

Q-1] Showing that Alice and Bob likely to disagree from factor  $\phi(A, B)$ , we can see that  $(a^0, b^0)$  has greater value that means that there is high chances that both will disagree, taking about  $(a^0, b^1)$  it state that Bob agrees and Alice disagree and contrary  $(a^1, b^0)$  which state that Alice agree and Bob disagree taking about this  $(a^0, b^1) > (a^1, b^0)$  its more means Bob weight of agree is more i.e B we can see from table them disagreeing of both has more weight than both being agreeable i.e.  $(a^0, b^0) > (a^1, b^1)$ .

Q.2]  $\rightarrow$  A markov network, also known as Markov Random Field is a probabilistic graphical model used in computer vision to represent the relationship between pixels in an image, where neighbour pixels are more likely to similar values.

1] Image segmentation: By modelling the relationship between neighbouring pixel, markov network can effectly group pixel belonging to the same object.

2] Image Denoising: By incorporating local dependency, markov network can be used to smooth out noise while preserving image detail.

Q.3] Independence condition:

$$P(X_1 = x_1, X_3 = x_3) = P(X_1 = x_1) \cdot P(X_3 = x_3)$$

for all  $x_1, x_3 \in \{0, 1\}$

8 configurations:

(0, 0, 0, 0)

(0, 0, 0, 1)

(1, 0, 0, 0)

(0, 0, 1, 1)

(1, 1, 0, 0)

(0, 1, 1, 1)

(1, 1, 1, 0)

(1, 1, 1, 1)

all configurations have  $P = 1/8$ ,  
configurations,  $P = 0$ .

$$P(X_1 = 0) = \frac{\text{no of lines } (X_1 = 0)}{\text{total configurations}} = \frac{4}{8} = \frac{1}{2}$$

$$P(X_1 = 1) = \frac{4}{8} = \frac{1}{2}$$

$$P(X_3 = 0) = \frac{4}{8} = \frac{1}{2}, \quad P(X_3 = 1) = \frac{4}{8} = \frac{1}{2}$$

$$P(X_1 = 0, X_3 = 0) = \frac{\text{no of times } (X_1, X_3)}{\text{total configs}} = \frac{2}{8} = \frac{1}{4}$$

now,

$$P(X_1 = 0, X_3 = 0) = 1/4; \quad P(X_1 = 0) \cdot P(X_3 = 0) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(X_1 = 0, X_3 = 1) = 1/4; \quad P(X_1 = 0) \cdot P(X_3 = 1) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$\therefore$  Condition holds true,  
 $\therefore X_1$  is independent of  $X_3$ .



Q.4] For each variable  $X$ , markov blanket condition,  $P(X | MB(X)) = P(X | \text{all other vars except } X)$

The distribution  $P(a_1, b_1, c_1, d_1) = 0.5$  and  $P(a_0, b_0, c_0, d_0) = 0.5$

This implies that the only possible configurations for  $(A, B, C, D)$  are  $(1, 1, 1, 1)$  and  $(0, 0, 0, 0)$  with each configuration having equal probability 0.5.

- ∴ All variables are perfectly correlated.
- ∴ Markov Blanket Condition.

$$P(A | B, C, D) = 1; B = C = D = 1$$

$B = C = D = 0, A = 0$  with probability 1.

$$P(A | B, C, D) = P(A | B, C, D) = P(A | MB(A))$$

$$X \in \{A, B, C, D\}$$

Q.6] For calculating canonical energy function for a clique  $D$  is given as :-

$$\epsilon^+ D^{(d)} = \sum_{Z \subseteq D} (-1)^{|D-Z|} e(d_Z, \epsilon^+ Z)$$

where sum is all subset of  $D$   
 $d_{AB} = a^0 b^0$  taking all subset  
 $\{A, B\}, \{A\}, \{B\}, \{\emptyset\}$

$$\epsilon(a^0, b^0) = e(a^0, b^0) - e(a^0, \epsilon_B) - e(\epsilon_A, b^0) + e(\epsilon_A, \epsilon_B)$$

$$\epsilon_{AB}(a^0, b^0) = \ln(80) - \ln(30) - \ln(30) + \ln(30) = 0$$

$$\epsilon_{AB}(a^0, b^1) = 0 \quad (\text{Similarly})$$

$$\epsilon_{AB}(a^1, b^0) = 0$$

$$\epsilon_{AB}(a^1, b^1) = 4.09$$

for  $\epsilon_{BC}(d_{BC})$  all subset  $\{B\}, \{B\}, \{Y\}$

$$\begin{aligned} \epsilon_{BC}(b^0 c^0) &= e(b^0 c^0) - e(b^0 c^0) \\ &= \ln(100) - \ln(100) = 0 \end{aligned}$$

$$\epsilon_{BC}(b^0 c^1) = 0$$

for $\epsilon_A(A)$	$\epsilon_B(B)$	$\epsilon_C(C)$	$\epsilon_D(D)$	$\epsilon(\phi)$
$a_0 = 0$	$b_0 = 0$	$c_0 = 0$	$d_0 = 0$	-3.18
$a_1 = -8.01$	$b_1 = -6.4$	$c_1 = 0$	$d_1 = 0$	