ME 322: Introduction to Robotics (Fall'17)
Assignment 3

Kinematics and Dynamics of a Serial-Chain Planar Manipulator

As part of this assignment, develop a Matlab program to analyze the kinematics and dynamics of a serial-chain planar manipulator. This work will help you revise the course content covered so far, and prepare you for the upcoming topics. Stick to symbolic programming, the program should be sufficiently commented and should be developed with modules corresponding to following sections. (You are encouraged to use Matlab Symbolic toolbox but it's not mandatory)

Let's consider a planar 3-DOF, 3R robot (all revolute joints), with link lengths l_1, l_2 and l_3. Joint variables are denoted as θ_i, i=1-3, and end-effector position and orientation are denoted as (p_x and p_y) and ϕ respectively.

1. **Forward Kinematics:** Assign clearly the coordinate frames and define the DH parameters.
   a. Formulate the forward kinematics transformation to calculate the end-effector position and orientation.
   b. Use your program to solve for any two cases, assume L_1 = 4m, L_2 = 3m and L_3 = 2m.

2. **Inverse Kinematics:**
   a. Develop equations to solve for the joint variables when the end-effector position and orientation are available.
   b. Use your program to solve for any two cases, assume L_1 = 4m, L_2 = 3m and L_3 = 2m.

3. **Workspace:** Workspace is defined as the set of points reachable by the end-effector of a manipulator.
   a. Plot the workspace of the 3R manipulator when $\phi = 0^0$, L_1 = 4m, L_2 = 3m and L_3 = 2m.
   b. Plot the workspace of the 3R manipulator when $\phi = 45^0$, L_1 = 4m, L_2 = 3m and L_3 = 2m.
   c. Is it possible to calculate the workspace of the 3R manipulator when there is no constraint on ϕ? If so then plot it. Discuss your results in comparison with part (a and b).

4. **Jacobian:** Develop the Jacobian matrix to analyze the velocity kinematics of the 3R planar manipulator.

5. **Singularities:** Using the Jacobian matrix, find all possible singular configurations of the 3R manipulator. Discuss your results and provide intuitive interpretation. (if needed assume L_1 = 4m, L_2 = 3m and L_3 = 2m)

6. **Statics:** For a configuration inside the workspace, say the end-effector applies force (F_x and F_y) and moment (M_z) on an object. Also, let's say T_i, i=1-3, are the corresponding joint torques. Assume that the manipulator is not moving and ignore gravity.
   a. Develop the relations between the joint torques and the end-effector force and moment.

b. Compare the obtained relations with the Jacobian matrix (section 4), and discuss your observations.

7. **Dynamics:**
   a. Finally, use any method to develop the equations of motion for the 3R manipulator.
   b. Discuss how to approach the forward and inverse dynamics problem?

Submit following in a zipped folder (with your name):

1. **Matlab program** - should have comments to run and to verify your results.
2. **Report** describing sections 1 to 7 in readable form to provide equations, explanation, figures and solution to above questions.
3. **LaTex file** used for the report.