Problem set 4.5

(1) $F(s) = \dfrac{e^{-3s}}{s^2}$

$f(t) = (t-3)\,u(t-3)$

(2) $F(s) = \dfrac{e^{-s} - e^{-3s}}{s^2}$

$F(s) = \dfrac{e^{-s}}{s^2} - \dfrac{e^{-3s}}{s^2}$

$= u(t-1)(t-1) - u(t-3)(t-3)$

(3) $F(s) = \dfrac{e^{-s}}{s+2}$

$F(s) = \dfrac{e^{-s}}{s+2} \Rightarrow u(t-1)\,e^{-2(t-1)}$

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$$F(s) = \frac{e^{-s} - e^{2-2s}}{s-1}$$

$$F(s) = \frac{e^{-s}}{s-1} - e^{2} \cdot \frac{e^{-2s}}{s-1}$$

$$= u(t-1)\, e^{t-1} - u(t-2) \cdot e^{2} \cdot e^{t-2}$$

5

$$F(s) = \frac{e^{-\pi s}}{s^2+1}$$

$$= e^{-\pi s} \cdot \frac{1}{s^2+1}$$

$$= u(t-\pi)\, \sin(t-\pi)$$

6

$$F(s) = \frac{s e^{-s}}{s^2+\pi^2}$$

$$f(t) = u(t-1) \cos \pi (t-1)$$

(7)

$$F(s) = \frac{1 - e^{-2\pi s}}{s^2+1}$$

$$= \frac{1}{s^2+1} - \frac{e^{-2\pi s}}{s^2+1}$$

$$= \sin t - u(t-2\pi)\sin(t-2\pi)$$

(8)

$$F(s) = \frac{s(1-e^{-2s})}{s^2+\pi^2}$$

$$= \frac{s}{s^2+\pi^2} - e^{-2s}\frac{s}{s^2+\pi^2}$$

$$f(t) = \cos \pi t - u(t-2)\cos \pi(t-2)$$

(9)

$$F(s) = \frac{s(1+e^{-3s})}{s^2+\pi^2}$$

$$F(s) = \frac{s}{s^2+\pi^2} + \frac{se^{-3s}}{s^2+\pi^2}$$

$$f(t) = \cos \pi t + u(t-3)\cos \pi(t-3)$$

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$$f(t) = 2 \cdot u_{03}(t)$$

$$f(t) = 2\left[u(t) - u(t-3)\right]$$

$$F(s) = 2\ \mathcal{L}(u(t)\cdot 1) - 2\mathcal{L}(u(t-3)\cdot 1)$$

$$= \frac{2}{s} - 2 \cdot e^{-3s} \cdot \frac{1}{s}$$

$$= \frac{2}{s}\left[1 - e^{-3s}\right]$$

12

$$f(t) = u_{14}(t)$$

$$f(t) = u(t-1) - u(t-4)$$

$$F(s) = \frac{e^{-s}}{s} - \frac{e^{-4s}}{s}$$

13

$$f(t) = \begin{cases} \sin t & \text{if } 0 \le t \le 2\pi \\ 0 & t > 2\pi \end{cases}$$

$$f(t) = \sin t\,[u(t) - u(t-2\pi)]$$

$$= u(t)\sin t - u(t-2\pi)\sin t$$

$$= \frac{1}{s^2+1} - e^{-2\pi s}\mathcal{L}(\sin(t+2\pi))$$

$$= \frac{1}{s^2+1} - e^{-2\pi s}\mathcal{L}(\sin t)$$

$$= \frac{1}{s^2+1}\left[1 - e^{-2\pi s}\right]$$

14

$$f(t) = \begin{cases} \cos\pi t & 0 \le t \le 2 \\ 0 & t > 2 \end{cases}$$

$$f(t) = [u(t) - u(t-2)]\cos\pi t$$

$$= u(t)\cos\pi t - u(t-2)\cos\pi t$$

$$= \frac{s}{s^2+\pi^2} - e^{-2s}\mathcal{L}(\cos(\pi t + 2\pi))$$

$$= \frac{s}{s^2+\pi^2}\left[1 - e^{-2s}\right]$$

(15)

$$f(t) = \begin{cases} \sin t & 0 < t < 3\pi \\ 0 & \text{o.w} \end{cases}$$

$$= \left[u(t) - u(t-3\pi) \right] \sin t$$

$$= \frac{1}{1+s^2} - e^{-3\pi} \mathcal{L}(\sin(t+3\pi))$$

$$= \frac{1}{1+s^2} - e^{-3\pi} \mathcal{L}[\sin t \underbrace{\cos 3\pi}_{=-1} + \cos t \underbrace{\sin 3\pi}_{=0})$$

$$= \frac{1}{1+s^2} \left[1 - e^{-3\pi} \right]$$

(16)

$$f(t) = \begin{cases} \sin 2t & \text{if } \pi \le t \le 2\pi \\ 0 & t < \pi, \ t > 2\pi \end{cases}$$

