ROBOTICS ASSIGNMENT 4

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1 Solution

For a planar 2-DOF , 2R robot.
lengths are $l_1 and l_2$.
masses are $m_1 and m_2$.
Inertias are $I_1 and I_2$
Motor parameters are
$J_{m1} = J_{m2} = 0.4x10^{-4} kgm^2$
$K_{m1} = K_{m2} = 2.32x10^{-2} Nm/A$
$B_{m1} = B_{m2} = 4.77x10^{-5} Nm/(rad/sec)$
$R_1 = R_2 = 0.365 Ohm$
$K_{b1} = K_{b2} = 0.0232 V/(rad/sec)$
$Gearration : r1 = r2 = 1/100$
$Intialstate[0000]$
$Finalstate[\pi/6\pi/300]$
$Dampingrationsare \zeta_1 = \zeta_2 = 1$
$\omega_1 = \omega_2 = 4 rad/sec$
Joint positions are denoted by q_i
Joint velocities are denoted by q_{di}
Joint acceleration are denoted by q_{ddi}, where i = 1,2.
Torques are denoted by T_i.
We know,
$v_{c1} = J_{v_{c1}} \dot{q}$

$$J_{c_{v1}} = \begin{bmatrix} -l_{c1} sinq_1 & 0 \\ l_{c1} cosq_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$v_{c2} = J_{v_{c2}} \dot{q}$

$$J_{c_{v2}} = \begin{bmatrix} -l_1 sinq_1 - l_{c2} sin(q_1 + q_2) & -l_{c2} sin(q_1 + q_2) \\ l_1 cosq_1 + l_{c2} cos(q_1 + q_2) & l_{c2} cos(q_1 + q_2) \\ 0 & 0 \end{bmatrix}$$

$\omega_1 = \dot{q}_1 k, \omega_2 = (\dot{q}_1 + \dot{q}_2) k$
$\frac{1}{2} \dot{q}^T I_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \dot{q} + I_2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \dot{q}$
$D(q) = m_1 J_{v_{c1}}^T J_{v_{c1}} + m_2 J_{v_{c2}}^T J_{v_{c2}} + \begin{bmatrix} I_1 + I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}$
$d_{11} = m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} cosq_2) + I_1 + I_2$
$d_{12} = d_{21} = m_2 (l_{c2}^2 + l_1 l_{c2} cosq_2) + I_2$
$d_{22} = m_2 l_{c2}^2 + I_2$
Christoffel symbols
$c_{111} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_1} = 0$
$c_{121} = c_{211} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = -m_2 l_1 l_{c2} sinq_2 = h$

$c_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = h$
$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -h$
$c_{122}c_{212} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$
$c_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$
Potential Energy
$V_1 = m_1 g l_{c1} sinq_1$
$V_2 = m_2 g(l_1 sinq_1 + l_{c2} sin(q_1 + q_2))$
$V = V_1 + V_2 = (m_1 l_{c1} + m_2 l_1) g sinq_1 + m_2 l_{c2} g sin(q_1 + q_2)$
$\phi_1 = \frac{\partial V}{\partial q_1} = (m_1 l_{c1} + m_2 l_1) g cosq_1 + m_2 l_{c2} g cos(q_1 + q_2)$
$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 l_{c2} cos(q_1 + q_2)$
$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_1\dot{q}_2 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2^2 + \phi_1 = \tau_1$
$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 = \tau_2$

$$C = \begin{bmatrix} h\dot{q}_2 & h\dot{q}_2 + h\dot{q}_1 \\ -h\dot{q}_1 & 0 \end{bmatrix}$$

For the following initial condition at t=0. $IC = [q_1 q_2 q_{d1} q_{d2}]$
$l_1 = l_2 = 1$
$m_1 = m_2 = 1$
$I_1 = I_2 = 1/12$
$[t_i t_f] = [010]$
$K = K_m/R$
$B_{eff} = B_m + K_b K_m/R$
$J_{eff} = J_m + r_k^2 d_{kk}(q)$
$K_D = 2\zeta\omega J_{eff} - B_{eff}/K$
$V(s) = K_P(\theta^d(s) - \theta(s)) - K_D s\theta(s)$
$V(t) = K_P(\theta^d(t) - \theta(t)) - K_D\dot{\theta}(t)$
By substituting the above equation in,
$J_{eff}\ddot{\theta}_{mk} + B_{eff}\dot{\theta}_{mk} = V(t) - r_k d_k$
We get,
$J_{eff}\ddot{\theta}_{mk} + B_{eff}\dot{\theta}_{mk} = K(K_P(\theta^d(t)) - \theta(t)) - K_D\dot{\theta}(t) - r_k d_k$
By using the above obtained equation we solve and get the results. From the figure From the figure From

