7.1

7.2

Exercises

(a) $S_n = \frac{1}{3n+1}$

$\Rightarrow S_n = \frac{\frac{1}{n}}{3+\frac{1}{n}} \longrightarrow 0$

First Five terms $= \left(\frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \frac{1}{13}, \frac{1}{16}\right)$

(b) $b_n = \frac{3n+1}{4n-1}$

$\Rightarrow b_n = \frac{3+\frac{1}{n}}{4-\frac{1}{n}} \longrightarrow \frac{3}{4}$

First Five term's $= \left(1, 1, \frac{10}{11}, \frac{13}{15}, \frac{16}{19}\right)$

(c) $C_n = \frac{n}{3^n}$

$\Rightarrow 3^n$ grow's faster than n

$C_n = \frac{n}{3^n} = \frac{\frac{d}{dn} n}{\frac{d}{dn} 3^n} = \frac{1}{3^n \ln 3}$

$C_n \longrightarrow 0$

First Five term's $\left(\frac{1}{3}, \frac{2}{9}, \frac{3}{27}, \frac{4}{81}, \frac{5}{243}\right)$

(d) $\sin\left(\frac{n\pi}{4}\right)$ do not converse

First five term's

$$\left(\sin\frac{\pi}{4}, \sin\frac{2\pi}{4}, \sin\frac{3\pi}{4}, \sin\frac{4\pi}{4}, \sin\frac{5\pi}{4}\right)$$

$$= \left(\frac{1}{\sqrt{2}}, 1, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$$

7.3

(a) $a_n = \frac{n}{n+1}$

$$\Rightarrow a_n = \frac{1}{1+\frac{1}{n}} \longrightarrow 1$$

(b) $b_n = \frac{n^2+3}{n^2-3}$

$$\Rightarrow b_n = \frac{1+\frac{3}{n^2}}{1-\frac{3}{n^2}} = 1$$

(c) $c_n = \frac{1}{2^n}$ $\quad c_n \longrightarrow 0$

(d) $t_n = 1+\frac{2}{n}$ $\quad t_n \longrightarrow 1$

(e) $x_n = 73 + (-1)^n$

Set $\{x_n = 73 + (-1)^n : n \in \mathbb{N}\} = \{72, 74\}$

do Not converge

(f) $S_n = 2^{\frac{1}{n}}$

$S_n = 2^{\frac{1}{n}}$ as $n \to \infty$

$S_n = 2^{\frac{1}{\infty}} = 1$

(g) $y_n = n!$ do not converge

(h) $d_n = (-1)^n \cdot n$ do not converge

(i) $\frac{(-1)^n}{n}$

as $n \to \infty$ $\frac{(-1)^n}{n} = \frac{+1 \text{ or } -1}{\infty} = 0$

(j) $\frac{7n^3 + 8n}{2n^3 - 3}$

$\Rightarrow \frac{7 + \frac{8}{n^2}}{2 - \frac{3}{n^3}} = \frac{7}{2}$

(k) $\dfrac{9n^2 - 18}{6n + 18}$

$$\frac{9 - \frac{18}{n^2}}{\frac{6}{n} + \frac{18}{n^2}} \longrightarrow \frac{9 - 0}{0 + 0} \longrightarrow \infty$$

do Not Converge

(l) $\sin\left(\frac{n\pi}{2}\right)$

fluctuates DO Not Converse

(m) $\sin(n\pi)$ Do Not Converge

(n) $\sin\left(\frac{2\pi n}{3}\right)$ DO Not Converse

(o) $\frac{1}{n}\sin n$

$$\Rightarrow \quad \frac{\sin n}{n} = \frac{(-1, 1)}{\infty} = 0$$

(p) $\dfrac{2^{n+1} + 5}{2^n - 7}$

$$\frac{2^{n+1}+5}{2^n-7} = \frac{1+\frac{5}{2^{n+1}}}{\frac{1}{2}-\frac{7}{2^{n+1}}} = \frac{1}{1/2} = 2$$

