Assignment 2

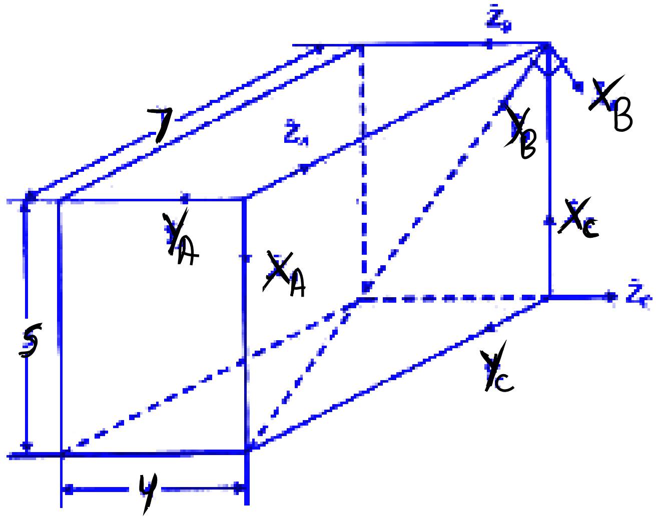
1. Write a Matlab subroutine that changes representation of orientation from rotation-matrix form to equivalent angle-axis form. Point out if there are some special cases and that the subroutine takes care of them.
2. Under what condition do two rotation matrices representing finite rotations commute? A proof is not required.
3. Two frames $\{A\}$ and $\{B\}$ are related by the following matrix:

$$T_A^B = \begin{bmatrix} 0 & 1 & 0 & 15 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & -9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

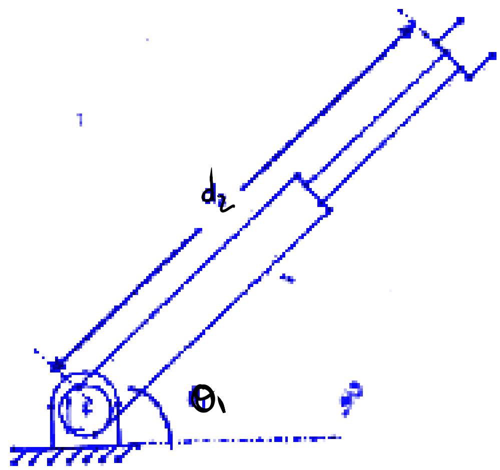
   a) Where is the origin of the frame $\{B\}$ located when seen in frame $\{A\}$?
   b) If the orientation vector of a parameter is given in frame $\{B\}$ as, $V_B = [10\ 10\ 10]^T$. What is the orientation in frame $\{A\}$?
   c) If there exists a point P whose coordinates in frame $\{A\}$ are given as $P_A = [10\ 10\ 10]^T$, find its coordinates in frame $\{B\}$.
4. A vector must be mapped through three rotation matrices: $P_A = R_A^B R_B^C R_C^D P_D$. One choice is to multiply the three rotation matrices together, to form R_A^D in the expression: $P_A = R_A^D\ P_D$ Another choice is to transform the vector through matrices one at a time, that is,

$$P_A = R_A^B R_B^C R_C^D P_D$$
$$P_A = R_A^B R_B^C P_C$$
$$P_A = R_A^B P_B$$
$$P_A = P_A$$

   If P_D is changing at 100 Hz, we would have to recalculate P_A at the same rate. However the three rotation matrices are also changing, as reported by a vision system that gives us new values for R_A^B, R_B^C and R_C^D at 30 Hz. What is the best way to organize the computation to minimize the calculation efforts (multiplications and additions)?
5. Referring to Figure(I)
   a) Write the matrix T_C^A , T_B^A and T_C^B .
   b) Write the matrix T_A^C from T_C^A and verify your answer from Figure(I)
6. Imagine two unit vectors, v_1 and v_2 , embedded in a rigid body. Note that, no matter how the body is rotated, the geometric angle between these two vectors is preserved (i.e., rigid body rotation is an “angle preserving” operation). Use this fact to give a concise proof that the inverse of rotation matrix must equal its transpose and a rotation matrix is orthonormal.
7. Figure(II) shows a two-link RP planar manipulator. Obtain the transformation matrix T_0^2 and find the coordinates of the point P in the base reference frame if its is $[2\ 3\ 4]^T$ in the end-effector frame.

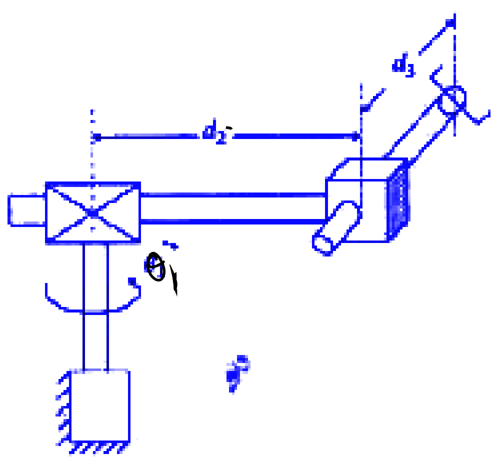


Figure(I): Frames $\{A\}$, $\{B\}$ and $\{C\}$ are at the corners of the prism (dimensions: 4, 5 and 7m).



Figure(II):Two-link RP manipulator. θ_1 and d_2 are variables

8. Assign link frames RPP spatial robot shown in Figure(III) and give the D-H parameters.



Figure(III):RPP spatial manipulator, θ_1, d_2 and d_3 are joint variables

9. For a planar 3-DOF, 3R robot (all revolute joints), with link lengths L_1, L_2 and L_3.

a) Assign coordinate frames and derive the DH parameters.

b) Derive the neighbouring homogenous transformation matrices $T_{i-1}^{i},\ i = 1, 2, 3$. These are functions of the joint-angle variables $\theta_i,\ i = 1, 2, 3$.

c) Use symbolic Matlab to derive the forward-pose kinematics solution T_0^3 symbolically (as a function of θ_i).

d) Calculate the forward-pose kinematics results via Matlab for the following input cases ($L_1 = 4m$, $L_2 = 3m$ and $L_3 = 2m$)

   i) $\Theta = [\theta_1\ \theta_2\ \theta_3]^T = \{0\ 0\ 0\}^T$.

   ii) $\Theta = [10^o\ 20^o\ 30^o\ ]^T$.

   iii) $\Theta = [90^o\ 90^o\ 90^o\ ]^T$.