Exercise: section 11

Sub sequences

(11.1) Let $a_n = 3+2(-1)^n \quad \forall n \in \mathbb{N}$

(a) List 1st 8 term's of the seq

$(1, 5, 1, 5, 1, 5, 1, 5)$

(b) Given a subseq^n that is constant. Specify the selection function σ

$\sigma(k) = 2k \qquad \lim S_{2k} = 5$

$\sigma(k) = 2k+1 \qquad \lim S_{2k+1} = 1$

(11.2) Consider the seq^n defined as follows

$a_n = (-1)^n, \quad b_n = \frac{1}{n}, \quad c_n = n^2$

$$d_n = \frac{6n+4}{7n-3}$$

(a) $\forall$ seq^n, Give an example of monotone subseq^n.

$a_n = (-1)^n \quad (a_{2k})_{k\in\mathbb{N}} = 1$

$b_n = \frac{1}{n} \quad (b_{2k})_{k\in\mathbb{N}} = \frac{1}{2k}$

$c_n = n^2 \quad (c_{2k})_{k\in\mathbb{N}} = 4k^2$

$d_n = \frac{6n+4}{7n-3} \quad (d_{2k})_{k\in\mathbb{N}} = \frac{12k+4}{14k-3}$

(b) $\forall$ seq^n, give its set of subseq^n limits

$a_n = (-1)^n \quad a = \{-1, 1\}$

$b_n = \frac{1}{n} \quad b = \{0\}$

$c_n = n^2 \quad c_n = \{+\infty\}$

$d_n = \frac{6n+4}{7n-3} \quad d_n = \{\frac{6}{7}\}$

(c) $\forall$ seq^n, give its limsup, lim inf

$\limsup_{n\to\infty} a_n = 1$ $\liminf_{n\to\infty} a_n = -1$

$\limsup_{n\to\infty} b_n =$ $\liminf_{n\to\infty} b_n = 0$

$\limsup_{n\to\infty} c_n =$ $\liminf_{n\to\infty} b_n = +\infty$

$\limsup_{n\to\infty} d_n =$ $\liminf_{n\to\infty} d_n = \frac{6}{7}$

(d) which of the seq^n converges? diverges to $+\infty$? diverges to $-\infty$?

b_n converges

c_n diverges to $+\infty$

d_n converges to $\frac{6}{7}$

(e) which of the seq^n is bounded?

a_n, b_n, d_n are Bounded

11.3 repeat Exercise 11.2 for the seq^n

* $s_n = \cos\left(\frac{n\pi}{3}\right)$

* $S = \{-1, -\frac{1}{2}, \frac{1}{2}, 1\}$

* $\limsup_{n\to\infty} s_n = 1$ $\liminf_{n\to\infty} s_n = -1$

* Bounded

$t_n = \frac{3}{4n+1}$

* $\lim_{n\to\infty} t_n = 0$ $t = \{0\}$

* $\liminf_{n\to\infty} t_n = \limsup_{n\to\infty} t_n = 0$

* Converges to 0

* Bounded

$U_n = \left(-\frac{1}{2}\right)^n$

* $(U_{2k})_{k\in\mathbb{N}} = \left(\left(-\frac{1}{2}\right)^{2k}\right)_{k\in\mathbb{N}} = \left(\left(\frac{1}{2}\right)^{k}\right)_{k\in\mathbb{N}}$

monotone sub seq^n

* $U = \{0\}$

* $\limsup_{n\to\infty} U_n = \liminf_{n\to\infty} U_n = 0$

* Converges to 0

* Bounded

$V_n = (-1)^n + \frac{1}{n}$

$(V_{2k})_{k\in\mathbb{N}} = \left(1 + \frac{1}{2k}\right)_{k\in\mathbb{N}}$

$(V_{2k+1})_{k\in\mathbb{N}} = \left(-1 + \frac{1}{2k+1}\right)_{k\in\mathbb{N}}$

* monotonic subseq^n

* $V = \{-1, 1\}$

* $\limsup_{n \to \infty} V_n = -1$ $\quad \liminf_{n \to \infty} V_n = -1$

* Bounded