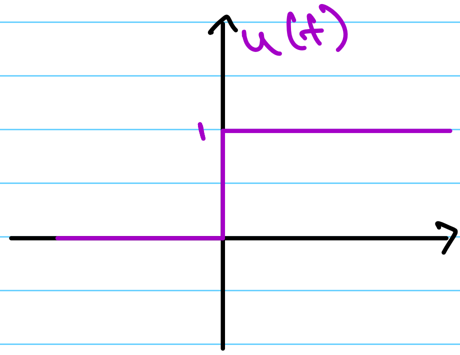
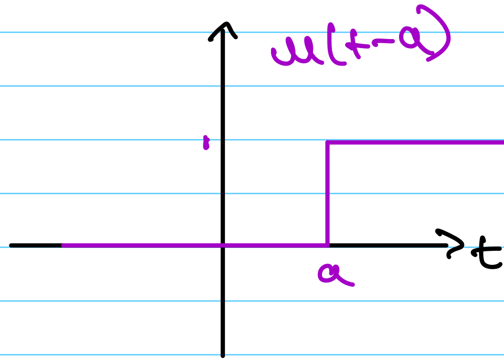
Lec 22: Jump discontinuties

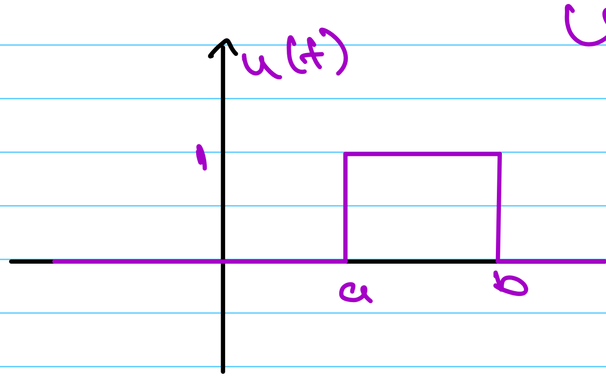


$u(t)$ "unit step"

$u(0)$ undefined



$u_a(t) = u(t-a)$



$u_{ab}(t) =$ "unit Box"

$u_{ab}(t) = u_a(t) - u_b(t)$

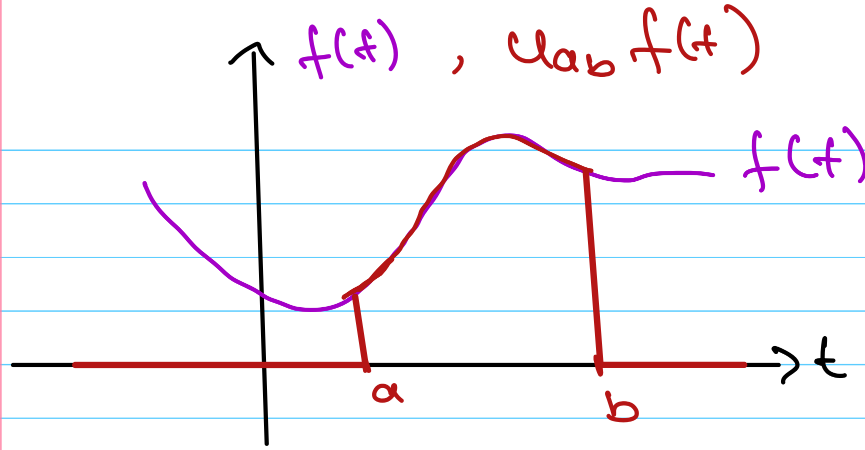
$= u(t-a) - u(t-b)$

what so good about these function?

when we use them in multiPlecation

they transform other function's (they operate

on other function's)



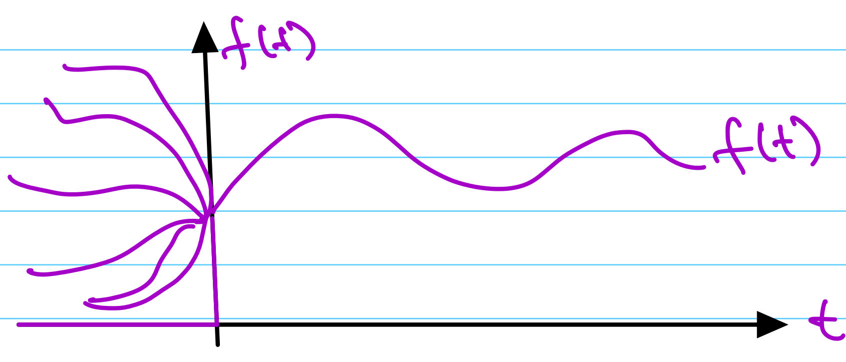
we wiped away entire f(t) graph except for b/w a, b

$$\mathcal{L}(u(t)) = \int_0^\infty e^{-st}\,dt = \frac{1}{s} \qquad s>0$$

$$\mathcal{L}(1) = \frac{1}{s} \qquad s>0$$

Both u(t) and 1 have same Laplace transform. what the big deal?

