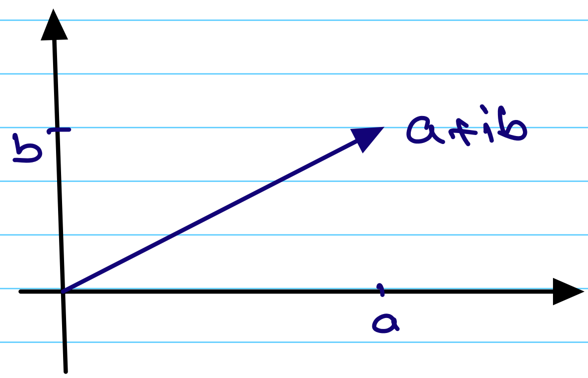
Lec 06: Complex Number's and Complex exponential's



$i^2 = -1$

$z = a + bi \Rightarrow \bar{z} = a - bi$

$z\bar{z} = a^2 + b^2$

Polar Co-ordinates

$a + ib = r\cos\theta + i r\sin\theta$

$= r(\cos\theta + i\sin\theta)$

Euler: $e^{i\theta} = \cos\theta + i\sin\theta$

Exponential: $(a^x \cdot a^y = a^{x+y})$

1. Exponential law
2. e^{at} solution to $\frac{dy}{dt} = ay \quad y(0) = 1$

(1)

$$e^{i\theta_1} \cdot e^{i\theta_2} = e^{i\theta_1 + i\theta_2}$$

$$= e^{i(\theta_1+\theta_2)}$$

$$\Rightarrow (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$$

$$\Rightarrow \cos\theta_1 \cdot \cos\theta_2 - \sin\theta_1 \cdot \sin\theta_2 + i(\cos\theta_1 \cdot \sin\theta_2 + \cos\theta_2 \sin\theta_1)$$

$$\Rightarrow \cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)$$

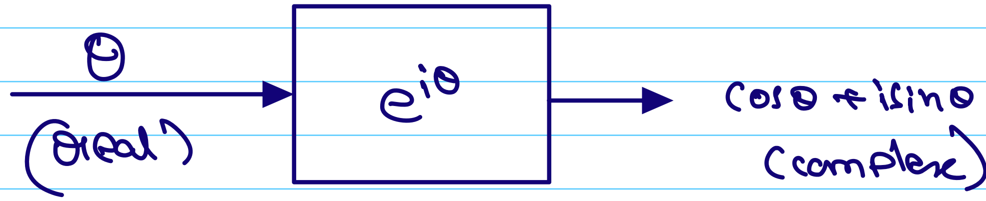
$$\Rightarrow e^{i(\theta_1+\theta_2)}$$

(2)

$$\frac{d}{d\theta} e^{i\theta} = e^{i\theta} \cdot \frac{d}{d\theta}(i\theta) = i \cdot e^{i\theta}$$

$$\Rightarrow \frac{d}{d\theta} e^{i\theta} = i e^{i\theta}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$



Complex valued function of a real variable. CV-VF of real θ

$e^{it} = \cos\theta + i\sin\theta$

A general complex valued real variable looks line

$$t \longrightarrow \boxed{\text{cv-vd}} \longrightarrow u(t) + iv(t)$$

($e^{i\theta}$ is special case)

$$D(u+iv) = D(u) + iD(v)$$

$$\frac{d}{dt} e^{it} = \frac{d}{dt}(\cos t + i\sin t)$$

$$= -\sin t + i\cos t$$

$$= i(\cos t + i\sin t)$$

$$= ie^{it}$$

$$e^{a+ib} = e^{a} \cdot e^{ib}$$