The Set $\mathbb{Q}$ of Rational Number's

$$\mathbb{N} = \{1, 2, 3, \dots\}$$

subtraction is Introduced

$\Downarrow$

Include 0, −ve integer's

$$\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, \dots\}$$

division is introduced

$\Downarrow$

include all fraction's

$$\mathbb{Q} = \left\{ \frac{m}{n} \;\middle|\; m, n \in \mathbb{Z},\ n \neq 0 \right\}$$

* The $\mathbb{Q}$ is a very nice algebraic system until one tries to solve eqn like $x^2 = 2$
* No Rational number satisfy $x^2 = 2$

It is evident that there are lots of rational number's and yet there are "gaps" in $\mathbb{Q}$

Definition:

A number is called an algebraic number if it satisfies a polynomial eq^n

$$c_n x^n + c_{n-1} x^{n-1} + \cdots + c_1 x + x_0 = 0$$

where the co-efficients $c_0, c_1, \ldots c_n$ are integer's, $c_n \neq 0$ and $n \geq 1$

Ex:- Algebraic number's

ⓐ $\frac{4}{17} \Rightarrow 17x - 4 = 0$

ⓑ $\sqrt{3} \Rightarrow x^2 - 3 = 0$

ⓒ $\sqrt{2 + \sqrt[3]{5}} \Rightarrow x^2 - 2 = \sqrt[3]{5}$

$\Rightarrow (x^2 - 2)^3 - 5 = 0$

ⓓ $\sqrt{\frac{4 - 2\sqrt{3}}{7}} \Rightarrow \left(\frac{4 - 7x^2}{2}\right)^2 - 3 = 0$

ⓔ