EE2001 : Tutorial 2 Solution

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1. (a) $F(x, y, z) = \Sigma(2, 3, 6, 7)$

x \ yz	00	01	11	10
0	0	0	1	1
1	0	0	1	1

$$F(x, y, z) = y$$

(b) $F(A, B, C, D) = \Sigma(4, 6, 7, 15)$

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	0	1	1
11	0	0	1	0
10	0	0	0	0

$$F(A, B, C, D) = A'BD' + BCD = B(A'D' + CD)$$

(c) $F(A, B, C, D) = \Sigma(3, 7, 11, 13, 14, 15)$

AB \ CD	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	1	1	1
10	0	0	1	0

$$F(A, B, C, D) = CD + ABD + ABC = CD + AB(C + D)$$

(d) $F(w, x, y, z) = \Sigma(2, 3, 12, 13, 14, 15)$

wx \ yz	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	1	1	1	1
10	0	0	0	0

$$F(w, x, y, z) = wx + w'x'y$$

(e) $F(w, x, y, z) = \Sigma(11, 12, 13, 14, 15)$

wx \ yz	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	0	0	1	0

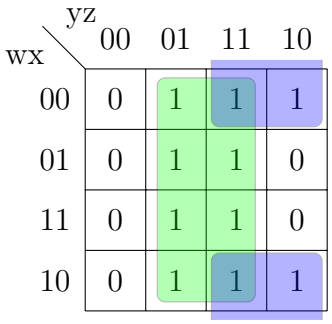
$$F(w, x, y, z) = wx + wyz = w(x + yz)$$

(f) $F(w, x, y, z) = \Sigma(8, 10, 12, 13, 14)$

wx \ yz	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	0	1
10	1	0	0	1

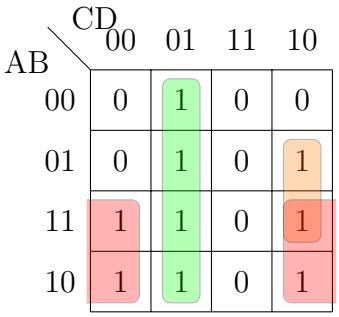
$$F(w, x, y, z) = wz' + wxy' = w(z' + xy')$$

2. (a) $F(w, x, y, z) = w'z + xz + x'y + wx'z$



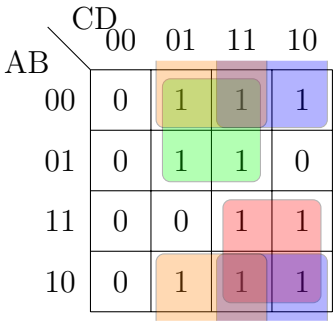
$$F(w, x, y, z) = z + x'y$$

(b) $F(A, B, C, D) = AD' + B'C'D + BCD' + BC'D$



$$F(A, B, C, D) = C'D + AD' + BCD'$$

(c) $F(A, B, C, D) = AB'C + B'C'D + BCD + ACD' + A'B'C + A'BC'D$



$$F(A, B, C, D) = A'D + AC + B'C + B'D$$

(d) $F(w, x, y, z) = wxy + xz + wx'z + w'x$

wx \ yz	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	1	1	1
10	0	1	1	0

$$F(w, x, y, z) = w'x + wz + xy$$

3. (a) $F(x, y, z) = xy + yz + xy'z$

x \ yz	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$F(x, y, z) = \Sigma(3, 5, 6, 7)$$

(b) $F(A, B, C, D) = C'D + ABC' + ABD' + A'B'D$

AB \ CD	00	01	11	10
00	0	1	1	0
01	0	1	0	0
11	1	1	0	1
10	0	1	0	0

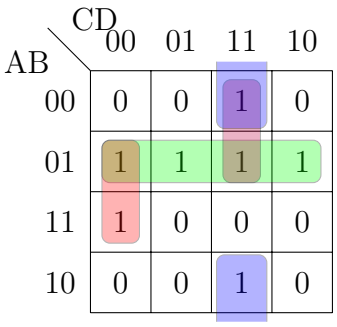
$$F(A, B, C, D) = \Sigma(1, 3, 5, 9, 12, 13, 14)$$

