Ch3: Vector Spaces & Subspaces

* The main operation we do with vector's is add them, multiply them with scaler's.

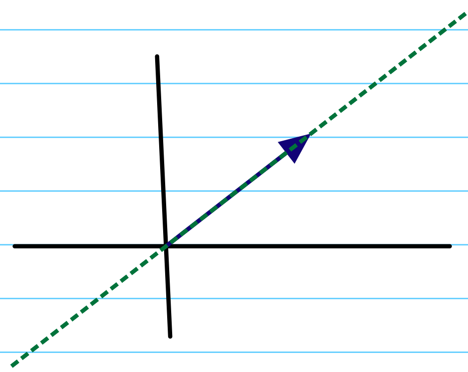
Vector Space :-

Ex: $\mathbb{R}^2$ = all 2-Dim vector's.

= x-y Plane.

Subspace: a vector space inside $\mathbb{R}^2$

Ex:



a line in $\mathbb{R}^2$ (through origin) is a subspace.

all Possible's subspace in $\mathbb{R}^2$

① $\mathbb{R}^2$

② A Line through origin $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

③ $\{0\}$ zero vector.

where do these subspace come from?

How do they come out of matrices?

Let's create some subspace out of matrices.

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix}$$

coloumn's are in $\mathbb{R}^3$

I want those coloumn's in subspace

$\Downarrow$

we must be able to all these coloum's, multiple with scalar & still be inside the subspace.

$\Rightarrow$ All Linear Combination's of coloumn's (This form a subspace)

we call it coloumn space $C(A)$

The whole coloumn space from col 1 & col 2 will be Plane.

Lec 06: Column space & Null space

vector space requirement's

$v+w$ and cv are in the space

all combination's $cv+dw$ are in the space

Ex: in $\mathbb{R}^3$

Subspaces:

1. $\mathbb{R}^3$
2. A Plane through origin $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ P
3. A Line through origin L
4. $\{0\}$ Just origin.

is the two subspaces P & L

$P \cup L =$ all vector's in P or L or Both

is this a subspace? NO

$P \cap L$ = all vector's in Both P and L

yes

General Question:

Subspaces S and T

$S \cap T$ = is a subspace.

$S \cup T$ = Not a subspace (Not always)

Coloumn space:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}_{4\times 3}$$

The coloumn space of this matrix A is a subspace of $\mathbb{R}^4$

$A_{4\times 3}$

How many row's (How many component's in a coloumn)

So, what's in the coloumn space?

All the linear combination's of 3 coloumn's i.e all linear combinatio of $\begin{bmatrix}1\\2\\3\\4\end{bmatrix}, \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}2\\3\\4\\5\end{bmatrix}$

we are interested in this coloumn space.

what's in that space?

How Big is that space?

Does $Ax=b$ have a solution for every b? NO

$Ax=b$ (4 eq^n, 3 unknown's)

which right hand side b have solution?

$$Ax = \begin{bmatrix}1 & 1 & 2\\2 & 1 & 3\\3 & 1 & 4\\4 & 1 & 5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix}$$

The linear combination's of these coloumn's don't fill the whole 4 Dimensional space, there will be a lot of vector's b, that are not linear combination's of these 3 coloumn's.

which vector's b allow this $Ax=b$ to be solved??

$$b \in C(A)$$

Are those three coloumn's independent?

does each coloumn contribute new? or Not?

Actually only 2 coloumn's are contributing something new.

col 3 not contributining new.

Null space:

Totally different subspace

Null space of A = all solution's of

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ to } Ax = 0$$

we are interested in solution's x, not b

$\Rightarrow$ so null space is a subspace of $\mathbb{R}^3$

$$Ax = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Null space: $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, c\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

Null space = line in $\mathbb{R}^3$

How do we know Null space is a subspace?

Check that solution's to $Ax=0$ always's give a subspace.

if $Av=0$ and $Aw=0$

then $A(v+w)=0$

& $A(c_1v+c_2w)=0$

$\Rightarrow$ Null space is a subspace

* for $Ax=b$

① do the solution's form a subspace?

because zero vector is not a solution.

(It's not a subspace, it's a line not going through origin)