Robotics Mid Term

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1 Solution

For a planar 2-DOF , 2R robot.
lengths are $l_1 and l_2$.
Joint positions are denoted by q_i
Joint velocities are denoted by q_{di}
Joint acceleration are denoted by q_{ddi}, where i = 1,2.
Torques are denoted by T_i.
We know,
$v_{c1} = J_{v_{c1}}\dot{q}$

$$J_{c_{v1}} = \begin{bmatrix} -l_{c1}sinq_1 & 0 \\ l_{c1}cosq_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$v_{c2} = J_{v_{c2}}\dot{q}$

$$J_{c_{v2}} = \begin{bmatrix} -l_1 sinq_1 - l_{c2}sin(q_1+q_2) & -l_{c2}sin(q_1+q_2) \\ l_1 cosq_1 + l_{c2}cos(q_1+q_2) & l_{c2}cos(q_1+q_2) \\ 0 & 0 \end{bmatrix}$$

Hence,
Translational Kinetic Energy
$\frac{1}{2}m_1 v_{c1}^T v_{c1} + \frac{1}{2}m_2 v_{c2}^T v_{c2} = \frac{1}{2}\dot{q}^T m_1 J_{v_{c1}}^T J_{V_{c2}^T} J_{V_{c2}}\dot{q}$

$\omega_1 = \dot{q}_1 k, \omega_2 = (\dot{q}_1 + \dot{q}_2)k$
$\frac{1}{2}\dot{q}^T I_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \dot{q} + I_2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \dot{q}$
$D(q) = m_1 J_{v_{c1}}^T J_{v_{c1}} + m_2 J_{v_{c2}}^T J_{v_{c2}} + \begin{bmatrix} I_1 + I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}$

$d_{11} = m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2} cosq_2) + I_1 + I_2$
$d_{12} = d_{21} = m_2(l_{c2}^2 + l_1 l_{c2} cosq_2) + I_2$
$d_{22} = m_2 l_{c2}^2 + I_2$
Christoffel symbols
$c_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$
$c_{121} = c_{211} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -m_2 l_1 l_{c2} sinq_2 = h$
$c_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = h$
$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -h$
$c_{122}c_{212} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$
$c_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$
Potential Energy
$V_1 = m_1 g l_{c1} sinq_1$
$V_2 = m_2 g(l_1 sinq_1 + l_{c2} sin(q_1+q_2))$
$V = V_1 + V_2 = (m_1 l_{c1} + m_2 l_1) g sinq_1 + m_2 l_{c2} g sin(q_1+q_2)$
$\phi_1 = \frac{\partial V}{\partial q_1} = (m_1 l_{c1} + m_2 l_1) g cosq_1 + m_2 l_{c2} g cos(q_1+q_2)$

$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 l_{c2} cos(q_1 + q_2)$
$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_1\dot{q}_2 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2^2 + \phi_1 = \tau_1$
$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 = \tau_2$

$$C = \begin{bmatrix} h\dot{q}_2 & h\dot{q}_2 + h\dot{q}_1 \\ -h\dot{q}_1 & 0 \end{bmatrix}$$

For the following initial condition at t=0. $IC = [q_1 q_2 q_{d1} q_{d2}]$
$l_1 = l_2 = 1$
$m_1 = m_2 = 1$
$I_1 = I_2 = 1/12$
$[t_i t_f] = [010]$