(c) $F(w, x, y, z) = wxy + w'x' + wxz'$

wx \ yz	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	1	0	1	1
10	0	0	0	0

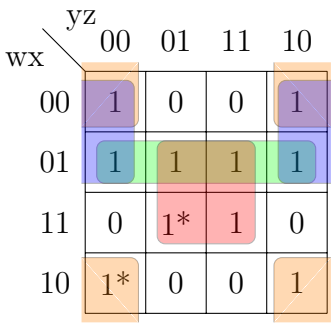
$$F(w,x,y,z) = \Sigma(0,1,2,3,12,14,15)$$

(d) $F(A,B,C,D) = A'B + A'CD + B'CD + BC'D'$



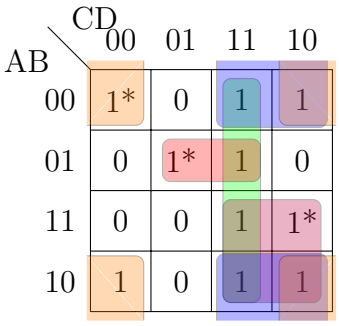
$$F(A,B,C,D) = \Sigma(3,4,5,6,7,11,12)$$

4. (a) $F(w,x,y,z) = \Sigma(0,2,4,5,6,7,8,10,13,15)$



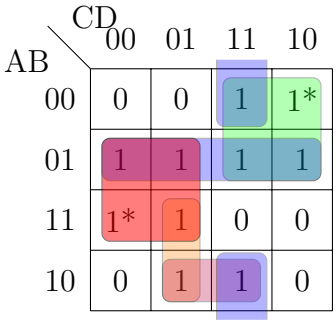
Prime Implicants :$w'z', x'z', w'x, xz$
Essential Implicants : $x'z', xz$

(b) $F(A,B,C,D) = \Sigma(0,2,3,5,7,8,10,11,14,15)$



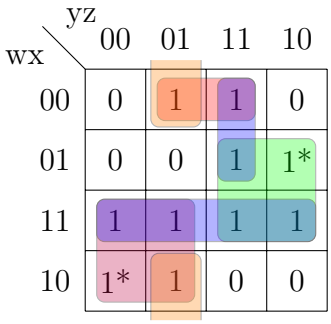
Prime Implicants :$B'D', B'C, CD, AC, A'BD$
Essential Implicants :$B'D', A'BD, AC$

(c) $F(A,B,C,D) = \Sigma(2,3,4,5,6,7,9,11,12,13)$



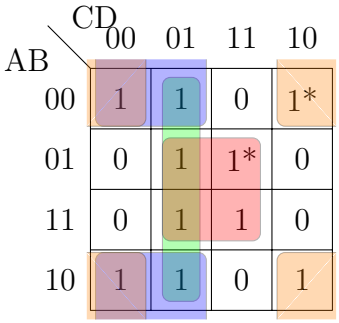
Prime Implicants :$A'C, A'B, BC', B'CD, AB'D, AC'D$
Essential Implicants :$A'C, BC'$

(d) $F(w, x, y, z) = \Sigma(1, 3, 6, 7, 8, 9, 12, 13, 14, 15)$



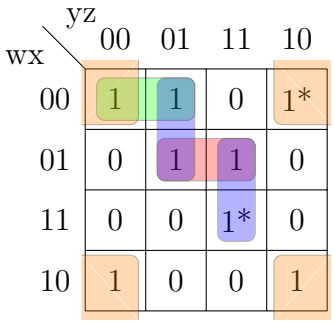
Prime Implicants : $wx, wy', xy, x'y'z, w'yz, w'x'z$
Essential Implicants :wy', xy

