Discrete random variables

A random variable X is said to be Discrete if $\exists$ a countable $\mathbb{E} \subseteq \mathbb{R}$ such that $\mathbb{P}_X(\mathbb{E}) = 1$

Borel set

* in other words, X takes value in a countable set.

$$\mathbb{E} = \{e_1, e_2, \ldots\} \quad \text{singleton's}, e_i \in \mathbb{R}$$

$$1 = \mathbb{P}_X(\mathbb{E}) = \sum_{i=1}^{\infty} \mathbb{P}_X(\{e_i\})$$

$$= \sum_{i=1}^{\infty} \mathbb{P}_X(X = e_i)$$

Probability mass function: (PMF)

if X is a discrete r.v, the function

$P_X : \mathbb{R} \to [0,1]$, defined by $P_X(x) = \mathbb{P}(X=x)$

$\forall x$ is called the PMF of X

Continuous r.v

* a random variable is set to be continuous r.v if the probability law P_X assigned to zero measure Borel set's is 0.

* if you take all Borel sets of lebesgue measure 0, on the Real line, i.e

$$\mathbb{P}_X(0 \text{ lebesgue measure sets})$$

$$= 0$$

then that r.v is said to be continuous.

Def: Let μ, ν be measure defined on $(\Omega, \mathcal{F})$, then we say ν is absoolutly continuous w.r.t μ if for every $N \in \mathcal{F}$, s.t $\nu(N) = 0$, then we have $\mu(N) = 0$