What's the inverse Laplace transform of $\frac{1}{s}$? i.e $\mathcal{L}^{-1}\left(\frac{1}{s}\right)$?



all these function produce same laplace transform. $\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\, dt$

because laplace T does not care about $f(t)$ before 0, (for -ve value's of t)

$$f(t) \xrightarrow{\mathcal{L}} F(s)$$

$$F(s) \xrightarrow{\mathcal{L}^{-1}}$$

* Laplace transform is only useful for problem's for future time.

* if the problem is starting now and go onto the future, and you don't have to know anything about the past that's a L.T problem

* if we also have to know about

Post, then F.T Problem.

* we will make L.T unique by making $f(t) = 0 \;\; \forall t < 0$ (unique L.T)

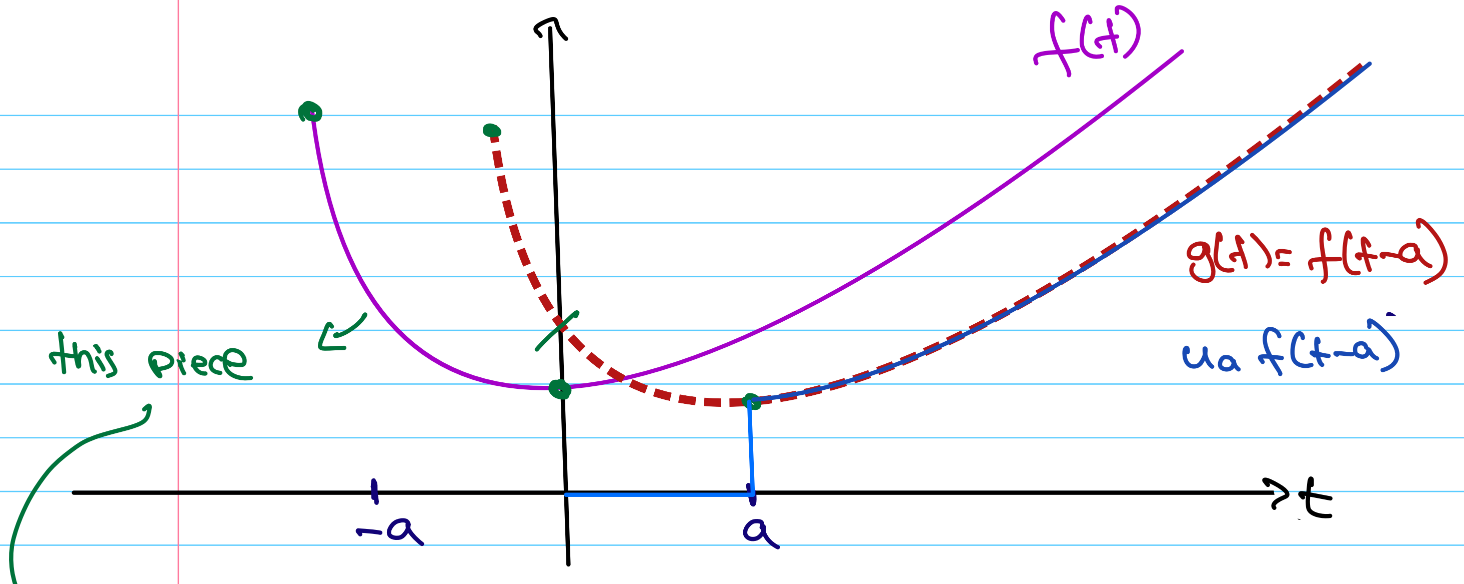
$$f(t) \xrightarrow{\mathcal{L}} F(s)$$

$$F(s) \xrightarrow{\mathcal{L}^{-1}} \underbrace{u(t)\, f(t)}_{\text{unique L.T}}$$

Want formula : $\mathcal{L}(f(t-a))$ in term's of $\mathcal{L}(f(t))$

* There is cannot be such a formula.