(e) $F(A, B, C, D) = \Sigma(0, 1, 2, 5, 7, 8, 9, 10, 13, 15)$



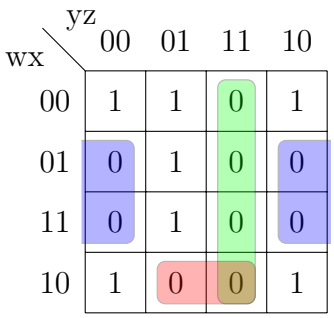
Prime Implicants : $B'D', C'D, BD, B'C'$
Essential Implicants :$BD, B'D'$

(f) $F(w, x, y, z) = \Sigma(0, 1, 2, 5, 7, 8, 10, 15)$



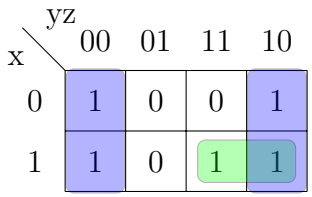
Prime Implicants : $x'z', w'x'y', w'y'z, w'xz, xyz$
Essential Implicants :$x'z', xyz$

5. (i) $F(w,x,y,z) = \Sigma(0,1,2,5,8,10,13) \implies \overline{F} = \Sigma(3,4,6,7,9,11,12,14,15)$



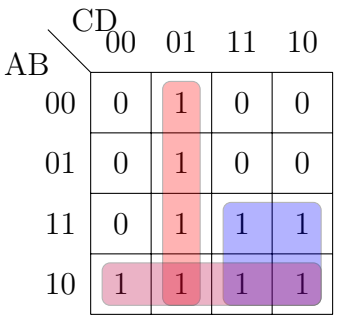
$\overline{F} = yz + xz' + wx'z \implies \overline{\overline{F}} = \overline{yz + xz' + wx'z} \implies F = (y'+z')(x'+z)(w'+x+z')$

(ii) (a) $x'z' + y'z' + yz' + xy$



$SOP = z' + xy \quad POS = (x'z + y'z)' = (x + z')(y + z')$

(b) $ACD' + C'D + AB' + ABCD$



$$SOP = AB' + C'D + AC$$
$$POS = (A'C + A'D' + BC'D')' = (A + C')(A + D)(B' + C + D)$$

(c) $(A+B+D')(A'+B'+C')(A'+B'+C)(B'+C+D')$

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	0	1	1
11	0	0	0	0
10	1	1	1	1

$$SOP = AB' + A'D' + A'BC$$
$$POS = (AB + A'B'D + BC'D)' = (A' + B')(A + B + D')(B' + C + D')$$

(d) $BCD' + ABC' + ACD$

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	1
11	1	1	1	1
10	0	0	1	0

$$SOP = AB + ACD + BCD'$$
$$POS = (B'D' + A'C' + A'D + B'C')' = (B + D)(A + C)(A + D')(B + C)$$

6. (a) $F(x, y, z) = \Sigma(0, 1, 4, 5, 6), d(x, y, z) = \Sigma(2, 3, 7)$

x \ yz	00	01	11	10
0	1	1	X	X
1	1	1	X	1

$$F = 1$$

(b) $F(A, B, C, D) = \Sigma(0, 6, 8, 13, 14), d(A, B, C, D) = \Sigma(2, 4, 10)$

AB \ CD	00	01	11	10
00	1	0	0	X
01	X	0	0	1
11	0	1	0	1
10	1	0	0	X

$$F = CD' + B'D' + ABC'D$$

(c) $F(A, B, C, D) = \Sigma(5, 6, 7, 12, 14, 15), d(A, B, C, D) = \Sigma(3, 9, 11)$