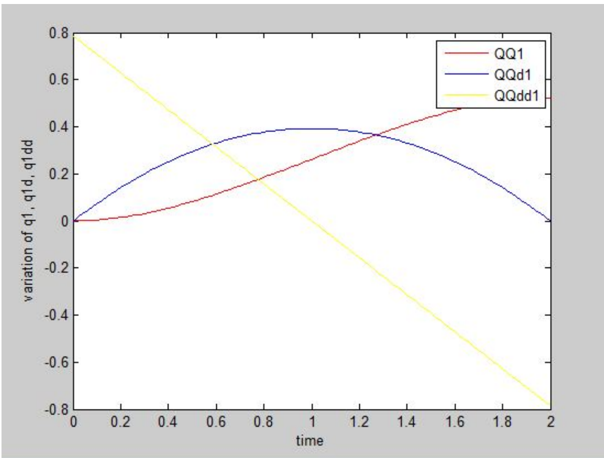

Figure 1: Plot represents the desired joint parameters of joint 1

the figure From the figure From the figure From the figure From the figure From the results obtained, we can observe the error in the values obtained by simulating the 2 R manipulator and there is considerable difference between the desired and simulated values with usage of PD controller.

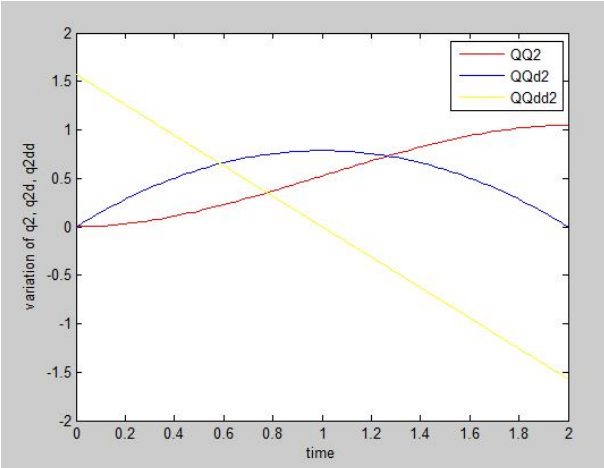

Figure 2: Plot represents the desired joint parameters of joint 2

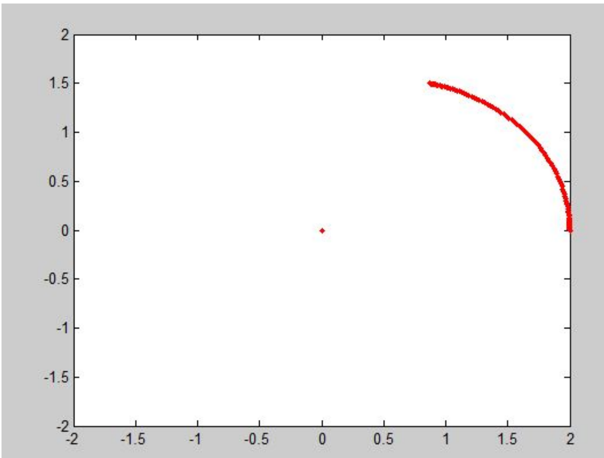

Figure 3: Plot represents the desired trajectory

