Exercises: Section 8

8.1 (a) $\lim \frac{(-1)^n}{n} = 0$

Sol^n Let $\varepsilon > 0$, then we want to prove for $n > N$

$$\left| \frac{(-1)^n}{n} \right| < \varepsilon$$

$$\Rightarrow \quad \frac{|(-1)^n|}{n} < \varepsilon$$

$$\Rightarrow \quad \frac{1}{n} < \varepsilon \Rightarrow \frac{1}{\varepsilon} < n$$

we can take $N = \frac{1}{\varepsilon}$

Formal Proof:

let $\varepsilon > 0$, Let $N = \frac{1}{\varepsilon}$, Then $n > N$

$$\Rightarrow \quad n > \frac{1}{\varepsilon}$$

$$\Rightarrow \quad \frac{1}{n} < \varepsilon$$

$$\Rightarrow \frac{|(-1)^n|}{n} < \varepsilon$$

$$\Rightarrow \left| \frac{(-1)^n}{n} - 0 \right| < \varepsilon$$

therefore $\lim_{n\to\infty} \frac{(-1)^n}{n} = 0$

Q-1 b

$$\lim \frac{1}{n^{1/3}} = 0$$

Sol^n

Let $\varepsilon > 0$, we want to prove for $n > N \in \mathbb{N}$

$$\left| \frac{1}{n^{1/3}} - 0 \right| < \varepsilon$$

$$\Rightarrow \frac{1}{n^{1/3}} < \varepsilon \Rightarrow \frac{1}{\varepsilon^3} < n$$

Therefore Take $N = \frac{1}{\varepsilon^3}$

Formal Proof:

let $\varepsilon > 0$, let $N = \frac{1}{\varepsilon^3}$, then for

$n > N$ i.e $n > \frac{1}{\varepsilon^3}$

$\Rightarrow \frac{1}{n^{1/3}} < \varepsilon$

$= \left| \frac{1}{n^{1/3}} - 0 \right| < \varepsilon$

Therefore $\lim \frac{1}{n^{1/3}} = 0$

8.1 C

Prove $\lim \frac{2n-1}{3n+2} = \frac{2}{3}$

Proof: Let $\varepsilon > 0$, we want to prove for $n > N$

$$\left| \frac{2n-1}{3n+2} - \frac{2}{3} \right| < \varepsilon$$

$$\Rightarrow \left| \frac{6n-3-6n-6}{3(3n+2)} \right| < \varepsilon$$

$$\Rightarrow \left| \frac{-9}{3(3n+2)} \right| < \varepsilon$$

$$\Rightarrow \quad \left| \frac{-3}{3n+2} \right| < \varepsilon$$

$$\Rightarrow \quad \frac{3}{3n+2} < \varepsilon \Rightarrow \left(\frac{3}{\varepsilon} - 2\right)\frac{1}{3} < n$$

$$\Rightarrow \quad n > \frac{1}{\varepsilon} - \frac{2}{3}$$

Formal Proof:

Let $\varepsilon > 0$, and $N = \frac{1}{\varepsilon} - \frac{2}{3}$, $\forall n > N$

i.e $\quad n > \frac{1}{\varepsilon} - \frac{2}{3}$

$$\Rightarrow \quad n + \frac{2}{3} > \frac{1}{\varepsilon}$$

$$\Rightarrow \quad \frac{3}{3n+2} < \varepsilon$$

$$\Rightarrow \quad \left| \frac{-9}{3(3n+2)} \right| < \varepsilon$$

$$\Rightarrow \quad \left| \frac{2n-1}{3n+2} - \frac{2}{3} \right| < \varepsilon$$

Hence Proved.

8.1 d $\quad \lim \frac{n+6}{n^2-6} = 0$

Sol^n Let $\varepsilon > 0$, we want to prove

$$\left| \frac{n+6}{n^2-6} - 0 \right| < \varepsilon$$

$$\Rightarrow \quad \left| \frac{n+6}{n^2-6} \right| < \varepsilon$$

It is difficult to solve for or isolate n.

we need not need to find least N

therefore: $\Rightarrow$ find upper bound to numerator or

Lower bound to denominator.

i.e $\quad n+6 \leq 7n$

& $\quad n^2 - 6 \geq \frac{n^2}{2}$ for $n > 3$

therefore

$\forall n > 2$

$$\left|\frac{n+6}{n^2-6}\right| \le \left|\frac{7n}{\frac{n^2}{2}}\right| < \varepsilon$$

$$\Rightarrow \quad \left|\frac{14}{n}\right| < \varepsilon$$

$\Rightarrow \quad n > \frac{14}{\varepsilon} \quad \Rightarrow$ take $N = \frac{14}{\varepsilon}$

Formal Proof:

let $\varepsilon > 0$, let $N = \max\left\{\frac{14}{\varepsilon}, 2\right\}$, the $\forall n > N$

$$n > \frac{14}{\varepsilon}$$

$$\Rightarrow \left|\frac{14}{n}\right| < \varepsilon$$

$$\Rightarrow \left|\frac{7n}{\frac{n^2}{2}}\right| < \varepsilon$$

$\Rightarrow \left|\frac{n+6}{n^2-6}\right| < \varepsilon$ for $n > 2$

Hence proved