AB \ CD	00	01	11	10
00	0	0	X	0
01	0	1	1	1
11	1	0	1	1
10	0	1	X	0

$$F = BC + ABD' + A'BD$$

(d) $F(A, B, C, D) = \Sigma(4, 12, 7, 2, 10), d(A, B, C, D) = \Sigma(0, 6, 8)$

AB \ CD	00	01	11	10
00	X	0	0	1
01	1	0	1	X
11	1	0	0	0
10	X	0	0	1

$$F = C'D' + B'D' + A'BC$$

7. (i) $f = abc' + c'd + a'cd' + b'cd'$

AB \ CD	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	1	1	0	0
10	0	1	0	1

$$g = (a + b + c' + d')(b' + c' + d)(a' + c + d')$$

AB \ CD	00	01	11	10
00	1	1	0	1
01	1	1	1	0
11	1	0	1	0
10	1	0	1	1

$$f.g = (abc' + c'd + a'cd' + b'cd').(a + b + c' + d')(b' + c' + d)(a' + c + d')$$

AB \ CD	00	01	11	10
00	0	1	0	1
01	0	1	0	0
11	1	0	0	0
10	0	0	0	1

$$f.g = b'cd' + a'c'd + abc'd'$$

(ii) $F(A, B, C, D) = \Sigma(0, 4, 8, 9, 10, 11, 12, 14)$

AB \ CD	00	01	11	10
00	1	0	0	0
01	1	0	0	0
11	1	0	0	1
10	1	1	1	1

$$F = C'D' + AB' + AD'$$

$F'(A, B, C, D) = \Sigma(1, 2, 3, 5, 6, 7, 13, 15)$

AB \ CD	00	01	11	10
00	0	1	1	1
01	0	1	1	1
11	0	1	1	0
10	0	0	0	0

$$F' = A'D + BD + A'C$$
$$F = (F')' = (A'D + BD + A'C)' = (A'D)'(BD)'(A'C)'$$

Circuit Implementation:

Figure 1: NAND-AND

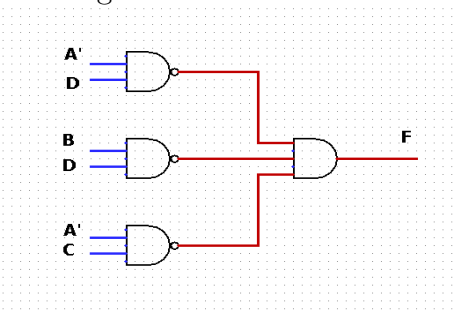


Figure 2: AND-NOR

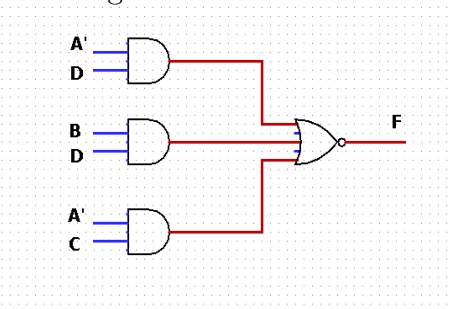


Figure 3: OR-NAND

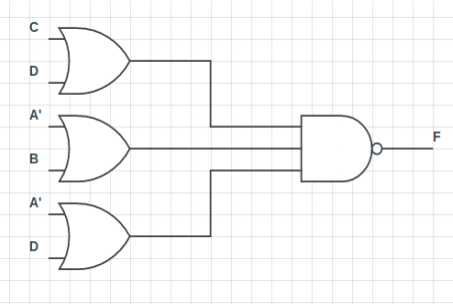
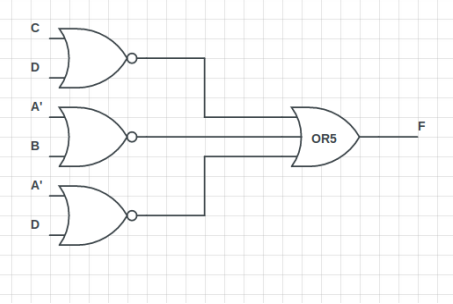


