Introduction to Robotics Assignment 3

S Sai Rohit (14110126)

Assumption:

The RRR serial chain manipulator lies on x-y plane.

Given parameters:

1. Lengths of all the links of serial chain manipulator which are$l_1, l_2 and l_3$.
2. Three joint angles of the serial chain manipulator, which are $\theta_1, \theta_2 and \theta_3$ respectively.
3. End factor position and orientation are denoted by $p_x and p_y and \phi$.

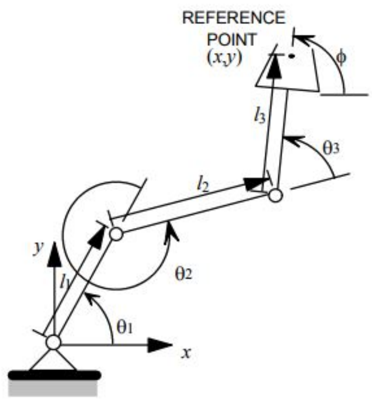

Figure 1: The joint variables and link lengths of the 3R serial manipulator[1]

1 Solution:

To find: To formulate the forward kinematics transformation to calculate the end-effector position and orientation.To calculate when$L_1 = 4m, L_2 = 3m and L_3 = 2m$.

Formula:

$p_x = l_1 * cos(\theta_1) + l_2 * cos(\theta_1 + \theta_2) + l_3 * cos(\theta_1 + \theta_2 + \theta_3)$
$p_y = l_1 * sin(\theta_1) + l_2 * sin(\theta_1 + \theta_2) + l_3 * sin(\theta_1 + \theta_2 + \theta_3)$
$\phi = \theta_1 + \theta_2 + \theta_3$
all the angles to be entered in degrees.
Solved with $\theta_1 = 0$ degrees, θ_2 =0 degrees and $\theta_3 = 0$ degrees
Output is $p_x = 9$, $p_y = 0$ and $\phi = 0$ degrees
Solved with $\theta_1 = 30$ degrees, θ_2 =60 degrees and $\theta_3 = 30$ degrees
Output is $p_x = 2.4641$, $p_y = 6.7321$ and $\phi = 120$ degrees

2 Solution

To find: To develop simple equations to solve for joint variables when end effector position and orientation are known.To calculate when$L_1 = 4m, L_2 = 3mandL_3 = 2m$.

Method:

After considering the inputs l_1, l_2 and l_3. p_x, p_y and ϕ. We calculate the position of joint between the link 2and link 3.
x = p_x - (l_3 * cos(ϕ))
y = p_y - (l_3 * sin(ϕ))
Calculate θ_2
$\theta_2 = \cos^{-1}(x^2 + y^2 - l_1^2 - l_2^2)/(2 * l_1 * l_2)$
sin $\theta_2 = \sqrt{(1 - cos\theta_2^2)}$
$k_1 = l_1 + l_2 cos\theta_2$
$k_2 = l_2 sin\theta_2$
$\gamma_1 = tan^{-1}(k_2/k_1)$
$\theta_1 = tan^{-1}(p_y/p_x) - \gamma_1$
$\theta_3 = \phi - \theta_1 - \theta_2$
second case
$\theta_2^{'}$ = - $\cos^{-1}(x^2 + y^2 - l_1^2 - l_2^2)/(2 * l_1 * l_2)$
sin $\theta_2{'}$ = - $\sqrt{(1 - cos\theta_2^2)}$
$k_1 = l_1 + l_2 cos\theta_2$
$k_2^{'} = -l_2 sin\theta_2$
$\gamma_1^{'} = tan^{-1}(k_2/k_1)$
$\theta_1^{'} = tan^{-1}(p_y/p_x) - \gamma_1$
$\theta_3^{'} = \phi - \theta_1 - \theta_2$
By using the above equations we get two solutions of joint angles.
$Solvedexamples$
Inputs: $p_x = 9, p_y = 0and\phi = 0$
Outputs: $\theta_1 = 0, \theta 2 = 0and\theta_3 = 0$.
$\theta_1 = 0, \theta 2 = 0and\theta_3 = 0$.
Inputs: $p_x = 5, p_y = 6and\phi = 30$
Outputs: $\theta_1 = 23.465, \theta 2 = 63.57and\theta_3 = -57.04$.
$\theta_1 = 76.92, \theta 2 = -63.57and\theta_3 = 16.6542$.

3 Solution

Plots for different cases are shown by Figure 2, Figure 3 and Figure4 respectively and the blue shaded region is the space where the end factor can reach under the given condition.
When,$\phi = 0, L_1 = 4m, L_2 = 3mandL_3 = 2m$

Yes, it is possible to find out the workspace covered without any constraint on ϕ. It appears similar to the above mentioned image.

