Lec 01:

# The Geometrical view of $y' = F(x, y)$

## FIRST ORDER ODE

$$y' = f(x, y)$$

Ex: $y' = \frac{x}{y}$ $\Rightarrow$ can be solved using seperation of variables

unsolvable (nonlinear) $\leftarrow$ $y' = x - y^2$

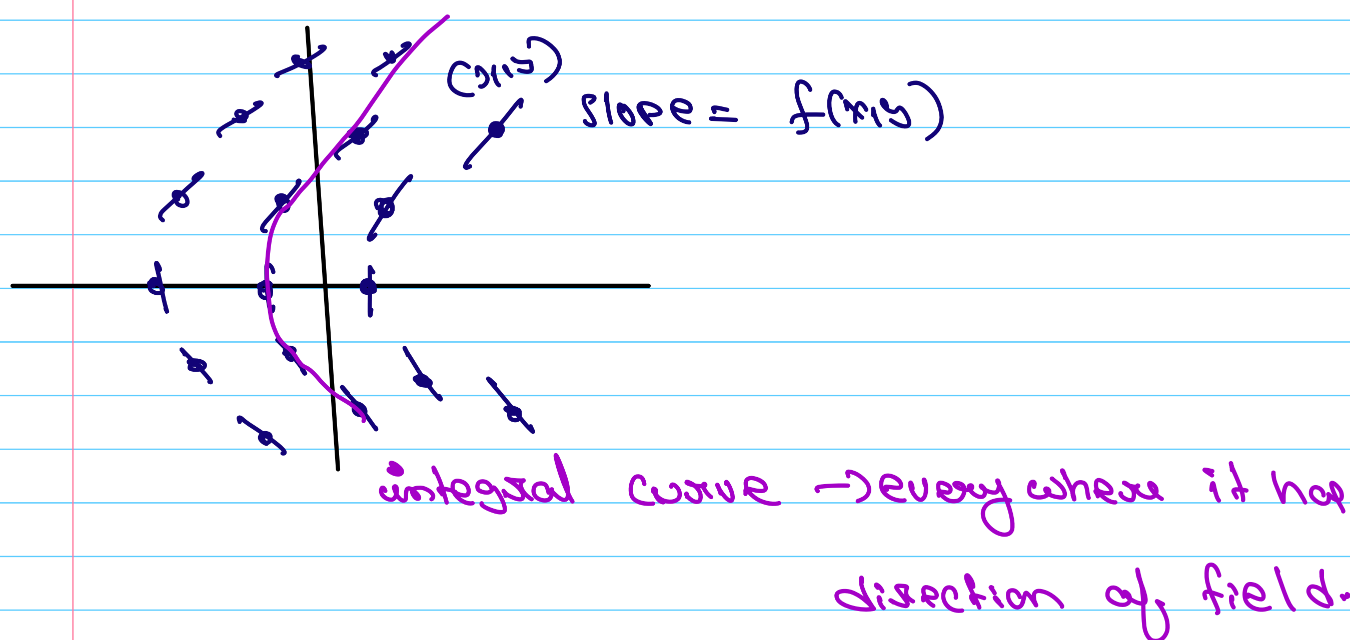
$\underbrace{y' = y - x^2}_{\text{easily solvable}}$

these two look extremly similar, But they are extremly dissimillar.

$y' = x - y^2$ $\Rightarrow$ even for the simplest possible differential eqn, those only involve 1st derivative, it's impossible to write down extremely looking simple guys

# Geometric view of diff eq$^n$

| Analytic view | | Geometric view |
|---|---|---|
| $y' = f(x,y)$ | $\longleftrightarrow$ | Direction field |
| $y_1(x)$ is sol$^n$ | $\xleftrightarrow{\text{solution}}$ | integral curve |




* The integral curve is the graph of the solution to differential eq$^n$.

* in other words, writing down analytically the differential eq$^n$ is same as geometrically as drawing this direction field.

* Solving analytically for the solution of differential eqn is the same thing as geometrically drawing a integral curve.

$y_1(x)$ is a solution to $y' = f(x,y)$

$\Updownarrow$

graph of $y_1(x)$ is an integral curve.