$$f(t) = \left(u(t-\pi) - u(t-2\pi) \right] \sin 2t$$

$$= u(t-\pi) \sin 2t - u(t-2\pi) \sin 2t$$

$$F(s) = e^{-\pi s} \mathcal{L}(\sin(2t+2\pi)) - e^{-2\pi} \mathcal{L}(\sin(2t+4\pi))$$

$$= \frac{2}{s^2+4}\left(e^{-\pi s} - e^{-2\pi}\right)$$

17 $f(t) = \sin \pi t \qquad 2 \le t \le 3$

$$f(t) = \left\{u(t-2) - u(t-3)\right\}\sin \pi t$$

$$F(s) = e^{-2s}\mathcal{L}\left(\sin(\pi t + 2\pi)\right) - e^{-3s}\mathcal{L}\left(\sin(\pi t + 3\pi)\right)$$

$$= e^{-2s}\mathcal{L}(\sin \pi t) - e^{-3s}\mathcal{L}(-\sin \pi t)$$

$$= \frac{\pi}{s^2+\pi^2}\left[e^{-2s} - e^{-3s}\right]$$

18 $f(t) = \cos \frac{1}{2}\pi t \quad \text{if} \quad 3 < t < 5$

$$f(t) = \left\{u(t-3) - u(t-5)\right\}\cos \frac{1}{2}\pi t$$

$$F(s) = e^{-3s}\mathcal{L}\left(\cos\left(\frac{\pi t}{2} + \frac{3\pi}{2}\right)\right) - e^{-5s}\mathcal{L}\left(\cos\left(\frac{\pi t}{2} + \frac{5\pi}{2}\right)\right)$$

$$= e^{-3s}\,\mathcal{L}\left(\cos\frac{\pi t}{2}\cdot\cos\frac{3\pi}{2} - \sin\frac{\pi t}{2}\cdot\sin\frac{3\pi}{2}\right)$$

$$+ e^{-5s}\,\mathcal{L}\left(\cos\frac{\pi t}{2}\cdot\cos\frac{5\pi}{2} - \sin\frac{\pi t}{2}\cdot\cos\frac{5\pi}{2}\right)$$

$$= e^{-3s}\,\mathcal{L}\left(0 + \sin\frac{\pi t}{2}\right)$$

$$+ e^{-5s}\,\mathcal{L}\left(-\sin\frac{\pi t}{2}\right)$$

$$= \left(\frac{\frac{\pi}{2}}{s^2+\frac{\pi^2}{4}}\right)\left[e^{-3s} - e^{-5s}\right]$$

(19) $f(t) = 0$ if $t < 1$

$\qquad t$ if $t \geq 1$

$$f(t) = u(t-1)\,t$$

$$F(s) = e^{-s}\,\mathcal{L}(t+1)$$

$$= e^{-s}\left(\frac{1}{s^2} + \frac{1}{s}\right)$$

(21)

$$f(t) = \begin{cases} t & \text{if } t < 1 \\ 2-t & 1 < t \leq 2 \\ 0 & t > 2 \end{cases}$$

$$f(t) = \{u(t) - u(t-1)\} t + \{u(t-1) - u(t-2)\}(2-t)$$

$$F(s) = \frac{1}{s^2} - e^{-s}\left(\frac{1}{s} + \frac{1}{s^2}\right) + e^{-s}\left(\frac{1}{s} - \frac{1}{s^2}\right) - e^{-2s}\left(-\frac{1}{s^2}\right)$$

$$= \frac{1}{s^2} - e^{-s} \cdot \frac{2}{s^2} + \frac{e^{-2s}}{s^2}$$

$$= \frac{1}{s^2}\left[1 - 2e^{-s} + e^{-2s}\right]$$

(20) $f(t) = t$ if $t < 1$

1 if $t > 1$

$$f(t) = [u(t) - u(t-1)]\,t + u(t-1)$$

$$F(s) = \frac{1}{s} - e^{-s}\left(\frac{1}{s} + \frac{1}{s^2}\right) + e^{-s}\frac{1}{s}$$

$$F(s) = \frac{1}{s} - \frac{1}{s^2}e^{-s}$$

(22) $f(t) = t^3$ $1 \le t \le 2$

0 $0, \infty$

$$f(t) = [u(t-1) - u(t-2)]\,t^3$$

$$F(s) = e^{-s}\,\mathcal{L}((t+1)^3) - e^{-2s}\,\mathcal{L}((t+2)^3)$$

$$= e^{-s} \mathcal{L}\left(t^3 + 3t^2 + 3t + 1\right)$$

$$- e^{-2s} \mathcal{L}\left(t^3 + 8 + 6t^2 + 12t\right)$$

$$= e^{-s}\left(\frac{6}{s^4} + \frac{6}{s^3} + \frac{3}{s^2} + \frac{1}{s}\right)$$

$$- e^{-2s}\left(\frac{6}{s^4} + \frac{12}{s^3} + \frac{12}{s^2} + \frac{8}{s}\right)$$