LEC 24 : Introduction to First-order System of ODE's

one independent variable, several dependent variable.

1st-order
$$\begin{cases} x' = f(x, y; t) \\ y' = g(x, y; t) \end{cases}$$

x, y = dependent variables

t = independent variable

Linear System's:

The dependent variable's must occur linearly. It must look like

$$x' = a(t)x + b(t)y + r_1(t)$$
$$y' = c(t)x + d(t)y + r_2(t)$$

a, b, c, d Constant's $\rightarrow$ constant co-efficient system's.

a, d, c, d can be function's of t,
It's a linear system.

Linear homogenous mean $\pi_1(t)=0$

$\pi_2(t)=0$

$$\Rightarrow \quad x' = a(t)x + b(t)y$$

$$y' = c(t)x + d(t)y$$

initial condition's $\quad x(t_0),\ y(t_0)$

Ex:

$$\frac{dT_1}{dt} = a(T_2 - T_1)$$

$$\frac{dT_2}{dt} = a(T_1 - T_2) + b(Te - T_2)$$

$$\Rightarrow \quad \frac{dT_1}{dt} = -aT_1 + aT_2$$

$$\frac{dT_2}{dt} = aT_1 - (a+b)T_2 + bTe(t)$$

$T_e(t) = 100e^{-kt}$

$a = 2 \quad b = 3$

$$\left.\begin{aligned} T_1' &= -2T_1 + 2T_2 \\ T_2' &= 2T_1 - 5T_2 \end{aligned}\right\} \Rightarrow \text{eliminate } T_2$$

$$T_2 = \frac{T_1' + 2T_1}{2}$$

$$\left(\frac{T_1' + 2T_1}{2}\right)' = 2T_1 - 5\left(\frac{T_1' + 2T_1}{2}\right)$$

$$T_1'' + 7T_1' + 6T_1 = 0 \qquad (r^2 + 7r + 6 = 0)$$

$$(r+1)(r+6)$$

$$T_1(t) = C_1e^{-t} + C_2e^{-6t}$$

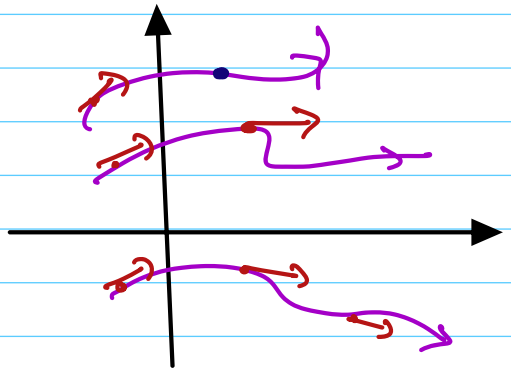
$$T_2(t) = \frac{1}{2}C_1e^{-t} - 2C_2e^{-6t}$$

Autonomous system's

$$\left.\begin{aligned} x' &= f(x,y) \\ y' &= g(x,y) \end{aligned}\right\} \text{No } t \text{ appears in RHS}$$

Solution's:

$x = x(t)$
$y = y(t)$
} Parametric curve (Parameterized curve)



$(x'(t), y'(t))$ is velocity vector (velocity of solution at time t)

System of 1st-order Autonomous ODE's = velocity field

Solution (A Parameterized Curve with the right velocity everywhere)