1

Problem set 3.2

(a)

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} \boxed{1} & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} \boxed{1} & 2 & 2 & 4 & 6 \\ 0 & 0 & \boxed{1} & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = U$$

$$R = \begin{bmatrix} \boxed{1} & 2 & 0 & 0 & 0 \\ 0 & 0 & \boxed{1} & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Pivot col, Free, Pivot col, Free, Free

$x_1, x_3 \rightarrow$ Pivot variable

$x_2, x_4, x_5 \rightarrow$ Free variable

(b)

$$B = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & -2 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} \boxed{1} & 0 & -1 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

2

(a) $Rx = 0$ $\quad [I \; F]\begin{bmatrix} -F \\ I \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$R = \begin{bmatrix} \boxed{1} & 2 & 0 & 0 & 0 \\ 0 & 0 & \boxed{1} & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -2 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b)

$$R = \begin{bmatrix} \boxed{1} & 0 & -1 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}.$$

3

(a) $R = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad$ (b) $R = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Both R & U has same Null space

4

Pivot variable + free variable = n

(a) $A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & 6 & 10 \end{bmatrix}$

$$R=$$

$$\begin{bmatrix} -1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

special solution $= \begin{bmatrix} 3 & 5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $B = \begin{bmatrix} -1 & 3 & 5 \\ -2 & 6 & 7 \end{bmatrix}$

$$\begin{bmatrix} -1 & 3 & 5 \\ 0 & 0 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} \boxed{1} & -3 & 0 \\ 0 & 0 & \boxed{1} \end{bmatrix}$$

special solution: $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

5

(a) A square matrix has no free variables? FAlse

(b) A invertable matrix has no free variable? TRue

(c) An mxn matrix has no more than n Pivot variables? True

(d) An mxn matrix has no more than m Pivot variables? True.

6

Put as many 1's as possible in a 4x7 echelon matrix U whose pivot coloumn's are

(a) 2, 4, 5

$$\begin{bmatrix} 0 & \boxed{1} & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & \boxed{1} & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & \boxed{1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) 1, 3, 6, 7

$$\begin{bmatrix} \boxed{1} & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & \boxed{1} & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \boxed{1} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \boxed{1} \end{bmatrix}$$

(c) 4 and 6

$$\begin{bmatrix} 0 & 0 & 0 & \boxed{1} & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \boxed{1} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(7) Put as many 1's as possible in a 4x8 reduced echelon matrix R so that the free coloumn's are

(a) 2, 4, 5, 6

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Free

(6) 1, 3, 6, 7, 8 $\longrightarrow$ Pivot = {2, 4, 5}

$$\begin{array}{cccccccc}
 & & & & & & & \\
\left[\begin{matrix} 0 & \boxed{1} & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & \boxed{1} & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & \boxed{1} & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}\right]
\end{array}$$

$\uparrow$ 1 $\quad \uparrow$ 3 $\quad \uparrow$ 6 $\quad \uparrow$ 7 $\quad \uparrow$ 8 Free

(8) free variable $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

(9) x_5 is free variable $\begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(10) The number of special solution's = $n - r$

(ii) $r = n$

(iii) $r = m$

(11) (i) 5 pivot's

(ii) 5 pivot's

(12) $x - 3y - z = 0$

$$\begin{bmatrix} 1 & -3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

x_2, x_3 are free variable's

$$A = \begin{bmatrix} 1 & -3 & -1 \end{bmatrix}$$

$$\text{rref}(A) = \begin{bmatrix} 1 & -3 & -1 \end{bmatrix}$$

special solution = $\begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

(13) $x - 3y - z = 12$ (plane)

$$\begin{bmatrix} 1 & -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 12$$

$x = 12 + 3y + z$

$y = y$

$z = z$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} y + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} z$$

(14) in a 4x5 matrix

col 1 + col 3 + col 5 = 0 with

4 pivot's.

which coloumn have no pivot? 5

what is the special solution?

$$\begin{bmatrix} +1 \\ 0 \\ +1 \\ 0 \\ +1 \end{bmatrix}$$

$$N(A) = c \begin{bmatrix} +1 \\ 0 \\ +1 \\ 0 \\ +1 \end{bmatrix}$$

(15)

$$N(A) = c_1 \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 2 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & -2 & -3 \\ 0 & 1 & -2 & -1 \end{bmatrix}$$

(16) Construct A so that $N(A)$ = all multiples of $(4,3,2,1)$. its rank is ?

Sol^n

$$N(A) = c\begin{bmatrix}4\\3\\2\\1\end{bmatrix}$$

$n - r = 1$

$\Rightarrow \boxed{r = 3}$

$$A_{3\times 4} = \begin{bmatrix}1 & 0 & 0 & -4\\0 & 1 & 0 & -3\\0 & 0 & 1 & -2\end{bmatrix}$$

$\boxed{\text{rank} = 3}$

(17) Construct a matrix whose column space contain's $(1,1,5)$ and $(0,3,1)$ and whose nullspace contain's $(1,1,2)$

Sol^n

$$N(A) = \begin{bmatrix}1\\1\\2\end{bmatrix} \qquad C(A) = \begin{bmatrix}1 & 0\\1 & 3\\5 & 1\end{bmatrix}$$

$n = 3$ $\qquad$ $m = 3$

$A_{3\times3}$ rank 2 matrix

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 1 & 3 & -2 \\ 5 & 1 & -3 \end{bmatrix}$$

18 $C(A) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ $N(A) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$

Impossible construction.

19 $C(A) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $N(A) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$r = 3$ $n = 4$, $m = 3$

$$A_{3\times4} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

(22) if $AB = 0$ then the coloumn space of B is contained in the ______ of A. why?

Sol:

$AB = 0$

each coloumn of B is in Null space of A

(Null)

(23) Identify.

$$R_{4\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(24) (a) A and A^T have the same Nullspace.

$$Ax = 0 \Rightarrow x \in N(A)$$

$$\Rightarrow x^T A^T = 0$$

$$A^T y = 0 \Rightarrow y \in N(A^T)$$

x, y cannot be same

because $x \in \mathbb{R}^n$

$y \in \mathbb{R}^m$

(25) $N(A) = c \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

what is R and what is its rank

$n = 4, \; r = 3$

$$R = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$