0.89\times0.9)+(0.16\times0.3) & (0.84\times0.1)+(0.16\times0.7) \\ (0.48\times0.9)+(0.52\times0.3) & (0.48\times0.1)+(0.52\times0.7) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.849 & 0.196 \\ 0.3678 & 0.37 \end{bmatrix}$$

Then $P^5 = P^2 \times P^3$

$$\begin{bmatrix} 0.89 & 0.16 \\ 0.48 & 0.52 \end{bmatrix} \begin{bmatrix} 0.849 & 0.196 \\ 0.3678 & 0.37 \end{bmatrix}$$

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$$\begin{bmatrix} 0.89 \times 0.849 + 0.16 \times 0.3678 & 0.89 \times 0.196 + 0.16 \times 0.37 \\ 0.48 \times 0.849 + 0.52 \times 0.3678 & 0.48 \times 0.196 + 0.52 \times 0.37 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.76944 & 0.23056 \\ 0.69168 & 0.30022 \end{bmatrix}$$

(b) Need to compute probability that car will start tomorrow if it starts today

i.e $P_1 = 0.90$

(c) Probability that car will start tomorrow if it doesn't start today

$P_2 = 0.30$

14.9) (a) from obtained probability matrix i.e, P^5.

$P = 0.23$

(b) from transition probability matrix from above question

$P = 0.30$

(c) need to compute stationary

i.e, $\pi_1 = 0.9\pi_2 + 0.3\pi_3$

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$\pi_1 + \pi_2 = 1$
$4\pi_3 = 1$
$\pi_3 = 1/4 = 0.25$
$\pi_2 = 0.75$

14.10 Let,
initial number of customers in the market = 300
Equally divided among 3 companies each get = 100
for Dress Rt = 100 (DR)
fashion line = 100 (FI)
luxury living = 100 (LL)

Given, Dressrite loses 10% to F.I & 20% of LL
fashion line loses 5% to DR & 10% of LL
Luxury Living loses 5% of FI & 5% DR.

After - 1 => one month

DR will have = 100 - 10 - 20 + 5 + 5 = 80
FI will have = 100 - 5 - 10 + 10 + 5 = 100
LL will have = 100 - 5 - 5 + 20 + 10 = 120

After t = 2 => 2nd month
DR will have = ~~80~~
80 - 8 - 10 + 5 + 6 = 67

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FI will have $= 100 - 5 - 10 + 8 + 6$
$= 9$

EL will have $= 120 - 6 - 6 + 16 + 10$
$= 134$

After $t = 3 \Rightarrow$ 3rd month

$DR = 67 - 6.7 - 13.4 + 4.95 + 6.97$
$= 58.55$

$FI = 99 - 4.945 - 9.9 + 6.7 + 6.7$
97.55

$LL = 134 - 6.7 - 6.7 + 13.4 + 9.9$
$= 143.90$

$\therefore$ market share after 3 months is

Dress Rate $: 58.55/300$
19.52%

Fashion Inc $= 97.55/300$
$= 32.52\%$

Luxury Living $= 143.9/300$
$= 47.96\%$

$\Rightarrow$ market shares will be 19.52%, 32.52%, & 47.96% for Dress Rate, fashion in & luxury Living

~~Hence~~ proved.

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