4 Solution

Jacobian matrix can be developed from the equations:
$p_x = l_1 * cos(\theta_1) + l_2 * cos(\theta_1 + \theta_2) + l_3 * cos(\theta_1 + \theta_2 + \theta_3)$
$p_y = l_1 * sin(\theta_1) + l_2 * sin(\theta_1 + \theta_2) + l_3 * sin(\theta_1 + \theta_2 + \theta_3)$
$\phi = \theta_1 + \theta_2 + \theta_3$
By differentiating on both sides the above equations we get,
$\dot{x} = -l_1\dot{\theta_1} sin(\theta_1) - l_2(\dot{\theta_1} + \dot{\theta_2}) sin(\theta_1 + \theta_2) - l_3(\dot{\theta_1} + \dot{\theta_2} + \dot{\theta_3}) sin(\theta_1 + \theta_2 + \theta_3)$

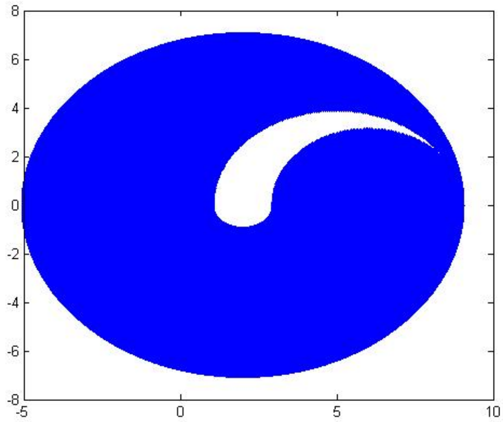

Figure 2: Space covered by end effector when $\phi = 0^o$

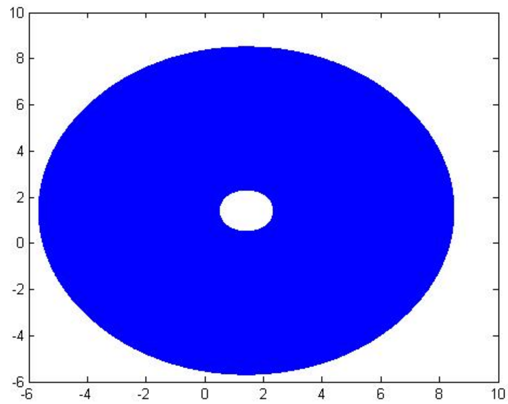

Figure 3: Space covered by end effector when $\phi = 45^o$

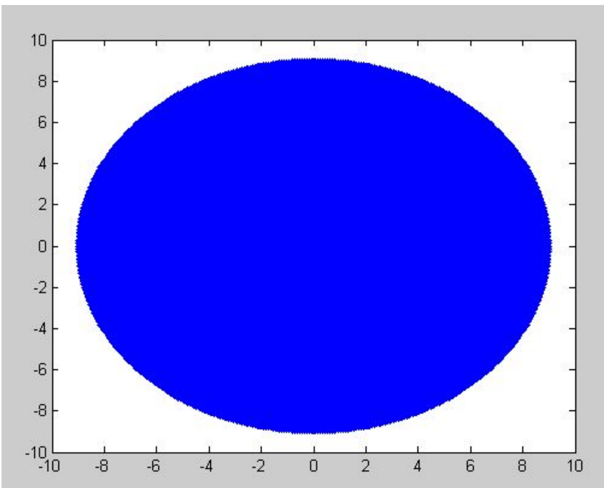

Figure 4: Space covered by end effector when there are no limitations

$\dot{y} = l_1\dot{\theta_1}cos(\theta_1) + l_2(\dot{\theta_1} + \dot{\theta_2})cos(\theta_1 + \theta_2) + l_3(\dot{\theta_1} + \dot{\theta_2} + \dot{\theta_3})cos(\theta_1 + \theta_2 + \theta_3)$
$\dot{\phi} = \dot{\theta_1} + \dot{\theta_2} + \dot{\theta_3}$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\phi}\end{bmatrix} = \begin{bmatrix}-l_1sin\theta_1 - l_2sin(\theta_1+\theta_2) - l_3sin(\theta_1+\theta_2+\theta_3) & -l_2sin(\theta_1+\theta_2) - l_3sin(\theta_1+\theta_2+\theta_3) & -l_3sin(\theta_1+\theta_2+\theta_3)\\ l_1cos\theta_1 + l_2cos(\theta_1+\theta_2) + l_3cos(\theta_1+\theta_2+\theta_3) & l_2cos(\theta_1+\theta_2) + l_3cos(\theta_1+\theta_2+\theta_3) & l_3cos(\theta_1+\theta_2+\theta_3)\\ 1 & 1 & 1\end{bmatrix}$$

$$J = \begin{bmatrix}-l_1sin\theta_1 - l_2sin(\theta_1+\theta_2) - l_3sin(\theta_1+\theta_2+\theta_3) & -l_2sin(\theta_1+\theta_2) - l_3sin(\theta_1+\theta_2+\theta_3) & -l_3sin(\theta_1+\theta_2+\theta_3)\\ l_1cos\theta_1 + l_2cos(\theta_1+\theta_2) + l_3cos(\theta_1+\theta_2+\theta_3) & l_2cos(\theta_1+\theta_2) + l_3cos(\theta_1+\theta_2+\theta_3) & l_3cos(\theta_1+\theta_2+\theta_3)\\ 1 & 1 & 1\end{bmatrix}$$

Using the above relation, we can develop the Jacobian matrix.

