

Exercises : section 9

9.11

(a) $\lim \frac{n+1}{n} = 1$

Soln

$$\lim \frac{n+1}{n} = \lim \frac{1 + \frac{1}{n}}{1}$$

$$= \frac{1 + \lim \frac{1}{n}}{1} = 1$$

(b)

$$\lim \frac{3n+7}{6n-5}$$

$$= \frac{3 + 7 \cdot \lim \frac{1}{n}}{6 - 5 \cdot \lim \frac{1}{n}} = \frac{3}{6} = \frac{1}{2}$$

(c)

$$\lim \frac{17n^5 + 73n^4 - 18n^2 + 3}{23n^5 + 13n^3} = \frac{17}{23}$$

$$= \frac{17 + 73 \lim \frac{1}{n} - 18 \lim \frac{1}{n^2} + 3 \lim \frac{1}{n^5}}{23 + 13 \lim \frac{1}{n^2}}$$

$$= \frac{17 + 0 - 0 + 0}{23 + 0} = \frac{17}{23}$$

9.2

$$\lim x_n = 3, \quad \lim y_n = 7 \quad y_n \neq 0 \quad \forall n$$

$$\begin{aligned} \text{(a)} \quad \lim (x_n + y_n) \\ = \lim x_n + \lim y_n = 10 \end{aligned}$$

$$\text{(b)} \quad \lim \frac{3y_n - x_n}{y_n^2}$$

$$= \frac{\lim \left(\frac{3}{y_n} - \frac{x_n}{y_n^2} \right)}{1}$$

$$= \frac{3 \cdot \lim \frac{1}{y_n} - \frac{\lim x_n}{\lim y_n^2}}{1}$$

$$= \frac{\frac{3}{7} - \frac{3}{49}}{1} = \frac{18}{49}$$

9.3

$\lim a_n = a$, $\lim b_n = b$ and $S_n = \frac{a_n^3 + 4a_n}{b_n^2 + 1}$

$$\begin{aligned}\lim S_n &= \frac{\lim a_n^3 + 4 \lim a_n}{\lim b_n^2 + 1} \\ &= \frac{a^3 + 4a}{b^2 + 1}\end{aligned}$$

9.4

Let $S_1 = 1$, and $\forall n \geq 1$, let $S_{n+1} = \sqrt{S_n + 1}$

(a) List the first four terms of (S_n)
 $(1, \sqrt{2}, \sqrt{\sqrt{2}+1}, \sqrt{\sqrt{\sqrt{2}+1}+1})$

(b)

$\lim S_n$?

$$\text{as } n \rightarrow \infty \quad S_n = S_{n+1} = S$$

$$\Rightarrow S = \sqrt{S+1}$$

$$\Rightarrow S^2 - S - 1 = 0$$

$$\Rightarrow S = \frac{1 \pm \sqrt{5}}{2}$$

S cannot be \rightarrow ve

\Rightarrow therefore

$$\lim_{n \rightarrow \infty} S_n = \frac{1 + \sqrt{5}}{2}$$

(9.5)

let $t_1 = 1$ and $t_{n+1} = \frac{t_n^2 + 2}{2t_n}$ for $n \geq 1$

What is $\lim t_n$?

Solⁿ

as t_n converges,

as $n \rightarrow \infty$ $t_{n+1} = t_n = t$

$$t_{n+1} = t = \frac{t^2 + 2}{2t}$$

$$\Rightarrow 2t^2 = t^2 + 2$$

$$\Rightarrow t^2 = 2$$

$$\Rightarrow t = \sqrt{2}$$

Q.6 let $x_1 = 1$ and $x_{n+1} = 3x_n^2$ for $n \geq 1$

(a) Show that if $a = \lim x_n$, then
 $a = \frac{1}{3}$ or 0

Soln

$$\text{as } n \rightarrow \infty \quad x_n = x_{n+1} = a$$

$$\Rightarrow a = 3a^2$$

$$\Rightarrow a(1-3a) = 0$$

$$\Rightarrow a = 0 \text{ or } a = \frac{1}{3}$$

(b) Does $\lim x_n$ exist? Explain.

$$(x_1 = 1, x_{n+1} = 3x_n^2 : \forall n \geq 1)$$

$$= (1, 3, 3^3, 3^7, 3^{15}, \dots)$$

The seq is diverging.

(c) The contradiction is we are ignoring x_1 (1st element)

9.8

(a) $\lim n^2$ (Not Exist)

(b) $\lim (-n^2) = -\lim n^2$ (Not Exist)

(c) $\lim (-n)^n =$ (Not Exist)

(d) $\lim (1.01)^n$ (Not Exist)

$$\lim a^n = 0 \quad \text{if } |a| < 1$$

here $1.01 > 1$

(e) $\lim n^n$ (Not exist)