(q) $\frac{3^n}{n!}$

$n!$ grow's faster than 3^n

$$\frac{a_{n+1}}{a_n} = \frac{\frac{3^{n+1}}{(n+1)!}}{\frac{3^n}{n!}} = \frac{3}{n+1}$$

as $n \to \infty$ $\frac{a_{n+1}}{a_n} \to 0$

$\Rightarrow$ a_n is a decreasing seq^n

$\Rightarrow$ $a_n \xrightarrow{n\to\infty} 0$

(r) $\left(1+\frac{1}{n}\right)^2 = 1^2 + \frac{1}{n^2} + \frac{2}{n}$

$$\lim_{n\to\infty}\left(1^2+\frac{1}{n^2}+\frac{2}{n}\right) = 1$$

(s)

$$\frac{4n^2+3}{3n^2-2}$$

$$\Rightarrow \frac{4+\frac{3}{n^2}}{3-\frac{2}{n^2}} = \frac{4}{3}$$

(t)

$$\frac{6n+4}{9n^2+7} \Rightarrow \frac{\frac{6}{n}+\frac{4}{n}}{9+\frac{7}{n^2}} = \frac{0}{9} = 0$$

(7.4) Give examples of

(a) A seq^n (s_n) of irrational number's having a limit $\lim_{n\to\infty} s_n$ that is a rational number

Sol^n

$$(s_n)_{n\in\mathbb{N}} = \left(1+\frac{\sqrt{2}}{n}\right)_{n\in\mathbb{N}}$$

$$\lim_{n\to\infty} s_n = 1 \quad \text{(rational)}$$

(b) A seq^n (x_n) of rational number's having a limit $\lim x_n$ that is an irrational number.

Sol^n $\left(1+\frac{1}{n}\right)^n$ (rational numbers)

$$\lim \left(1+\frac{1}{n}\right)^n = e \quad \text{(irrational)}$$

7.5

(a) $\lim S_n$, where $S_n = \sqrt{n^2+1} - n$

Sol^n $\sqrt{n^2+1} - n \cdot \dfrac{(\sqrt{n^2+1} + n)}{(\sqrt{n^2+1} + n)}$

$$= \frac{\cancel{n^2}+1-\cancel{n^2}}{\sqrt{n^2+1}+n}$$

$$= \frac{1}{\sqrt{n^2+1}+n} \leq \frac{1}{2n} \longrightarrow 0$$

(b) $\lim \left(\sqrt{n^2+n} - n\right)$

$$\sqrt{n^2+n} - n \cdot \frac{\left(\sqrt{n^2+n} + n\right)}{\left(\sqrt{n^2+n} + n\right)}$$

$$= \frac{n}{\sqrt{n^2+n} + n}$$

$$= \frac{n}{\sqrt{n^2\left(1+\frac{1}{n}\right)} + n}$$

$$= \frac{n}{n\sqrt{1+\frac{1}{n}} + n} = \frac{1}{\sqrt{1+\frac{1}{n}} + 1}$$

$$= \frac{1}{2}$$

$$\Rightarrow \lim \left(\sqrt{n^2+n} - n\right) = \frac{1}{2}$$

(3)

$\lim \left(\sqrt{4n^2+n} - 2n\right)$

$$= \sqrt{4n^2+n} - 2n \cdot \frac{\left(\sqrt{4n^2+n} + 2n\right)}{\left(\sqrt{4n^2+n} + 2n\right)}$$

$$= \frac{4\cancel{n^2}+n-4\cancel{n^2}}{\sqrt{4n^2+n}+2n}$$

$$= \frac{n}{n\sqrt{4+\frac{1}{n}}+2n} = \frac{1}{\sqrt{4+\frac{1}{n}}+2}$$

$$= \frac{1}{4}$$