Lec09: Independence, Basis and Dimension.

Linear independence spanning a space
Basis and dimention

Suppose A is $m \times n$ with $m < n$.
Then there are nonzero solution's to
$Ax = 0$
more unknown's than eq^n

then the concolion is $N(A) \neq \phi$
there will be special solution's.

<u>Reason:</u> There will be free variable's !!

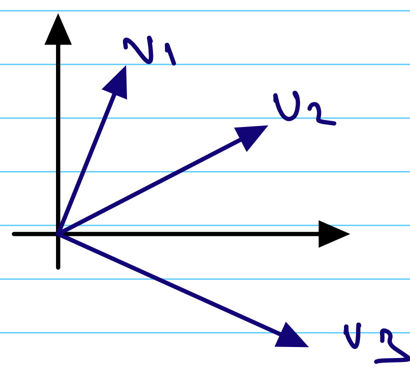
<u>Independence:</u>

vector's $x_1, x_2, \dots x_n$ are independent
Question do any of those combination give's 0 vector?

if some combination of those vector's give us zero vector, other than the combination of all zero's, then they are dependent.

* No combination give zero vector. (except the zero combination)
all $c_i = 0$

$\Rightarrow$ $c_1x_1 + c_2x_2 + \cdots + c_nx_n \neq 0$
if not the vector's are independent



How do we know v_1, v_2, v_3 are dependent? How do we know that some combination of v_1, v_2, v_3 will give zero vector?

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$$A = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$$

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$$\Rightarrow \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Ex:

$$\begin{bmatrix} 2 & 1 & 2.5 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ The coloumn's are dependent if there is something in the Null space.

Repeat when $v_1, v_2, v_3, \ldots v_n$ are coloumn's of A

rank $= n$ * They are independent if $N(A) = \{0\}$ (No free variable)

rank $< n$ * They are dependent if $Ac = 0$ for some nonzero $c \Rightarrow N(A)$ non empty

Put the vector's into a matrix.
And then the independence & dependence comes back to Null space.

SPANNING A SPACE:

What does it mean for a bunch of vector's to SPAN A SPACE.

⇒ Spanning a space mean's, vector's $v_1, v_2, \ldots v_l$ span a space mean's the space consists of all combination's of those vector's.

BASIS for a vector space is a seq^n of vector's $v_1, v_2, \ldots v_d$ with two properties

① they are independent

② They span the space.

for $\mathbb{R}^n$ if we have n vector's as BASIS, if the $n \times n$ matrix, then it need's to be invertable.

+ there are many many BASIS, But there is something in Common for all those BASIS.

They all have the same number of vector's. in $\mathbb{R}^3$ then we need 3 BASIS vector's

$\mathbb{R}^n \rightarrow n$ BASIS vector's.

Every BASIS for the space have same number of vector's.

* That number is telling us how big the space is.

That number = dimension of that space.