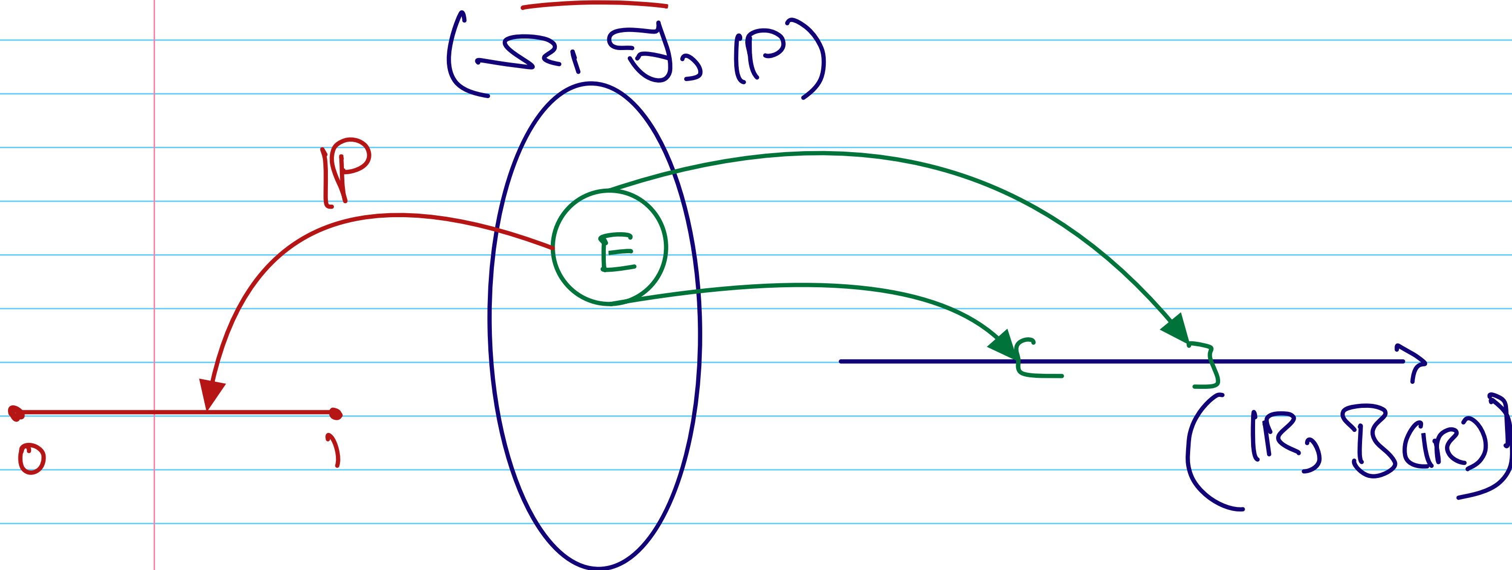
CDF



Probability law on X

$$\mathbb{P}_X(B) = \mathbb{P}(\{\omega \mid X(\omega) \in B\})$$
$$= \mathbb{P}(X \in B)$$

CDF:

$$\mathbb{F}_X(x) = \mathbb{P}_X((-\infty, x])$$
$$= \mathbb{P}(\{\omega \mid X(\omega) \leq x\})$$
$$= \mathbb{P}(X \leq x) \quad \text{(abuse of Notation)}$$

The Probability law $\mathbb{P}_X$ of a r.v X is uniquely defined by CDF $F_X(\cdot)$

Properties of CDF :-

$(\Omega, \mathcal{F}, \mathbb{P})$ is a Probability space, and $X : \Omega \longrightarrow \mathbb{R}$ is a r.v

(i) $$\lim_{x \to -\infty} F_X(x) = 0$$

(ii) $$\lim_{x \to \infty} F_X(x) = 1$$

(iii) monotonicity : (non-decreasing)