## Drawing Direction field

### Computer method

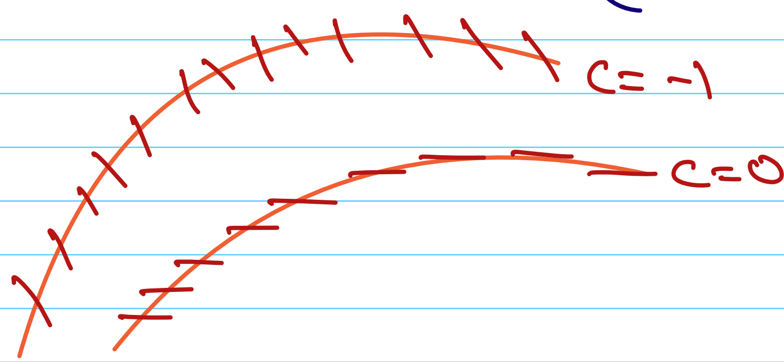
① Pick $(x,y)$ (equally spacing)

② $f(x,y) \rightarrow$ find

③ on screen Draw's $\not{}$ slope $f(x,y)$

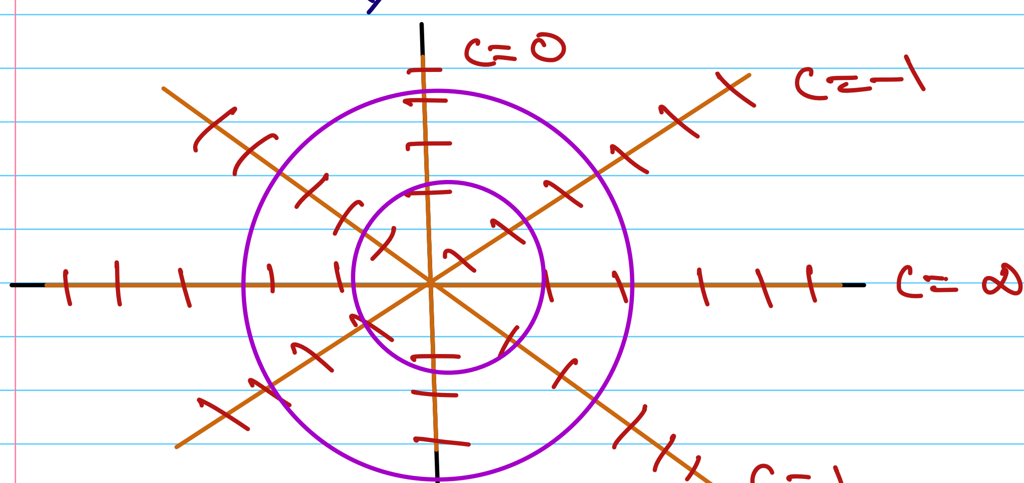
## Human does

1. Pick slope $= C$
2. find all the points $(x,y)$ where the slope $f(x,y) = C$. They will satisfy the equation $f(x,y) = c$ (curve)
3. Plot that curve. (isocline)




**Example:** $y' = -\frac{x}{y}$

$$\Rightarrow \quad -\frac{x}{y} = c \quad \Rightarrow \quad y = -\frac{1}{c} \cdot x$$