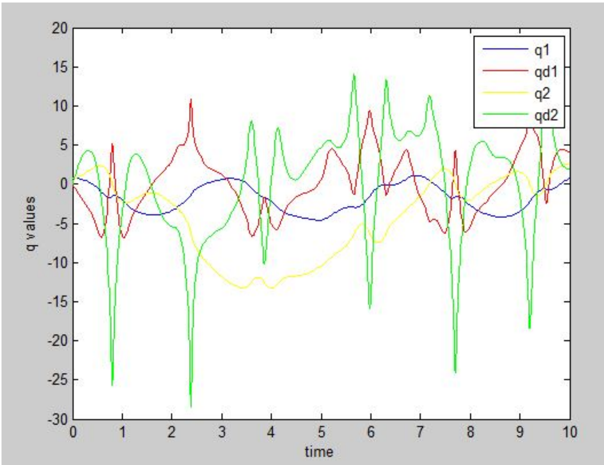

Figure 1: Plot of the joint variables q1, q2 ,qd1 and qd2

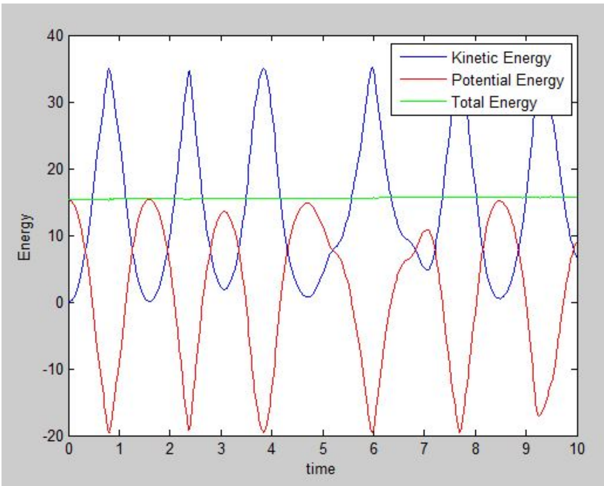

Figure 2: Plot of the Energy variables of the planar system

We can observe in figure 2 the slight increase in the Total energy as we move towards the time t=10.The Kinetic Energy and Potential Energy are conserved and can be seen oscillating w.r.t each other such that sum of them remains constant.Since most of the time center of mass of the manipulator is in the negative y axis the potential energy is mostly negative.

Due to the sinusoidal joint torques as shown in figure 3 we can see the results obtained by the joint torques are also nearly following the sinusoidal.This similar to constructive and destructive inference where when Torques are applied in the same direction then the magnitude of the joint variables reach maximum and vice versa.

