Lec 20:- Derivative formula's

* we will see how laplace Transform's are used to solve linear differential eqn. with constant coefficient's.

* The condition that makes the Laplace Transform definitely exist's for a function is $f(t)$ should not grow too rapidly.

* It can grow, because e^{-st} is pulling it down, all we have to do is guarantee that it doesn't grow so rapidly so that e^{-st} is powerless to put down.

How fast the function $f(t)$ is allowed to grow?

$f(t)$ of "exponential type"

$$|f(t)| \leq Ce^{kt}$$

$C > 0$
some +ve C

$k > 0$

$$y'' + Ay' + By = h(t)$$

* The Laplace Transform does not know how to solve these Just doesn't understand.

* The Laplace Transform must have an initial value problem.

$y(0) = y_0$ $\quad$ $y'(0) = y_0'$

$\Rightarrow$

$$\boxed{\begin{aligned} &y'' + Ay' + By = h(t) \\ &y(0) = y_0 \\ &y'(0) = y_0' \end{aligned}} \quad \text{IVP}$$

$y(t)$ soln $\rightsquigarrow$ $Y(s)$

$\downarrow \mathcal{L}$

Algebraic eqn in $Y(s)$ $\xrightarrow[Y(s)]{\text{solve for}}$ $Y = \dfrac{P(s)}{Q(s)}$ $\xrightarrow{\mathcal{L}^{-1}}$ $y = y(t)$

$$\mathcal{L}(f'(t)) = \int_0^\infty f'(t)\, e^{-st}\, dt$$

$$= e^{-st} \int f'(t)\, dt \Big]_0^\infty$$

$$- \int_0^\infty -s . e^{-s} \int f'(t)\, dt$$

$$= e^{-st} f(t) \Big]_0^\infty + s \int_0^\infty e^{-st} f(t)\, dt$$

$$= -f(0) + s\, F(s)$$

$$\mathcal{L}(f'(t)) = sF(s) - f(0)$$

$$f'(t) \rightsquigarrow sF(s) - f(0)$$

Laplace transform of 2nd derivative

$$f''(t) \rightsquigarrow s\,\mathcal{L}(f'(t)) - f'(0)$$

$$\rightsquigarrow s\,[\,sF(s) - f(0)\,] - f'(0)$$

$$\rightsquigarrow s^2F(s) - sf(0) - f'(0)$$

<u>Ex:</u> $y'' - y = e^{-t}$ $\quad y(0) = 1 \quad y'(0) = 0$

$$s^2\,Y(s) - s\,y(0) - y'(0)$$
$$- Y(s) = \mathcal{L}(e^{-t}) = \frac{1}{s+1}$$

$$\Rightarrow \quad Y(s)\,(s^2 - 1) = \frac{1}{s+1} + s$$

$$Y(s) = \frac{-\frac{1}{2}}{(s+1)^2} + \frac{\frac{1}{4}}{s+1} + \frac{3/4}{s+1}$$

$$\mathcal{L}^{-1}(Y(s)) = y(t) = -\frac{1}{2}te^{-t} + \cdot \quad \cdot \quad \cdot$$