Lec07 — 1st order Linear with constant co-efficient's: Behavior of solution's, use of complex method's.

Solve:

$$y' + ky = q(t)$$

I.F

$$e^{\int k\,t} = e^{kt}$$

$$e^{kt} y' + k e^{kt} y = q(t) e^{kt}$$

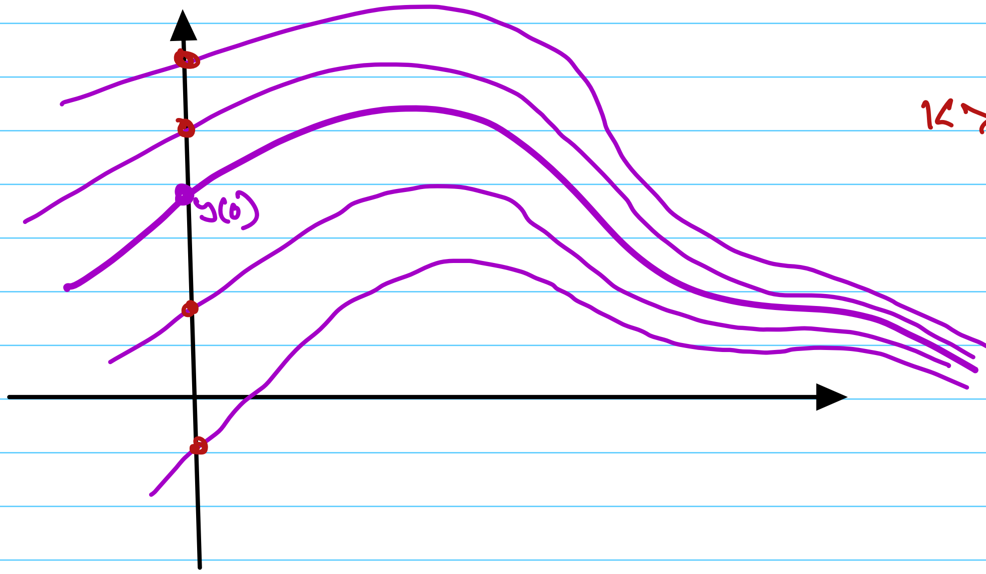
$$\left(e^{kt} y\right)' = q(t) e^{kt}\, dt$$

$$\Rightarrow \quad e^{kt} y(t) = \int q(t) e^{kt} dt + C$$

$$\Rightarrow \quad y(t) = \underbrace{Ce^{-kt}}_{\to 0 \text{ as } t \to \infty \text{ (transient state)}} + \underbrace{e^{-kt} \int_0^t q(t_1) e^{kt_1} dt_1}_{\text{steady-state solution (long term)}}$$

$K > 0$

$$y(t) = y(0)\, e^{-kt} + e^{-kt} \int_0^{t_1} q(t_1)\, e^{kt_1}\, dt_1$$



• other initial conditions

$K > 0$

As time goes on all solution's curves must approach same.

Which is the true steady state soln?

There is nothing special about the 'steady state solution. There isn't one steady state solution, they are many.

$\Rightarrow$ Pick the simplest function call it as steady state soln input: $q(t)$

response to the system: solution to diff eqn $y(t)$

$\Rightarrow$ The terminology, picture make sense only if $k > 0$ (+ve)

$\Rightarrow$ if k is -ve the term's steady state, transient is totally inappropriate if $k < 0$

Superposition of input's

$$q_1(t) \longrightarrow y_1(t)$$

$$q_2(t) \longrightarrow y_2(t)$$

$$q_1 + q_2 \longrightarrow y_1 + y_2$$

$$c q_1 \longrightarrow c y_1$$

USE'S: linearity of eq^n

Ex:

$y' + ky = k\, q_e(t)$

$q_e(t) = \cos\omega t$

$\omega =$ frequency. (angular freq)

$q_e = \cos \omega t$ find response $y(t)$ for $q_e = \cos\omega t$ input.

Complexification of the Problem:

(Since its easier to $\int$ exponential's)

$e^{i\omega t} = \cos\omega t + i\sin\omega t$

$Re(e^{i\omega t}) = \cos\omega t = q_e(t)$

$\tilde{y}' + k\tilde{y} = k e^{i\omega t}$

$\tilde{y} = y_1 + i y_2$ (complex sol^n)

Then $Re(\tilde{y}) = y_1$ is the solution.

$$\tilde{y}' + k\tilde{y} = ke^{i\omega t}$$

$$e^{kt}\tilde{y}' + ke^{kt}\tilde{y} = ke^{i\omega t} \cdot e^{kt}$$

$$\Rightarrow \left(e^{kt}\tilde{y}\right)' = ke^{(k+i\omega)t} dt$$

$$e^{kt}\tilde{y} = \int ke^{(k+i\omega)t} dt + C$$

$$\Rightarrow \tilde{y} = Ce^{kt} + \frac{ke^{(k+i\omega)t}}{(k+i\omega)}$$