* does not exist



* Laplace Transform Lost this info
* Not used for $\mathcal{L}(f(t))$ because L.T start's from $t=0$
* But that piece is needed for $\mathcal{L}(f(t-a))$. we will have to know what that is. Impossible to know.
* in other word's, when we took L.T of $f(t)$, we automatically lost all information about function $f(t)$ for −ve value of t.

we can only find L.T of

$U_a f(t-a)$ ie $u(t-a) f(t-a)$

$u(t-a) f(t-a) \rightsquigarrow e^{-as} F(s)$

$u(t-a) f(t) \rightsquigarrow e^{-as} \mathcal{L}(f(t+a))$

(t-Axis transulation formula)

Proof:

$$\int_0^\infty e^{-st} u(t-a) f(t-a)\, dt$$

$t-a = t_1$

$dt = dt_1$

$$\Rightarrow \int_{-a}^{\infty} e^{-s(t_1+a)} u(t_1) f(t_1)\, dt$$

$$= e^{-sa} \int_{-a}^{\infty} e^{-st_1} u(t_1) f(t_1)\, dt$$

$$= e^{-sa} \int_0^\infty e^{-st} f(t)\, dt$$

$$= e^{-sa} F(s)$$

$$\Rightarrow \quad u(t-a) f(t-a) \rightsquigarrow e^{-as} \mathcal{L}(f(t))$$

$$u(t-a) f(t) \rightsquigarrow e^{-as} \mathcal{L}(f(t+a))$$

Example's

$$u_{ab}(t) = u(t-a) - u(t-b)$$

$$= u(t-a)\cdot 1 - u(t-b)\cdot 1$$

$$\mathcal{L}(u_{ab}(t)) = \mathcal{L}(u(t-a)\cdot 1) - \mathcal{L}(u(t-b)\cdot 1)$$

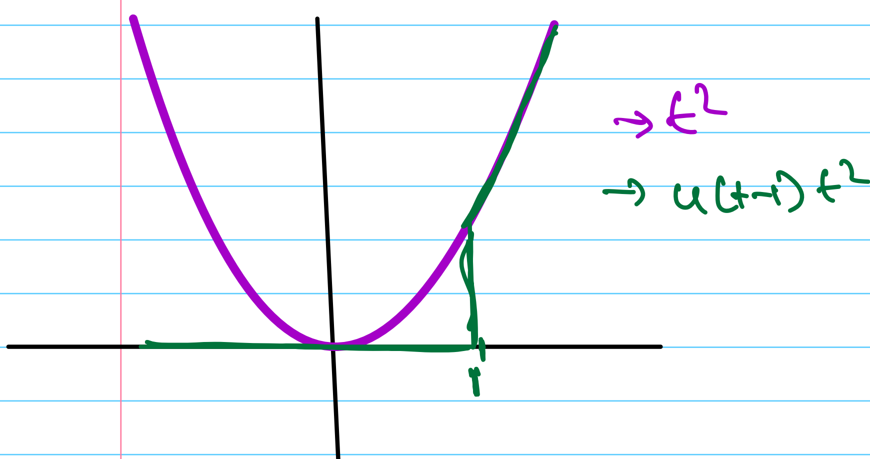
$$= \frac{e^{-as}}{s} - \frac{e^{-bs}}{s}$$

② $u(t-1)t^2$

$$\Rightarrow \mathcal{L}\left(u(t-1)t^2\right) = e^{-s}\mathcal{L}\left((t+1)^2\right)$$

$$= e^{-s}\mathcal{L}\left(t^2+2t+1\right)$$

$$= e^{-s}\left(\frac{2}{s^3}+2\cdot\frac{1}{s^2}+\frac{1}{s}\right)$$



$u(t-1)t^2$ this function is not a simple function. It's discontinuous, that's why its laplace transform has 3 term's in it times an exponential factor.

e^{-s} tell's it has a discontinuous at $t=1$

Ex:

$$\mathcal{L}^{-1}\left(\frac{1+e^{-\pi s}}{s^2+1}\right)$$

$$\Rightarrow \mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right) + \mathcal{L}^{-1}\left(\frac{e^{-\pi s}}{s^2+1}\right)$$

$$\Rightarrow u(t)\sin t + u(t-\pi)\sin(t-\pi)$$

$$f(t) = \begin{cases} \sin t & 0 \le t \le \pi \\ 0 & t > \pi \end{cases}$$