Figure 4: NOR-OR

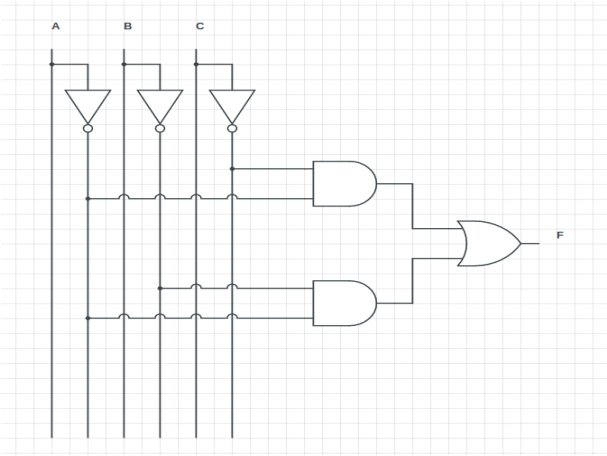


8. (i) (a) Output is 1 when the binary value of the inputs is less than 3.

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

C \ AB	00	01	11	10
0	1	1	0	0
1	1	0	0	0

$$F = A'C' + A'B'$$

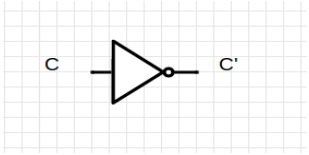


(b) Output is 1 when the binary value of the inputs is an even number.

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

C \ AB	00	01	11	10
0	1	1	1	1
1	0	0	0	0

$$F = C'$$

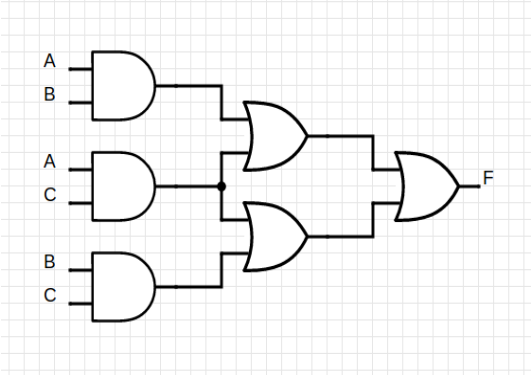


(ii) output is equal to 1 if the input variables have more 1s than 0s.

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

C \ AB	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$F = AB + AC + BC$$



9. (i) 4 bit gray code to binary code

$B_4 = A$

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

Table 1: Truth Table

Gray Code				Binary Code			
A	B	C	D	B_4	B_3	B_2	B_1
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	1	0	0	1	0
0	0	1	0	0	0	1	1
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	1
0	1	0	1	0	1	1	0
0	1	0	0	0	1	1	1
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	1
1	1	1	1	1	0	1	0
1	1	1	0	1	0	1	1
1	0	1	0	1	1	0	0
1	0	1	1	1	1	0	1
1	0	0	1	1	1	1	0
1	0	0	0	1	1	1	1

$B_3 = A'B + AB' = A \oplus B$

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	1	1	1	1

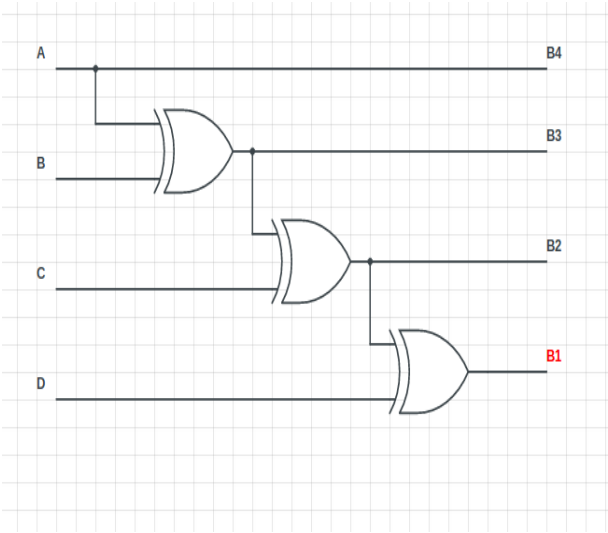
$B_2 = A'BC' + AB'C' + A'B'C + ABC = A(B'C' + BC) + A'(BC' + B'C)$