5 Solution

Using Jacobian matrix, the singularity cases can be possible when det(J)=0. By running the program we can compute the results for which det(J) = 0 using MATLAB.
Important observations
1. For satisfying the singularity condition, the determinant of Jacobian matrix should be equal to zero.
2. When any of the joint angle is zero then the singularity case satisfies and manipulator loses one degree of freedom.
3. This also happens when the joint between ground and link1, joint of link2 and link3 and end effector point lies on a straight.
4. By iterating the condition with different range of joint angles there are large set of triplets where the manipulator satisfies the singularity condition. The results can be seen by running the program although it is time taking program.

6 Solution

a. We know,
For static equilibrium, $\sum F = 0$
${}^if_i -^i f_{i+1} = 0$
Similary,
$\sum M = 0$
${}^in_i -^i n_{i+1} -^i O^i_{i+1}f_{i+1} = 0$
${}^if_i =^i_{i+1} [R]^{i+1}f_{i+1}$
${}^in_i =^i_{i+1} [R]^{i+1}n_{i+1} +^i O^i_{i+1}f_i$
At end effector if not in contact with environment will be zero.
3R manipulator applying force.
${}^0f_{Tool} = (f_x, f_y, 0)^T$
${}^0n_{Tool} = (0, 0, n_z)^T$

$$\begin{bmatrix}f'_x\\ f'_y\\ 0\end{bmatrix} = \begin{bmatrix}cos(\theta_1+\theta_2+\theta_3) & sin(\theta_1+\theta_2+\theta_3) & 0\\ -sin(\theta_1+\theta_2+\theta_3) & cos(\theta_1+\theta_2+\theta_3) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}f_x\\ f_y\\ 0\end{bmatrix}$$

and $(0, 0, n'_z)^T = (0, 0, N_Z)^T$
for i=3
${}^3f_3 = (f'_x, f'_y, 0)^T$
${}^3n_3 = (0, 0, n'_z + l_3f'_y)^T$
for i=2

$^2f_2 = (cos\theta_3 f'_x - sin\theta_3 f'_y, sin\theta_3 f'_x + cos\theta_3 f'_y, 0)^T$
$^2n_2 = (0, 0, n'_z + l_2(sin\theta_3 f'_x + cos\theta_3 f'_y) + l_3 f'_y)$
for i=1
$^1f_1 = (cos(\theta_3 + \theta_2)f'_x - sin(\theta_2 + \theta_3)f'_y, sin\theta_3 f'_x + cos\theta_3 f'_y, 0)^T$
$^1n_1 = (0, 0, n'_z + l_1(sin(\theta_2 + \theta_3)f'_x + cos(\theta_2 + \theta_3)f'_y) + l_2(sin\theta_3 f'_x + cos\theta_3 f'_y) + l^3 f'_y)^T$
$\tau_1 = n'_z + f'_x(l_1 sin(\theta_2 + \theta_3) + l_2 sin\theta_3) + f'_y(l_1 cos\theta_2 + \theta_3 + l_2 cos\theta_3 + l_3)$
$\tau_2 =^2 n_2^2 Z_2 = n'_z + f'_x l_2 sin\theta_3 + f'_y(l_2 cos\theta_3 + l_3)$
$\tau_3 = n'_z + f'_y l_3$

$$\tau = \begin{bmatrix} -l_1 sin\theta_1 - l_2 sin(\theta_1 + \theta_2) - l_3 sin(\theta_1 + \theta_2 + \theta_3) & l_1 cos\theta_1 + l_2 cos(\theta_1 + \theta_2) + l_3 cos(\theta_1 + \theta_2 + \theta_3) & 1 \\ -l_2 sin(\theta_1 + \theta_2) - l_3 sin(\theta_1 + \theta_2 + \theta_3) & l_2 cos(\theta_1 + \theta_2) + l_3 cos(\theta_1 + \theta_2 + \theta_3) & 1 \\ l_3 sin(\theta_1 + \theta_2 + \theta_3) & l_3 cos(\theta_1 + \theta_2 + \theta_3) & 1 \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ n_z \end{bmatrix}$$

From the above obtained equation[2] is the relation between the force on the body and torque joints

b. The relation matrix obtained in here is similar to the transpose of the Jacobian matrix.

7 Solution

There are many approaches for equations of motion for 3 R manipulator such as Newton Euler Form of Equation of motion, Lagrangian method of formulation of Equation of motion. Lagrangian Formulation is the easiest method as it easily solvable.

References

[1] *http://www.seas.upenn.edu/ meam520/notes/planar.pdf* .

[2] *http://nptel.ac.in/courses/112108093/module5/lecture.pdf*

[3] *https://ocw.mit.edu/courses/mechanical-engineering/2-12-introduction-to-robotics-fall-2005/lecture-notes/chapter7.pdf*

[4] *https://ocw.mit.edu/courses/mechanical-engineering/2-12-introduction-to-robotics-fall-2005/lecture-notes/chapter7.pdf*