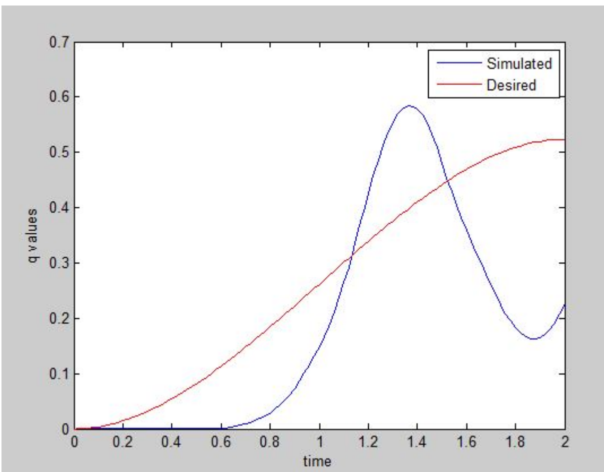

Figure 4: Plot represents Simulated and Desired values of joint 1 angle

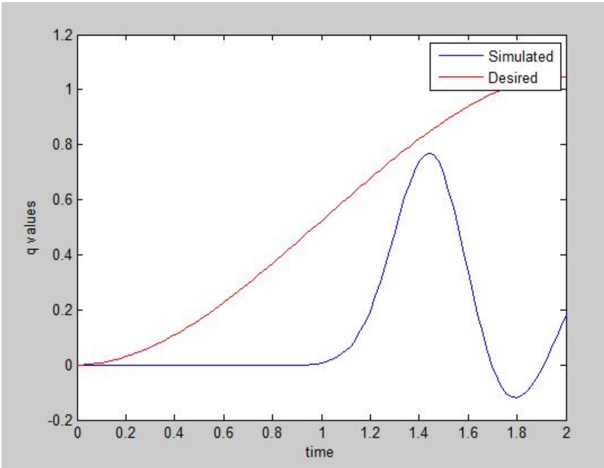

Figure 5: Plot represents Simulated and Desired values of joint 2 angle

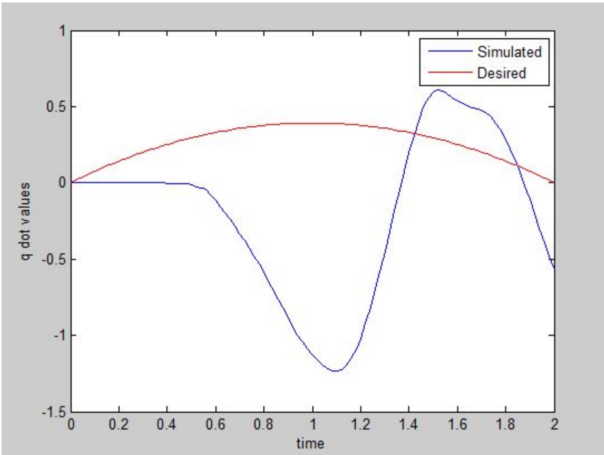

Figure 6: Plot represents the Simulated and Desired values of velocity of joint 1

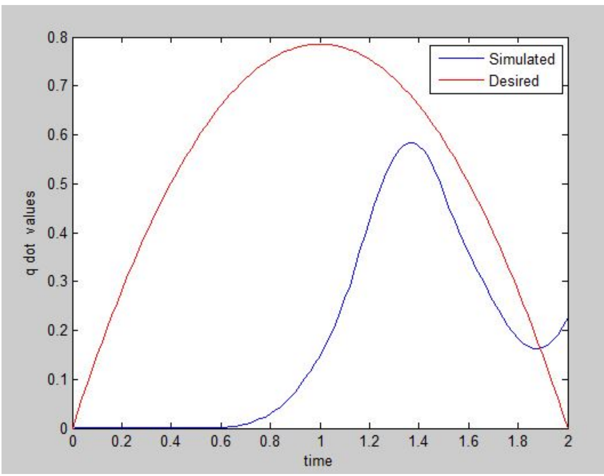

Figure 7: Plot represents the Simulated and Desired values of velocity of joint 2

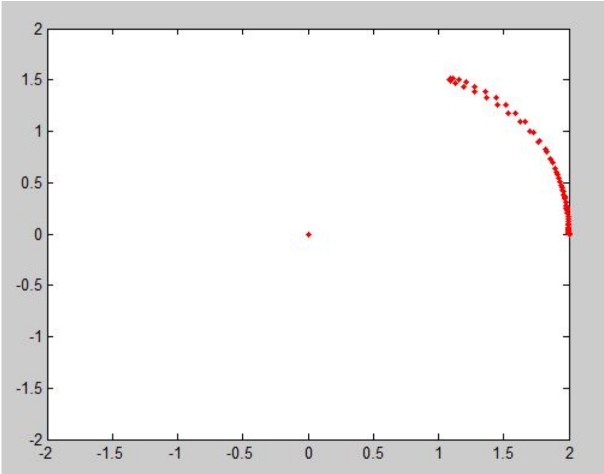

Figure 8: Plot represents the Simulated Trajectory

2 Solution

The results obtained when used PID in case of Q1 are. .

