Lecos:

$$\frac{dy}{dt} = f(y)$$ $\leftarrow$ no t on RHS

(autonomou's system's)
$\Downarrow$
No independent variable on RHS.

we can solve this using seperating variable.
they why are we even talking about this?

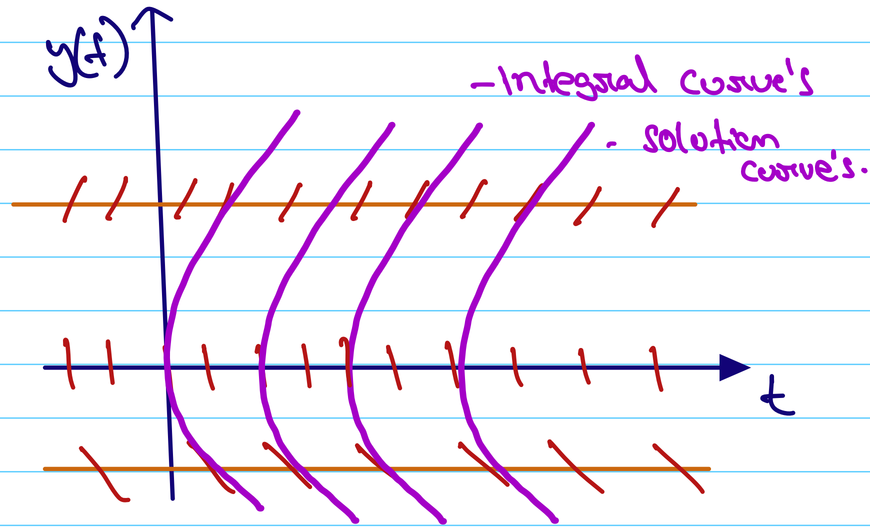
we will discuss about, How to get useful information out of the eqn, about the soln without solving the eqn.

$\Rightarrow$ Reason: FAST, use of lot of insight

Problem: to get Qualitative information about solution's without having to solve the solution.

① Direction Field

every horizontal line is a isocline



horizontal line $y = y_0$ is isoclines

$$\frac{dy}{dt} = f(y_0) = \text{constant.}$$

As we slide along t-axis the slope element's stay same. $\Rightarrow$ we can slide the curve (solution curve) horizontally, and still be integral curve (solution curve) everywhere.

$\Rightarrow$ Integral curve's are invariant under translation for an eq^n of this type (autonomous eq^n)

Critical Point:

$$f(y_0) = 0$$

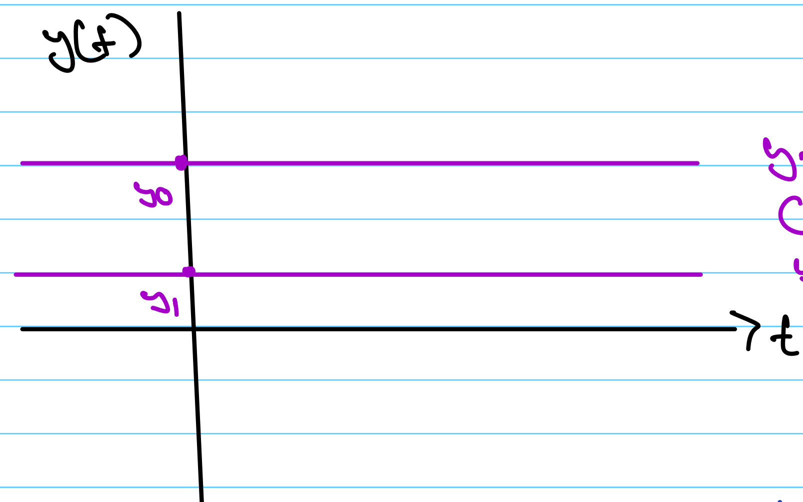
foor Critical Point, slope of the line element along the isocline $= 0$

isocline $=$ solution curve.