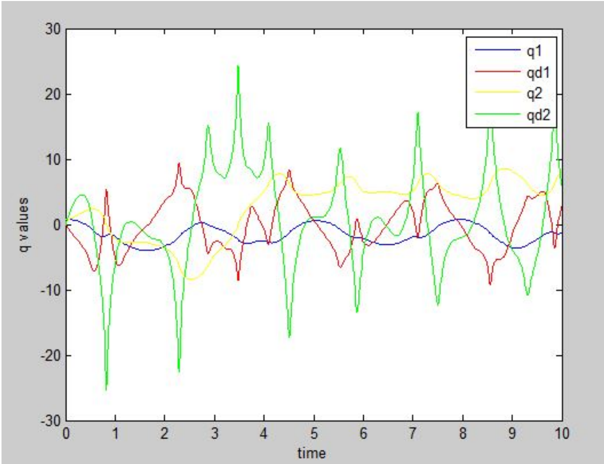

Figure 3: Plot represents different joint variables of manipulator

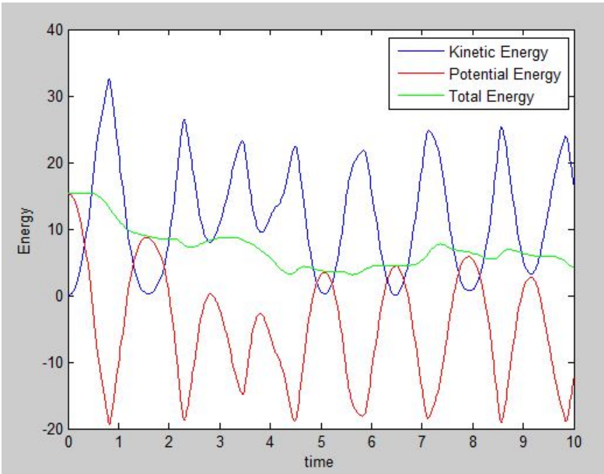

Figure 4: Plot represents the Energy terms of the 2R planar manipulator

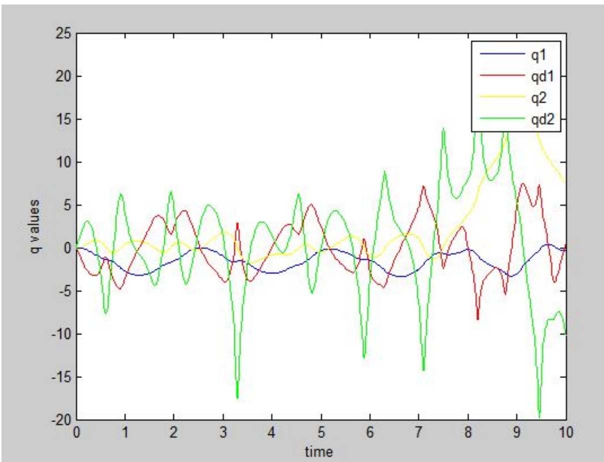

Figure 5:

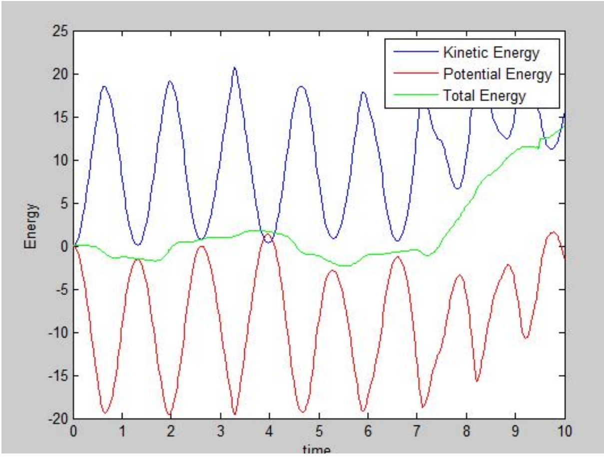

Figure 6: The joint variables and link lengths of the 3R serial manipulator[1]

From the figure 6 and figure 4 comparison of the change in energy terms we can observe depending on the initial state we can see that in case of figure 6 there is increase the total energy as it starts from lowest position compared to the initial position of the figure 4. In case of figure 4 there is decrease in the total energy observed.

2 Solution

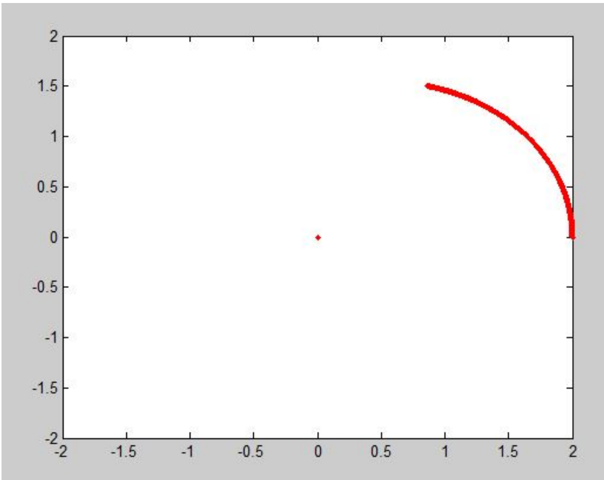

Figure 7: Path trajectory of the end effector of the 2R manipulator

3 Solution

As shown in figure 11 and figure 12 we can see the joint variables keeps on oscillating to and fro as we fit the joint variables using cubic equation this acts as a feed back effect because of which it angles keep on increasing constantly but the velocity terms keep on oscillating in order to control the motion in trajectory.

