Assignment 4

For a planar 2-DOF, 2R robot (all revolute joints), with link lengths l_1 and l_2. Joint positions, joint velocities and joint accelerations are denoted as q_i, q_{di} and q_{ddi}, $i=1-2$, respectively. Also, T_i, $i=1-2$, denote the joint torque values. Develop the equations of motion in following form,

$$D(q)\, q_{dd} + C(q,q_d)\, q_d + g(q) = T \qquad (1)$$

Independent Joint Control: Both joints are controlled using the independent joint control approach. Refer our lecture notes for the nomenclature and note the system parameters,

Links: $m_1 = m_2 = 1;\quad l_1 = l_2 = 1;\quad I_1 = I_2 = 1/12;$

Motors: $J_{m1} = J_{m2} = 0.4 \times 10^{-4}\ (kgm^2);\ K_{m1} = K_{m2} = 2.32 \times 10^{-2}\ (Nm/A)$

$B_{m1} = B_{m2} = 4.77 \times 10^{-5}\ (Nm/(rad/sec));\ R_1 = R_2 = 0.365\ (Ohm)$

$K_{b1} = K_{b2} = 0.0232\ (V/(rad/sec));\ r_1 = r_2 = 1/100$

1. **(PD Compensator)** Plan the desired joint trajectories when the states of the system, $[q_1\ q_2\ q_{d1}\ q_{d2}]$, change from $[0\ 0\ 0\ 0]$ at $t = 0$ sec to $[\pi/6\ \pi/3\ 0\ 0]$ at $t =$ **2** sec.
   Use PD compensator to plan the voltages for each motor, V_k (k = 1-2), to track the desired joint trajectories. Select the PD gains such that the response has following characteristics: damping ratios $\zeta_1 = \zeta_2 = 1$, and natural frequencies $\omega_1 = \omega_2 = 4$ (rad/sec).
   Use a numerical solver (ode45) for above conditions and parameters to simulate the system performance. Plot and compare the desired trajectories with the simulation results. Discuss the results and the role of PD gains. Animate the 2R manipulator.

2. **(PID Compensator)** For the desired joint trajectories and the PD gains as planned in Q:1, show the effect of adding an integral term in the control scheme on the joint trajectories tracking. You can select some values for the integral gain to simulate the system performance. Plot and compare the desired trajectories with the simulation results. Discuss the results and the role of PID gains. Animate the 2R manipulator.

Submit following in a zipped folder:

1. **Matlab program** - should have comments, we will run it to verify your results.
2. **Report** with answers, equations and figures to all above questions.
3. **LaTex file** used for the report.