Lec 21

Eigen values and eigen vector's

$$\det[A-\lambda I]=0$$

Matrices are square, we are now looking for some special number, eigenvalue, and some special vector's, the eigenvector's.

A : what does matrix do. it acts on vector's x.

A is like a function, in goes vector x, out comes vector Ax.

Ax parallel to x those are the eigen vector's

$\Rightarrow$ $\boxed{Ax=\lambda x}$ λ eigenvalue.

* what are the eigen vector's with eigen-value $=0$? they are the guy's in Null space. $Ax=0$

* if A is singular then $\lambda=0$ is an eigenvalue.

P = Projection matrix

what are the eigen values of Projection matrix.

* entire coloumn space is eigenvector's with eigen value $=1$

any x in $C(A)$ $\Rightarrow$ $Px=x$

$\Rightarrow$ eigenvalue $=1$

$\Rightarrow$ eigenvector $= C(A)$

if $x \in N(A^T)$ $\Rightarrow$ $Px=0$

$\Rightarrow$ eigenvalue $=0$

$\Rightarrow$ eigenvector $= N(A^T)$

$\Rightarrow$ Projection matrices only have eigenvalue 1 and 0

$\Rightarrow$ $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (Permutation matrix)

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\Rightarrow$ eigenvalue $= 1$ eigenvector: $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\Rightarrow$ $\lambda_2 = -1$ eigenvector: $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

* $n \times n$ matrices will have n eigen-values.

FACT:

$\Rightarrow$ Sum of the eigenvalue = trace.

How to solve $Ax = \lambda x$?

$Ax = \lambda x \Rightarrow (A - \lambda I)x = 0$

$\Rightarrow (A - \lambda I)\, x = 0$ (Singular)

$$\Rightarrow \quad x \in N(A-\lambda I)$$

$\Rightarrow \quad \det(A-\lambda I)=0$ find λ first.

$$A=\begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} \qquad A-\lambda I=\begin{bmatrix} 3-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix}$$

$$=(3-\lambda)^2=0$$

$$\lambda_1=3, \quad \lambda_2=3$$

$v_1=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v_2=$ No 2nd independent eigenvector

there is no 2nd eigenvector. This is called degenerate matrix. we only got one line of eigenvector's

This Possibility of repeated eigenvalue open this Further possibility of Shortage of eigenvector's.

Those are matrices that where eigenvector's don't give the complete story.