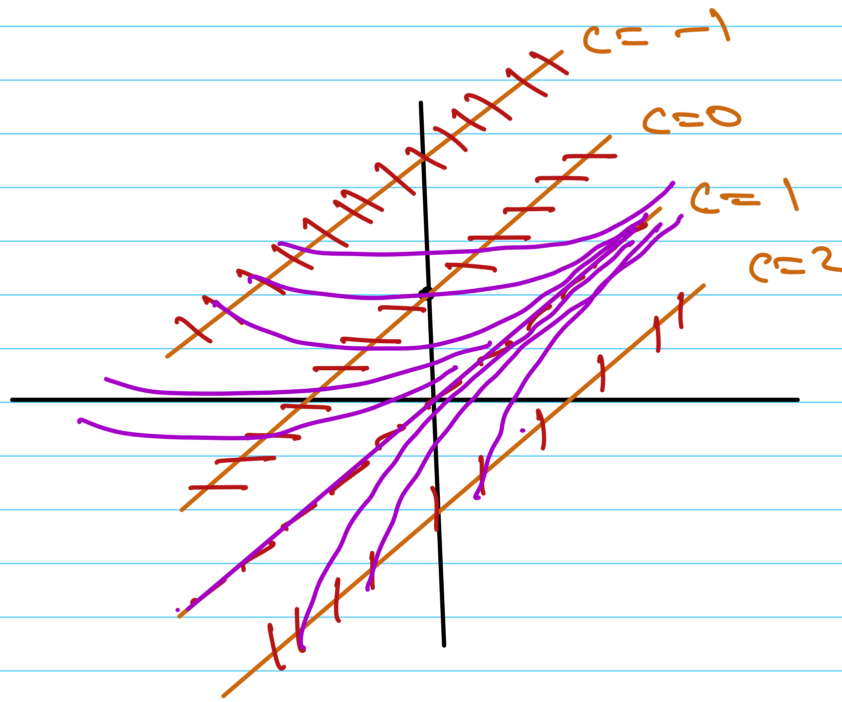



— integral curves are circle.

Ex: $y' = 1+x-y$

$C = 1+x-y$

$\Rightarrow y = x + 1 - C$



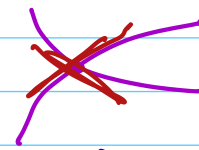

what's happening b/w $c=-1$, and $c=1$

$\Rightarrow$ The solution's are getting into that corridor, And there is no escape is possible. (It's A trAP)

Solution's can't ESCAPE

# Two Priciples

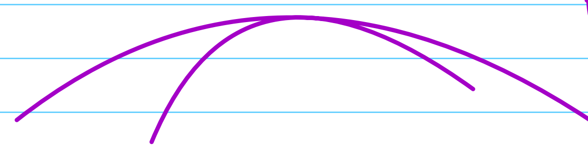
① two integral curve cannot cross



Because at Point $(x,y)$ it cannot have Both the slopes

② two integral curve cannot be tangent

NO!



because of Existence and uniqueness theorem.

It say's through a Point $(x_0,y_0)$ $y' = f(x_0,y_0)$ has one and only solution.

<u>Hypothesis</u> : $f(x,y)$ continous near $(x_0,y_0)$

also $f_y(x,y)$ should be continous $(x_0,y_0)$

$$xy' = 1-y$$

$$\Rightarrow \quad \frac{y'}{1-y} = \frac{1}{x}$$

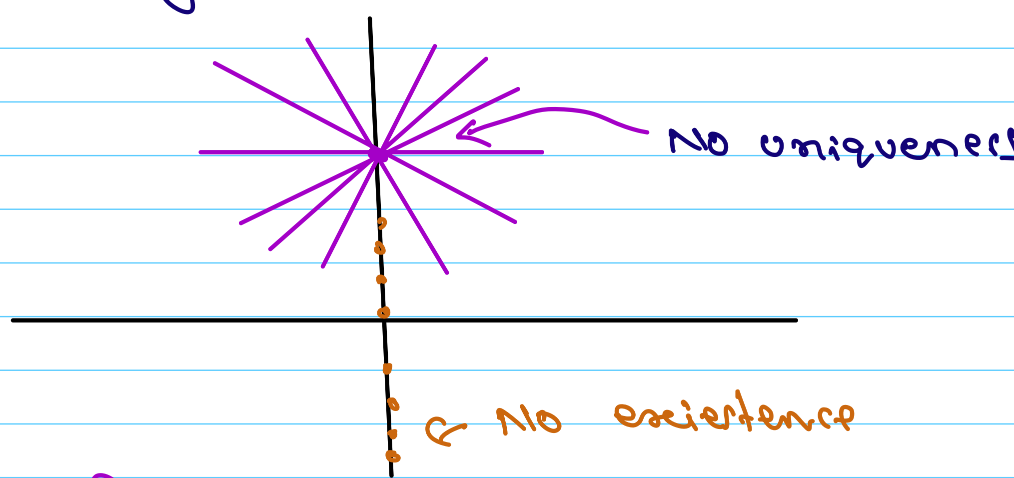
$$\Rightarrow \quad \frac{dy}{1-y} = \frac{dx}{x}$$

$$\int \frac{dy}{1-y} = \int \frac{dx}{x}$$

$$\ln|1-y| = \ln|x| + C$$

$$1-y = Cx$$

$$y = 1-Cx$$




What's wrong?

$$\frac{dy}{dx} = \frac{1-y}{x}$$ (Not continuous at $x=0$)

Existence & uniqueness Not gauranteed.