The Set $\mathbb{R}$ of Real Number's

The Basic algebraic operation's on $\mathbb{Q}$ are addition and multiplication.

$\Rightarrow a, b \in \mathbb{Q}$, then $a+b, ab \in \mathbb{Q}$

Properties:-

A1: $a + (b+c) = (a+b) + c \quad \forall a,b,c$

A2: $a + b = b+a \quad \forall a,b$

A3: $a+0 = a \quad \forall a$

A4: $\forall a, \exists -a$ s.t $a+(-a)=0$

M1: $a(bc) = (ab)c \quad \forall a,b,c$

M2: $ab = ba \quad \forall a,b$

M3: $a \cdot 1 = a \quad \forall a$

M4: $\forall a \neq 0, \exists a^{-1}$ s.t $a \cdot a^{-1} = 1$

DL: $a(b+c) = ab + ac \quad \forall a,b,c$

A1, M1 - associative Law's

A2, M2 - Commutative Law's

DL - Distributive Law

A system that has more than one element, and satisfies these nine properties is called Field

The set $\mathbb{Q}$ also has an order structure "$\leq$" satisfying

O1: Given a, b either $a \leq b$ or $b \leq a$

O2: if $a \leq b$, $b \leq a$ then $a = b$

O3: if $a \leq b$ and $b \leq c$, then $a \leq c$

O4: if $a \leq b$, then $a+c \leq b+c$

O5: if $a \leq b$, $0 \leq c$, then $ac \leq bc$

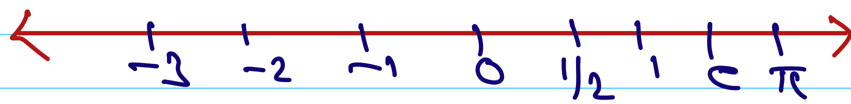
O3 – transitive Law

A Field with an ordering satisfying properties O1 through O5 is called an "Ordered field."

The mathematical system on which we will do our analysis will be the set $\mathbb{R}$ of all real number's.

$\mathbb{R} = \mathbb{Q}$ + algebraic number's + π, e..

$\mathbb{R}$ can be drawn as a Real number Line



* Unlike $\mathbb{Q}$, $\mathbb{R}$ will not have gap's.

* Real number's $\mathbb{R}$ satisfy the Field Properties, A1 to A4, M1 through M4, and DL

* $\mathbb{R}$ also has an order structure "$\leq$" that satisfy O1 through O5.

Thus, Like $\mathbb{Q}$, $\mathbb{R}$ is an Ordered Field.

Theorem 3.1 :-

The following are the consequences of the field properties:

(i) $a+c = b+c \implies a=b$

(ii) $a \cdot 0 = 0 \quad \forall a$

(iii) $(-a)b = -ab \quad \forall a, b$

(iv) $(-a)(-b) = ab \quad \forall a, b$

(v) $ac = bc$ and $c \neq 0 \implies a=b$

(vi) $ab=0 \implies a=0$ or $b=0 \quad \forall a, b, c \in \mathbb{R}$

Theorem 3.2 :-

The following are the consequences of the properties of an ordered field.

(i) if $a \leq b$, then $-b \leq -a$

(ii) if $a \leq b$, $c \leq 0$, then $ac \geq bc$

(iii) if $0 \leq a$, $0 \leq b$, then $0 \leq ab$

(iv) $0 \leq a^2 \quad \forall a$

(v) $0 < 1$

(vi) if $0 < a$, then $0 < a^{-1}$

(vii) if $0 < a < b$, then $0 < b^{-1} < a^{-1}$

Definition 3.3:

$|a| = a$ if $a \geq 0$ and $|a| = -a$

$|a| :=$ distance b/w 0 and a

Definition 3.4:

The number's a and b we define $dist(a,b) = |a-b|$; $dist(a,b)$ represents the distance b/w a and b

Theorem 3.5:

(i) $|a| \geq 0 \quad \forall a \in \mathbb{R}$

(ii) $|ab| = |a| \cdot |b| \quad \forall a,b \in \mathbb{R}$

(iii) $|a+b| \leq |a| + |b| \quad \forall a,b \in \mathbb{R}$

Proof: (iii)

$$-|a| \le a \le |a|$$

$$-|b| \le b \le |b|$$

$$\Rightarrow -|a| - |b| \le a+b \le |a| + |b|$$

$$\Rightarrow -(|a| + |b|) \le a+b \le |a| + |b| \quad \text{--- (1)}$$

&

$$-|a+b| \le a+b \le |a+b| \quad \text{--- (2)}$$

(1) & (2)

$$-|a+b| \le |a| + |b|$$

Corollary:

$$\text{dist}(a,c) \le \text{dist}(a,b) + \text{dist}(b,c)$$

$$\forall a,b,c \in \mathbb{R}$$