As shown in figure 13 and figure 14 we can see the joint variables keeps on oscillating to and fro as we fit the joint variables using cubic equation this acts as a feed back effect because of which it angles

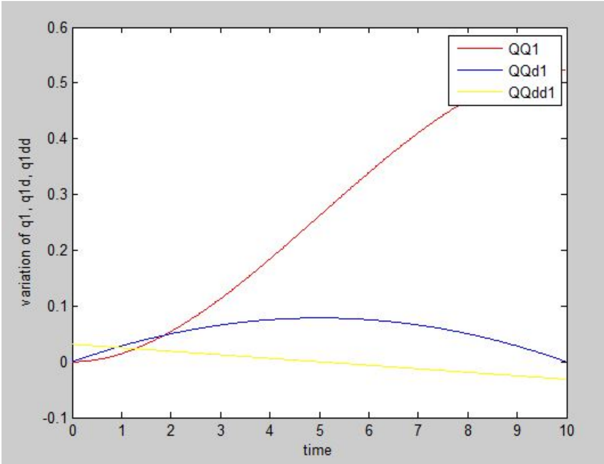

Figure 8: Trajectory of the joint variable1 w.r.t time

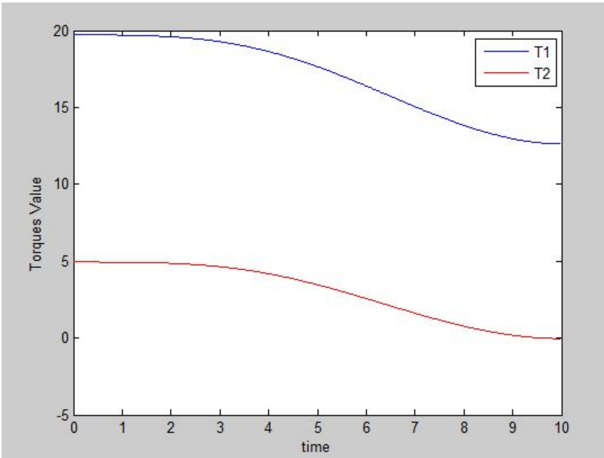

Figure 9: The desired joint torques w.r.t time for planned trajectory

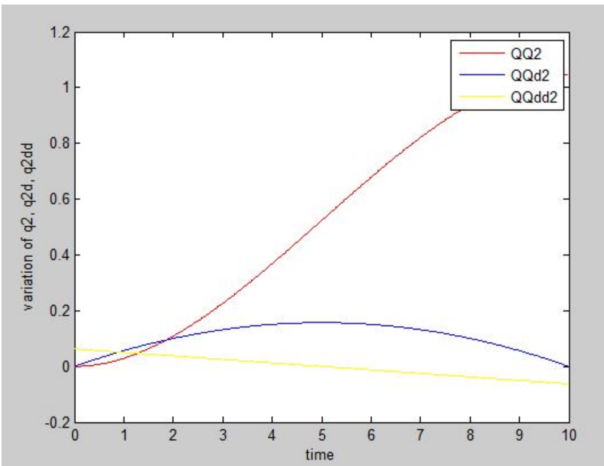

Figure 10: The joint variables 2 w.r.t time

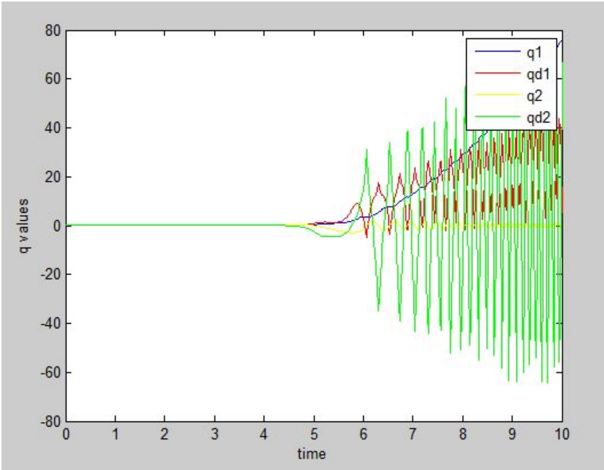

Figure 11: Plot represents the joint variable values w.r.t time

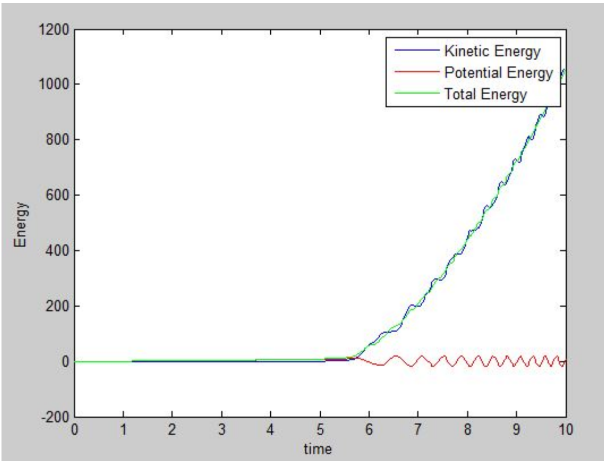

Figure 12: PLot represents the energy terms of manipulator w.r.t time

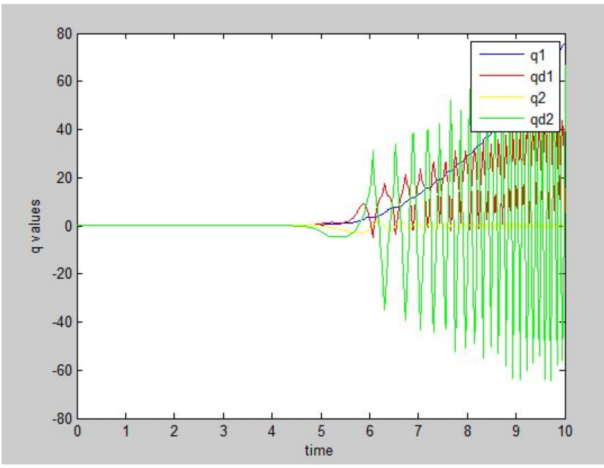

Figure 13: Plot represents joint variables w.r.t time when mass is added to second link

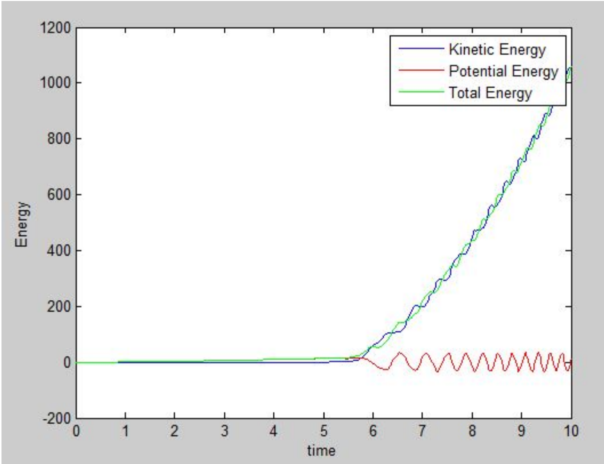