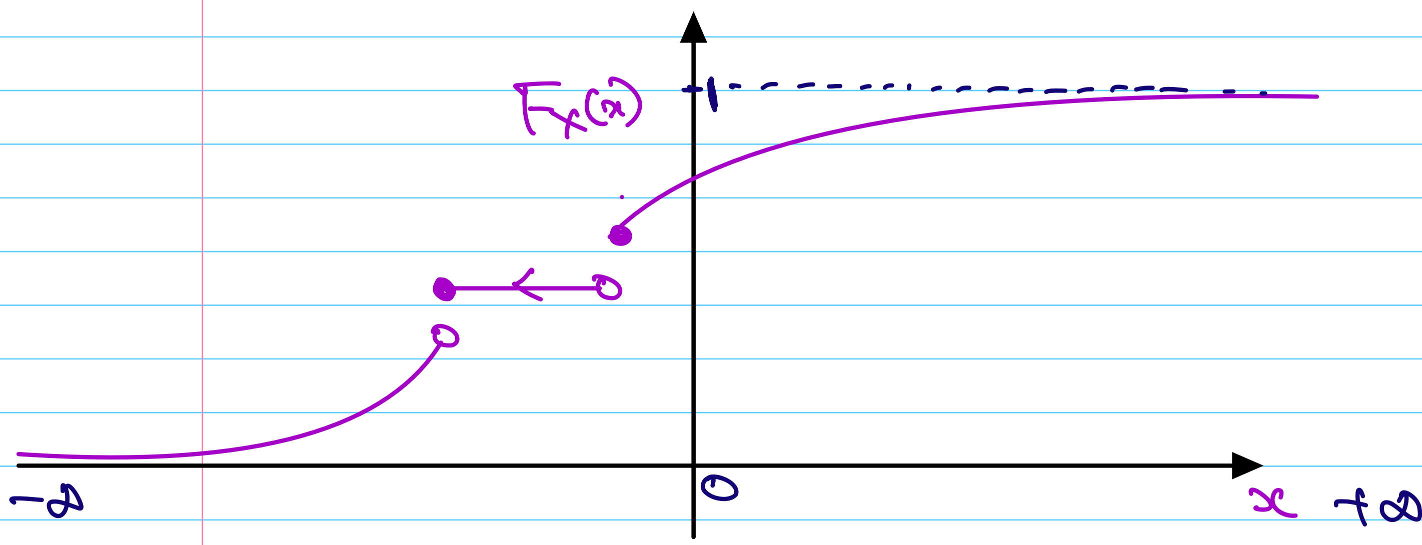
if $x \leq y$, $F_X(x) \leq F_X(y)$

(iv) $F_X(\cdot)$ is right continuous i.e

$$\lim_{\varepsilon \downarrow 0} F_X(x+\varepsilon) = F_X(x)$$

$\varepsilon \downarrow 0$: means ε is going to be 0

($\varepsilon \to 0^+$) from the side



CDF need not be left-continuous, But need to be right continuous.

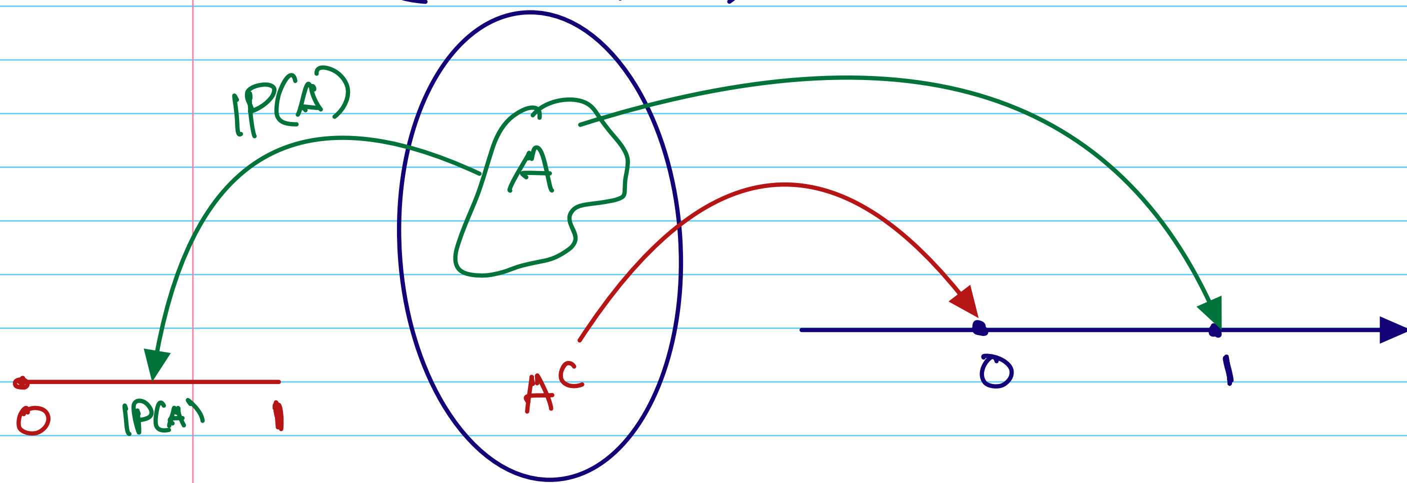
+ if it is Both right-continuous, and left-continuous, then it is continuous.

Indicator r.v

Let $A \in \mathcal{F}$. Define

$$I_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$$

$(\Omega, \mathcal{F}, \mathbb{P})$



$$\mathbb{P}(A) = \mathbb{P}_X(X = 1)$$

$$\mathbb{P}(A^c) = \mathbb{P}_X(X = 0)$$