$\Rightarrow$ $y = y_0$ is a solution to ODE

Proof: $$\left.\frac{dy}{dt}\right|_{y=y_0} = 0 = \frac{dy_0}{dt} = 0$$

$$\Rightarrow f(y_0) = 0$$



$y = y_0$
(integral curve
$y = y_1$

other integral curve cannot cross there

① **Step 1:** find Critical Point's

② **Step 2:** Graph $f(y)$ > 0 ?
< 0 ?

$$\frac{dy}{dt} = f(y)$$

$\frac{dy}{dt}$ is +ve $\Leftarrow$ V (+ve)

Slope is +ve, Solution is increasing.

$y =$ money in Bank account

$r =$ Continuous interest rate

$$\frac{dy}{dt} = ry - W$$

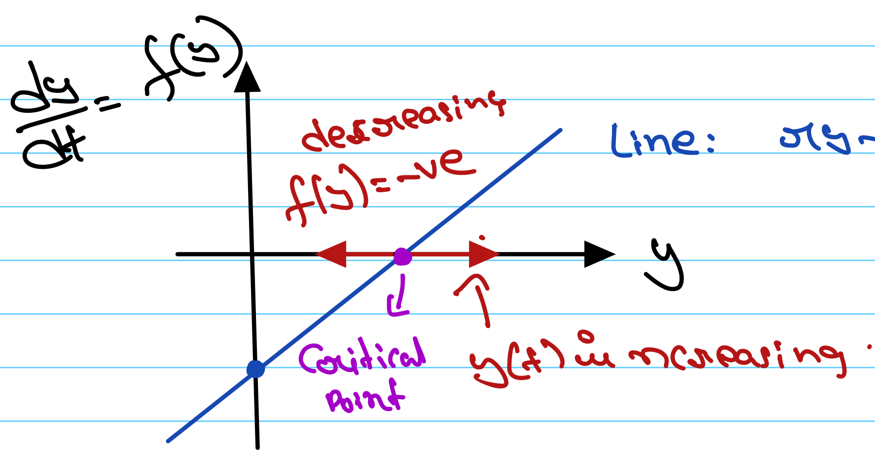
$W =$ rate of embezzlement

(every day money is sneaked out of your account)