$= A(B \oplus C)' + A'(B \oplus C) = A \oplus B \oplus C$

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	1	0	0
11	0	0	1	1
10	1	1	0	0

$B_1 = AB'C'D' + AB'CD + ABCD' + ABC'D + A'BC'D' + A'BCD + A'B'C'D + A'B'C'D'$
$= A(BC + B'C')D' + A(BC' + B'C)D + A'(BC + B'C')D + A'(BC' + B'C)D'$
$= (BC + B'C')(AD' + A'D) + (BC' + B'C)(AD + A'D')$
$= (B \oplus C)'(A \oplus D) + (B \oplus C)(A \oplus D)'$
$= A \oplus B \oplus C \oplus D$

AB \ CD	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0



(ii) Four-bit combinational circuit 2s complementer

Table 2: Truth Table

Binary Code				2's Complement			
A	B	C	D	C_4	C_3	C_2	C_1
0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0
0	0	1	1	1	1	0	1
0	1	0	0	1	1	0	0
0	1	0	1	1	0	1	1
0	1	1	0	1	0	1	0
0	1	1	1	1	0	0	1
1	0	0	0	1	0	0	0
1	0	0	1	0	1	1	1
1	0	1	0	0	1	0	0
1	0	1	1	0	1	0	1
1	1	0	0	0	1	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	0	1	0
1	1	1	1	0	0	0	1

$$C_4 = A'D + A'C + A'B + AB'C'D' = A'(B+C+D) + A(B+C+D)' = A \oplus (B+C+D)$$

CD \ AB	00	01	11	10
00	0	1	0	1
01	1	1	0	0
11	1	1	0	0
10	1	1	0	0

$$C_3 = B'D + B'C + BC'D' = B'(C+D) + B(C+D)' = B \oplus (C+D)$$

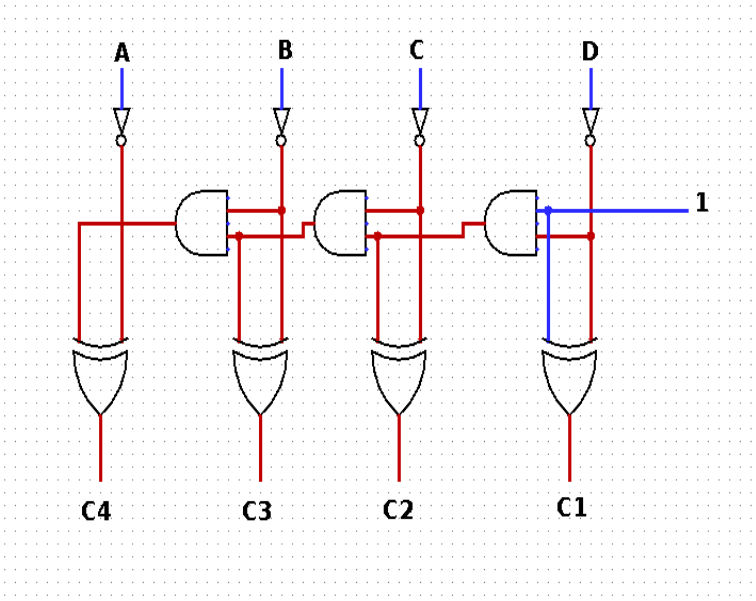
CD \ AB	00	01	11	10
00	0	1	1	0
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