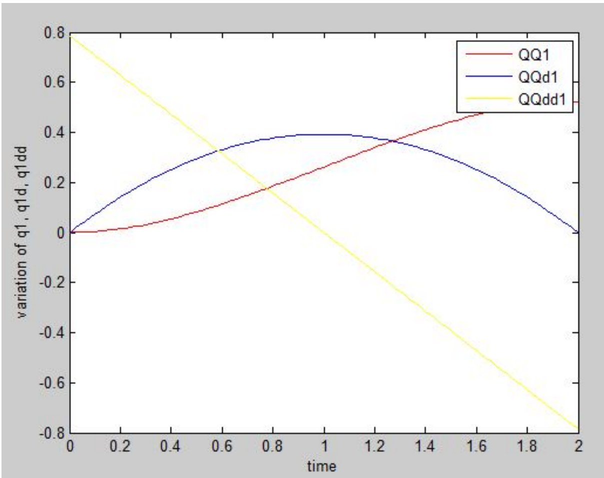

Figure 9: Plot desired values parameter of the joint1

From the results, we can see error value decreased in case of a joint1 compared PD control in the same case and by using the overall result we can observe that the PID is slightly better than PD controller. Due to some error in implementation the results of simulated are not good enough.

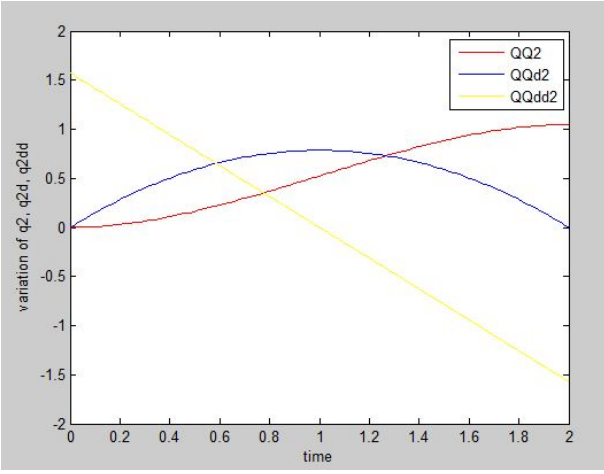

Figure 10: Plot represents desired values of parameters of the joint 2

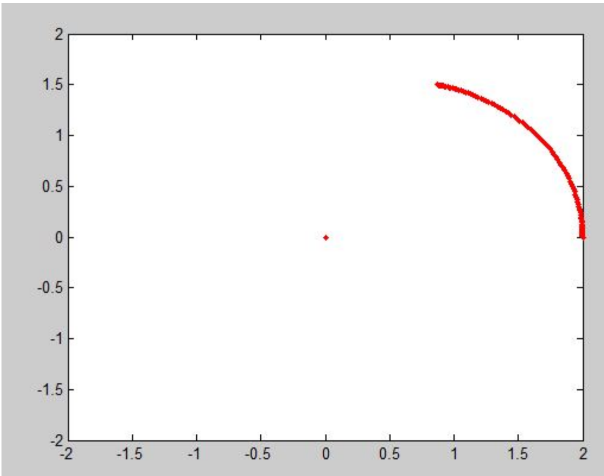

Figure 11: Plot represents the desired Trajectory

