Exercises: The Completeness Axiom

4.1, 4.2, 4.3, 4.4

(a) $S = [0,1]$

$\min S = 0$ Upper Bound = $1, 5, 10$

$\max S = 1$ Lower Bound = $-1, -5, 0$

$\sup S = 1$

$\inf S = 0$

(b) $S = (0,1)$

$\min S =$ No min Upper Bound = $1, 5, 10$

$\max S =$ No max Lower Bound = $-1, -5, 0$

$\sup S = 1$

$\inf S = 0$

(c) $S = \{2, 7\}$

$\min S = 2$ Upper Bound = $7, 10, 15$

$\max S = 7$ Lower Bound = $-5, 0, 2$

$\sup S = 7$ $\inf S = 2$

(e) $S = \{\frac{1}{n} : n \in \mathbb{N}\} = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$

$\min S$ = No min — Upper Bound = 2, 1, 3

$\max S = 1$ — Lower Bound = 0, −1, −2

$\sup S = 1$ — $\inf S = 0$

(f) $S = \{0\}$

$\min S = 0$ — Upper Bound = 0, 1, 2

$\max S = 0$ — Lower Bound = −1, 0, −2

$\sup S = 0$ — $\inf S = 0$

(g) $S = [0,1] \cup [2,3]$

$\min S = 0$ — Upper Bound = 3, 4, 5

$\max S = 3$ — Lower Bound = −1, −0.5, 0

$\sup S = 0$ — $\inf S = 3$

(h) $S = \bigcup_{n=1}^{\infty} [2n, 2n+1] = [2,3] \cup [4,5] \cup [6,7] \cup [8,9] \dots$

$\min S = 2$ — Upper Bound = No upper Bound.

$\max S$ = No max — Lower Bound = 2, 0, 1

$\sup S$ = No sup — $\inf S = 2$

(i) $S = \bigcap_{n=1}^{\infty} \left[-\frac{1}{n}, 1+\frac{1}{n}\right] = [-1, 2] \cap \left[-\frac{1}{2}, \frac{3}{2}\right] \cap \left[-\frac{1}{3}, \frac{4}{3}\right] \cap \left[-\frac{1}{4}, \frac{5}{4}\right] \cap \left[-\frac{1}{5}, \frac{6}{5}\right] \cdots$

$\min S =$ No min — Upper Bound $= 1, 2, 3$

$\max S =$ No max — Lower Bound $= -1, 2, 0$

$\sup S = 1$ — $\inf S = 0$

(j) $S = \left\{1 - \frac{1}{3^n} : n \in \mathbb{N}\right\} = \left\{\frac{2}{3}, \frac{8}{9}, \frac{26}{27}, \frac{80}{81}, \cdots\right\}$

$\min S = \frac{2}{3}$ — Upper Bound $= 1, 2, 3$

$\max S =$ No max — Lower Bound $= \frac{2}{3}, \frac{1}{2}, 0$

$\sup S = 1$ — $\inf S = \frac{2}{3}$

(K) $S = \left\{n + \frac{(-1)^2}{n} : n \in \mathbb{N}\right\} = \left\{0, \frac{5}{2}, \frac{6}{9}, \cdots\right\}$

$\min S = 0$ — Upper Bound $=$ No upper Bound

$\max S =$ No max — Lower Bound $= 0, -1, -2$

$\sup S =$ No sup — $\inf S = 0$

(l) $\{x \in \mathbb{Q} : x < 2\}$

min S = No min Upper Bound = 2, 3, 4

max S = No max Lower Bound = No lower Bound

sup S = 2 inf S = No inf

(m) $\{x \in \mathbb{Q} : x^2 < 4\}$

min S = No min Upper Bound = 2, 3, 4

max S = No max Lower Bound = $-2, -3, -4$

sup S = 2 inf S = -2

(n) $\{x \in \mathbb{Q} : x^2 < 2\}$

min S = No min Upper Bound = $\sqrt{2}, 3, 4$

max S = No max Lower Bound = $-\sqrt{2}, -3, -4$

sup S = No sup inf S = No inf

(o) $\bigcap_{n=1}^{\infty} \left(1 - \frac{1}{n}, 1 + \frac{1}{n}\right)$

$= (0, 2) \cap \left(\frac{1}{2}, \frac{3}{2}\right) \cap \left(\frac{2}{3}, \frac{4}{3}\right)$

$\cdots = \{1\}$

min S = 1 Upper Bound = 1, 2, 3

max S = 1 Lower Bound = 1, 0, -1

sup S = 1 inf S = 1