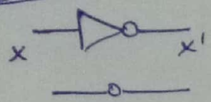
## DIGITAL SYSTEMS

24/01

### BASIC LOGICAL OPERATIONS

**NOT**



| $x$ | $x'(\bar{x})$ |
|---|---|
| 0 | 1 |
| 1 | 0 |

$x' \rightarrow$ "x - compliment"

**AND**

| $x$ | $y$ | $x \cdot y$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**OR**

| $x$ | $y$ | $x+y$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Logical constants – 0, 1.

Variables – $x, y, z$.

(Const) op (Const) $\rightarrow$ Pre defined by ~~the~~ truth tables.

(Const) op (Variable) $\rightarrow$ $x + 0 = x$ || $x \cdot 0 = 0$

$x + 1 = 1$ || $x \cdot 1 = x$ (Identity)

(Variable) op (Same variable) $\rightarrow$ $x + x = x$ || $x \cdot x = x$

**NOT**

$(x')' = x$ $\Rightarrow$ Odd No. of compliments – $x'$

Even No. of compliments – $x$.

### TWO VARIABLES

$x + y = y + x$. $\Rightarrow$ ('OR' is commutative)

$x \cdot y = y \cdot x$ $\Rightarrow$ ('$\cdot$' is commutative)

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### Evaluation Scheme

Quiz - 1 – 12

Quiz - 2 – 13

Tutorials – 10

End Sem – ~~30~~ 35

Lab – 30.

## THREE VARIABLES $(x, y, z)$.

$(x+y)+z = x+(y+z)$. $\Rightarrow$ '+' is Associative.

$= x+y+z$

## TRUTH TABLE.

| $(xyz)_2$ | x $(2^2)$ | y $(2^1)$ | z $(2^0)$ | x + y | y + z | (x + y) + z | x + (y + z) |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | | | |
| 1 | 0 | 0 | 1 | | | | |
| 2 | 0 | 1 | 0 | | | | |
| 3 | 0 | 1 | 1 | | | | |
| 4 | 1 | 0 | 0 | | | | |
| 5 | 1 | 0 | 1 | | | | |
| 6 | 1 | 1 | 0 | | | | |
| 7 | 1 | 1 | 1 | | | | |

$x \cdot y \cdot z = (x \cdot y) \cdot z = x \cdot (y \cdot z) \Rightarrow$ '·' is Associative.

## THREE VARIABLES & TWO OPERATIONS.

$x \cdot (y+z) = x \cdot y + x \cdot z \rightarrow$ '·' is distributive over the op. '+'.

$x + (y \cdot z) = (x+y)(x+z) \Rightarrow$ '+' is distributive over '·'.

$= x + yx + xz + yz$

$= x(\underbrace{1+y+z}) + yz$

$= x + yz \quad (\because (1+x = x))$.

# Boolean Algebra (George Boole)

A set of elements $S(\ldots\ldots)$ along with two operators.
$(\star, \square)$

→ The set $S$ is closed under $\star$ if $x, y \in S$ & $x \star y \in S$

" under $\square$ if $x, y \in S$ & $x \square y \in S$

## Commutative law

$$x \star y = y \star x ;$$

$$x \square y = y \square x .$$

$\star$ is distributive over $\square$ ⇒ $x \star (y \square z) = (x \star y) \square (x \star z)$

$\square$ " " " $\star$ ⇒ $x \square (y \star z) = (x \square y) \star (x \square z)$

## Inverse

For every $x \in S$ there is an element $\bar{x}$ such that

$$x \star \bar{x} = 1$$

$$x \square \bar{x} = 0 .$$

## Identity.

Identity element for $\star$ (say 0) → $x \star 0 = 0 \star x = x .$

for $\square$ (say 1) ⇒ $x \square 1 = 1 \square x = x .$

$(0, 1)$ $(+, \cdot)$

$$x + \bar{x} = 1$$

$$x \cdot \bar{x} = 0$$

## SIMPLIFICATION THEOREMS

$\rightarrow \quad x + xy = x. \quad \rightarrow \quad x(1+y) = x \cdot 1$

$xy + x'y = y \quad \Rightarrow \quad (x+x')y = 1 \cdot y.$

$(x+y')y = xy \quad \Rightarrow \quad xy + yy' = xy + 0 .$

## DEMORGAN'S LAW

$\Rightarrow \quad (x+y)' = x' \cdot y'.$

| $x$ | $y$ | $x+y$ | $(x+y)'$ | $x'$ | $y'$ | $x' \cdot y'$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0. |

IIly $\quad (x \cdot y)' = x' + y'.$

To generalize.

$$F'(x, y, z, 0, 1, +, \cdot) = F(x', y', z', 1, 0, \cdot, +)$$