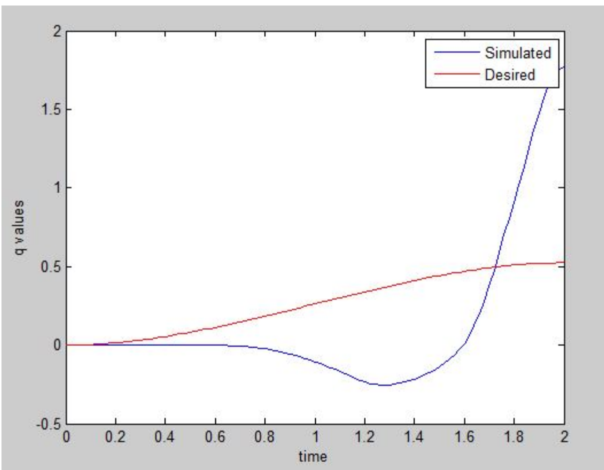

Figure 12: The Simulated and Desired values of joint angle 1

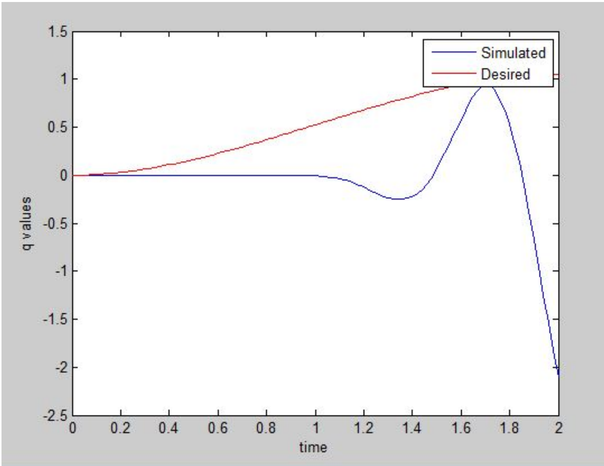

Figure 13: Plot represents the Simulated and Desired values of joint angle 2

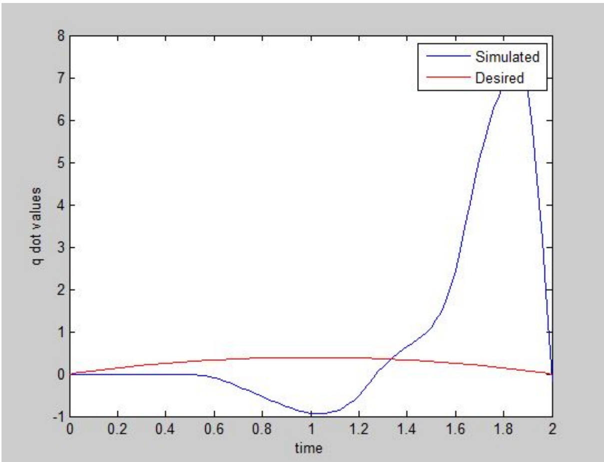

Figure 14: Plot represents the Simulated and Desired values of joint angle velocity of 1

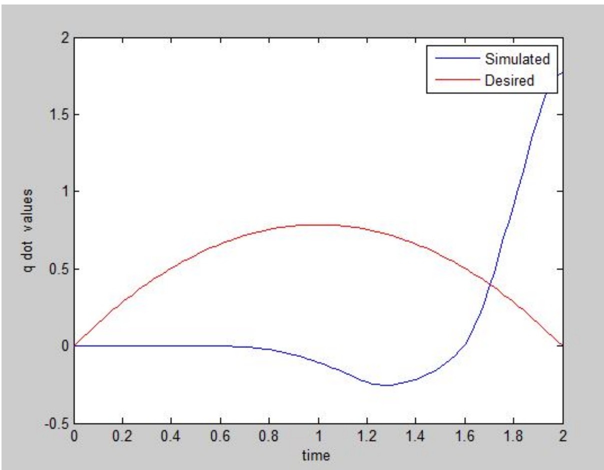

Figure 15: Plot represents the Simulated and Desired values of joint angle velocity of 2

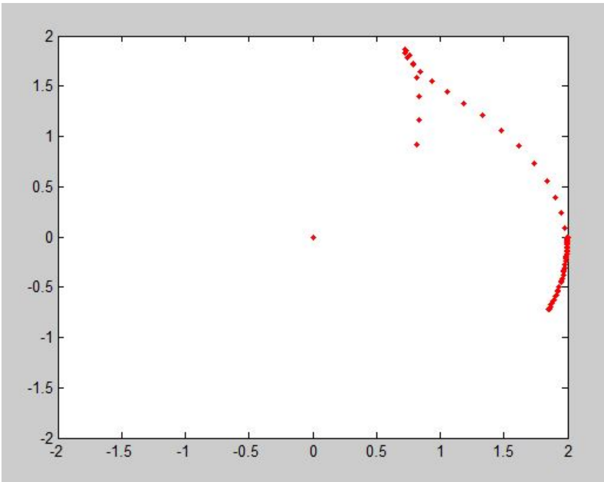

Figure 16: Plot represents the Simulated Trajectory incase of PID control