$$C_2 = C'D + CD' = C \oplus D$$

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	1	1	1	1

$$C_1 = D$$

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

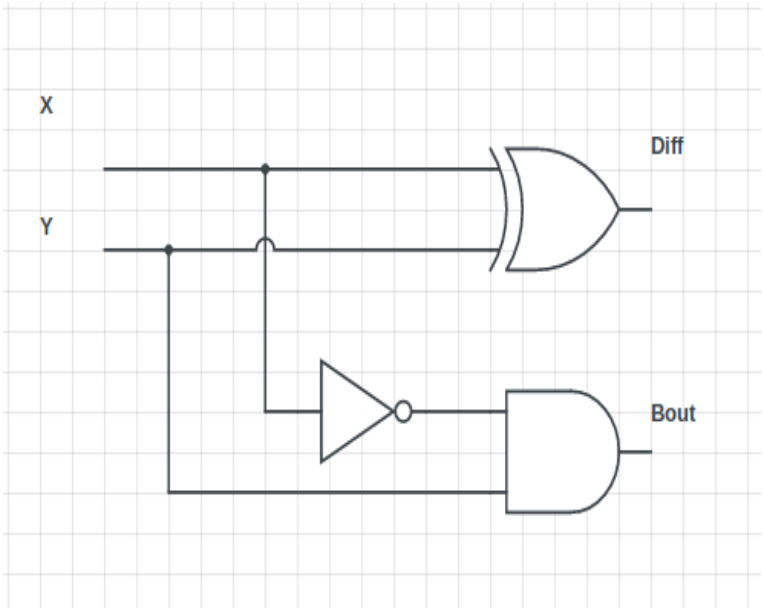


For 5 bit conversion, ABCDE $\rightarrow C_5C_4C_3C_2C_1$
$C_5 = A \oplus (B + C + D + E)$
$C_4 = B \oplus (C + D + E)$
$C_3 = C \oplus (D + E)$
$C_2 = D \oplus E$
$C_1 = E$

10. (i) Half subtractor
Input= X, Y Output= D_{iff}, B_{out}

X	Y	D_{iff}	B_{out}
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

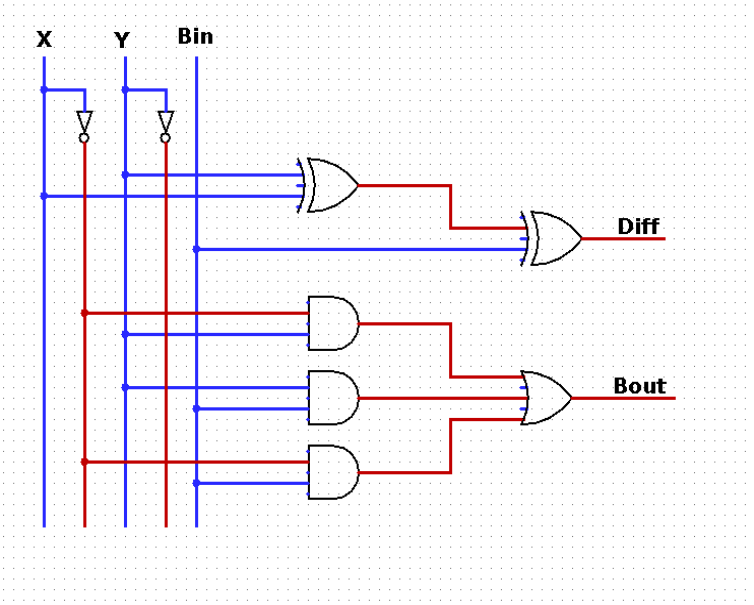
$D_{iff} = X \oplus Y, \qquad B_{out} = X'Y$



(ii) Full Subtractor
Input= X, Y, B_{in} Output= D_{iff}, B_{out}

X	Y	B_{in}	D_{iff}	B_{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$D_{iff} = X \oplus Y \oplus B_{in}$
$B_{out} = X'Y + Y B_{in} + X' B_{in}$


X
Y
Bin
Diff
Bout