Figure 14: Plot represents energy terms w.r.t time when mass is added to second link

keep on increasing constantly but the velocity terms keep on oscillating in order to control the motion in trajectory.There is not much effect of adding mass on the link 2 the result is similar to the previous one. Here we can observe one thing which is total energy keeps on increasing as potential energy oscillates and Kinetic energy increases.The joints torques keep on adjusting accordingly.

4 Solution

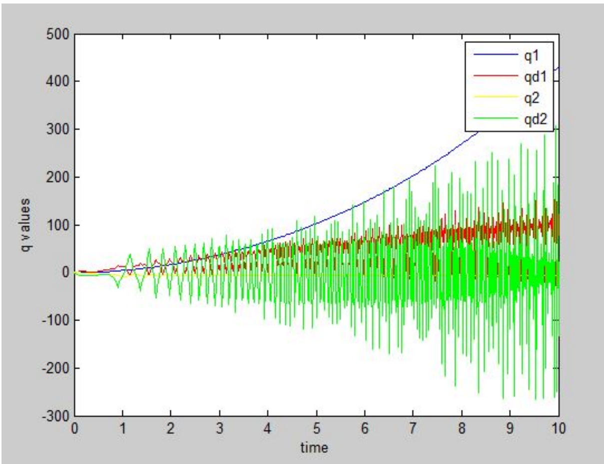

Figure 15: Plot represents the joint variables of the manipulator when underactuated

In this case the potential energy decreases because the underactuation of the joint makes it lose its potential energy as it tilts downwards due to less support.The angles keep on increasing whereas the velocity terms oscillate with high frequency.

