Exercises : section 9

9.1 (a) $\lim \frac{n+1}{n} = 1$

Sol^n

$$\lim \frac{n+1}{n} = \lim \frac{1+\frac{1}{n}}{1}$$

$$= \frac{1 + \lim \frac{1}{n}}{1} = 1$$

(b)

$$\lim \frac{3n+7}{6n-5}$$

$$= \frac{3 + 7 \cdot \lim \frac{1}{n}}{6 - 5 \cdot \lim \frac{1}{n}} = \frac{3}{6} = \frac{1}{2}$$

(c) $\lim \frac{17n^5 + 73n^4 - 18n^2 + 3}{23n^5 + 13n^3} = \frac{17}{23}$

$$= \frac{17 + 73 \lim \frac{1}{n} - 18 \lim \frac{1}{n^3} + 3 \lim \frac{1}{n^5}}{23 + 13 \lim \frac{1}{n^2}}$$

$$= \frac{17+0-0+0}{23+0} = \frac{17}{23}$$

9.2 $\lim x_n = 3$, $\lim y_n = 7$ $\quad y_n \neq 0 \ \forall n$

a) $\lim (x_n + y_n)$

$$= \lim x_n + \lim y_n = 10$$

b) $\lim \frac{3y_n - x_n}{y_n^2}$

$$= \frac{\lim \left(\frac{3}{y_n} - \frac{x_n}{y_n^2} \right)}{1}$$

$$= \frac{3 \cdot \lim \frac{1}{y_n} - \frac{\lim x_n}{\lim y_n^2}}{1}$$

$$= \frac{\frac{3}{7} - \frac{3}{49}}{1} = \frac{18}{49}$$

9.3 $\lim a_n = a$, $\lim b_n = b$ and $s_n = \dfrac{a_n^3 + 4a_n}{b_n^2 + 1}$

$$\lim s_n = \frac{\lim a_n^3 + 4 \lim a_n}{\lim b_n^2 + 1}$$

$$= \frac{a^3 + 4a}{b^2 + 1}$$

9.4 Let $s_1 = 1$, and $\forall n \geq 1$, let $s_{n+1} = \sqrt{s_n + 1}$

(a) List the first four term's of (s_n)

$\left(1, \sqrt{2}, \sqrt{\sqrt{2}+1}, \sqrt{\sqrt{\sqrt{2}+1}+1}\right.$

(b)

$\lim s_n$?

as $n \to \infty$ $\quad s_n = s_{n+1} = s$

$\Rightarrow \quad s = \sqrt{s+1}$

$\Rightarrow \quad s^2 - s - 1 = 0$

$\Rightarrow \quad S = \dfrac{1 \pm \sqrt{5}}{2}$

S cannot be -ve

$\Rightarrow$ therefore

$$\lim_{n \to \infty} S_n = \frac{1+\sqrt{5}}{2}$$

9.5 Let $t_1 = 1$ and $t_{n+1} = \dfrac{t_n^2 + 2}{2t_n}$ for $n \geq 1$

What is $\lim t_n$?

Sol^n as t_n converges.

as $n \to \infty$ $\quad t_{n+1} = t_n = t$

$$t_{n+1} = t = \frac{t^2+2}{2t}$$

$\Rightarrow \quad 2t^2 = t^2 + 2$

$\Rightarrow \quad t^2 = 2 \quad \Rightarrow \quad \boxed{t = \sqrt{2}}$

9.6 let $x_1 = 1$ and $x_{n+1} = 3x_n^2$ for $n \geq 1$

(a) Show that if $a = \lim a_n$, then $a = \frac{1}{3}$ or 0

Sol^n as $n \to \infty$ $\quad x_n = x_{n+1} = a$

$$\Rightarrow \quad a = 3a^2$$

$$\Rightarrow \quad a(1-3a) = 0$$

$$\Rightarrow \quad a = 0 \quad \text{or} \quad a = \frac{1}{3}$$

(b) Does $\lim x_n$ exists? Explain.

$$(x_1 = 1,\ x_{n+1} = 3x_n^2 : \forall n \geq 1)$$

$$= (1, 3, 3^3, 3^7, 3^{15}, \ldots)$$

The seq is diversinsins.

(c) The contradiction is we are ignoring x_1 (1st element)