Lec 12: General theory of inhomogenous ODE's.

$$y'' + P(x)y' + q(x) = f(x) \qquad (x = t \text{ independent})$$

$f(x)$: input signal, forcing term

Solution $y(x)$ is response, output.

$$\boxed{y'' + P(x)y' + q(x) = 0}$$

associated homogenous sol^n

Sol^n $y_c = c_1y_1 + c_2y_2$

$y_h = c_1y_1 + c_2y_2$

homogenous sol^n, complementary sol^n.

Theorem: $Ly = f(x)$

L is a linear operator

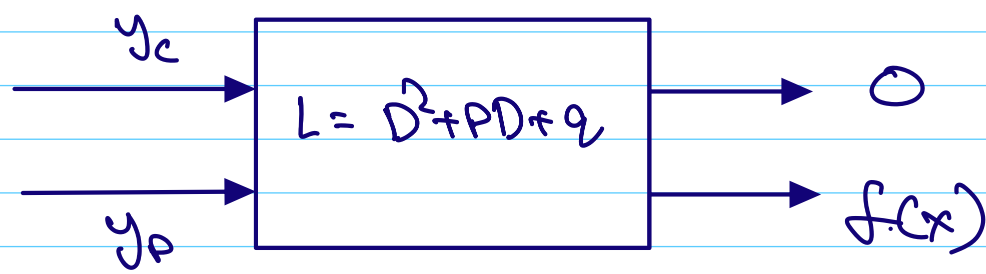
$$L := D^2 + P(x)D + q(x)$$

Sol^n $y(t) = y_p(t) + y_c(t)$

$$y(t) = \underbrace{y_p(t)}_{\text{Particular sol}^n} + \underbrace{y_c(t)}_{\text{complementary sol}^n}$$

$\Rightarrow \quad y(t) = y_p(t) + c_1 y_1(t) + c_2 y_2(t)$

y_p is Particular sol^n to $Ly = f(x)$



Particular mean's any one do the work.

To Prove: ① All the $y_p + c_1y_1 + c_2y_2$ are solution's

Proof:

$$L(y_p + c_1y_1 + c_2y_2)$$

$$= L(y_p) + c_1 L(y_1) + c_2(y_2)$$

$$= f(x) + 0 + 0 = f(x)$$

1st order:

$$y' + ky = q(t)$$

$$y = \underbrace{e^{-kt}\int q(t)e^{kt}\,dt}_{y_p} + \underbrace{ce^{-kt}}_{y_c}$$

Steady state, Transient state

if $k>0$

$y =$ Steady state + transient

y_p $\qquad$ $y_c \to 0$ as $t \to 0$

2nd order:

$$y'' + Ay' + By = f(t)$$

$$y = y_p + \underbrace{c_1y_1 + c_2y_2}_{\text{uses initial condition's}}$$

when does $c_1y_1 + c_2y_2 \to 0$ as $t \to \infty$

for all c_1, c_2

if this is so ODE is called stable

$$y = \underbrace{y_p}_{\text{steady state}} + \underbrace{c_1 y_1 + c_2 y_2}_{\text{transient}}$$