In this case where spring is attached, we find first joint angle increases where as other terms oscillated frequently due to the attached spring. We can find the energy terms are similar to the previous underactuated case.

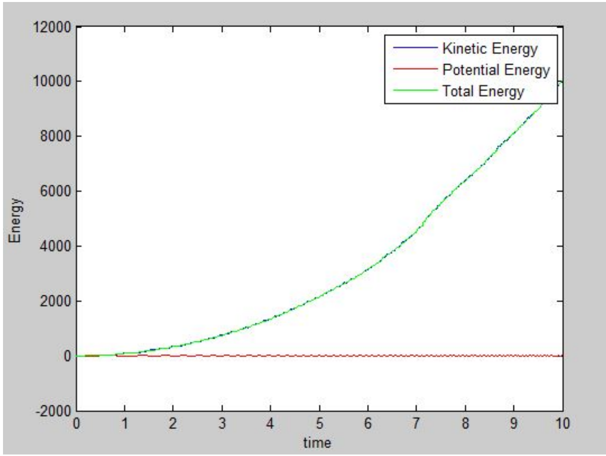

Figure 16: Plot represents the energy terms of the manipulator in the underactuated case

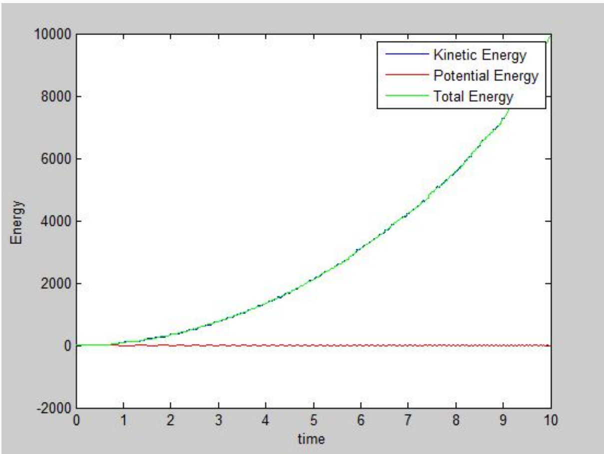

Figure 17: Plot represents the energy terms of the manipulator in the case where is spring is attached to the second joint

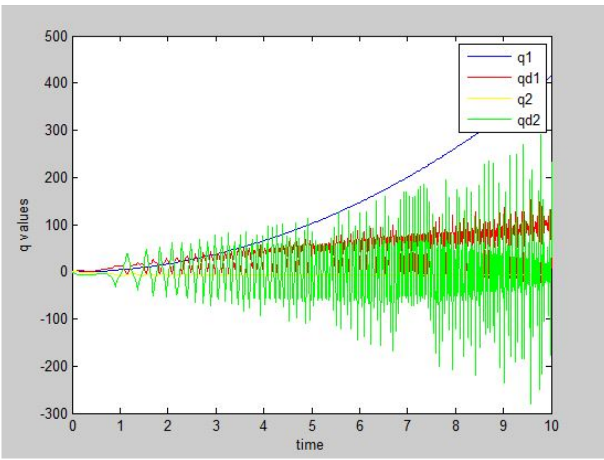

Figure 18: Plot represents the joint variables when spring is attached to the joint 2

5 Course Feedback

a. I am interested in Robotics and my interest led to take this course. I am more interested towards Robotic Vision and main reason to learn these topics is so that it will be handy when I develop certain robotic models. b. Course content is good and more topics can be covered.

References

[1] *Robotics Dynamics and Control by M. Vidyasagar and Mark W. Spong* .