$$\frac{dy}{dt} = ry - W = f(y) \quad (\text{autonomous eq}^n)$$

$$f(y_0) = 0 \Rightarrow ry_0 - W = 0$$

$$\Rightarrow y_0 = \frac{W}{r} \quad (\text{Critical Point})$$

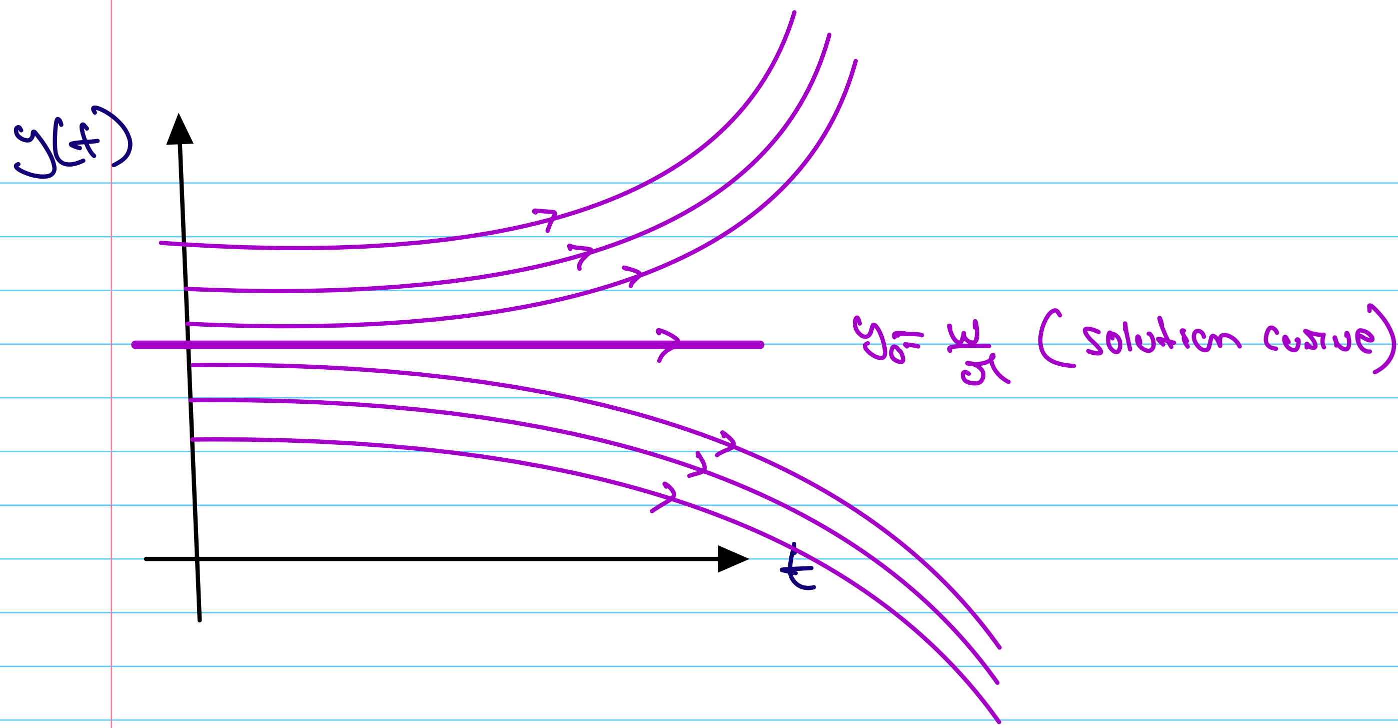


Line: $ry - W$ (slope r)

$y_0 = \frac{W}{r}$

$y > y_0$ ($\frac{dy}{dt}$ +ve)

$y < y_0$ ($\frac{dy}{dt}$ -ve)



Logistic eqn

Population Behavior $y(t)$

$$\frac{dy}{dt} = ky$$

k = growth rate = constant

Logistic growth

k declines when y increases

$$\frac{dy}{dt} = ay - by^2 = f(y)$$

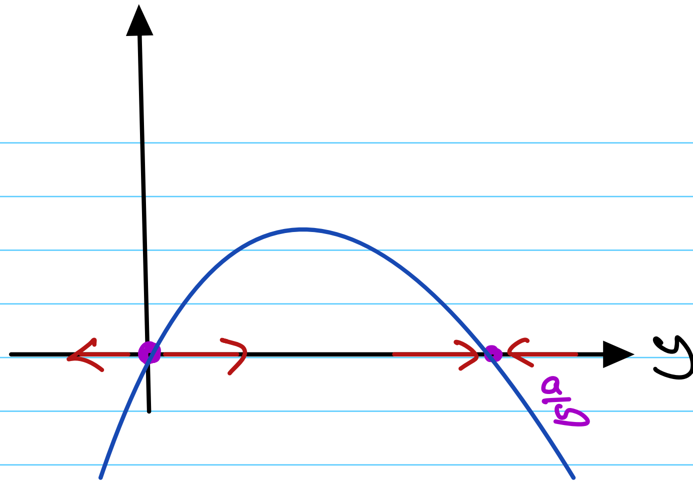
$$f(y_0) = 0 \Rightarrow ay_0 - by_0^2 = 0$$

$$\Rightarrow y_0 = \frac{a}{b}, \quad y_0 = 0$$

two critical point's.

$$\frac{dy}{dt} = f(y)$$

$$\frac{a^2}{2b} - b \cdot \frac{a^2}{4b^2}$$



Stable critical point

unstable critical point