Lec02 : Signal's & System's – I

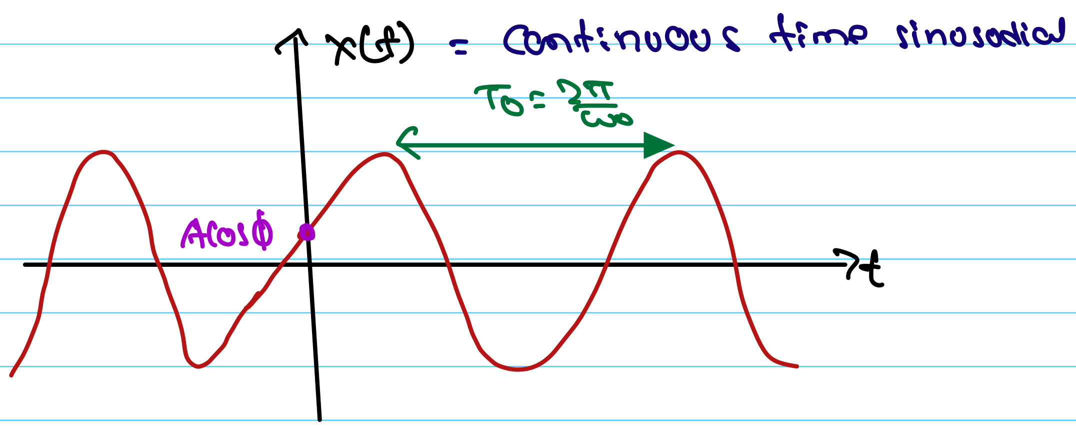
* we will discuss some of the BASIC signal's, Both continuous time & Discrete time that will form important Building Blocks As course Progess.

① Continuous – time Sinusoidal Signal

$$x(t) = A\cos(\omega_0 t + \phi)$$

Amplitude   freq   phase

Periodic $\Rightarrow$ $x(t+T_0) = x(t)$



* Sinusoidal signal has a number of important properties, that we will find it convenient to exploit as the course goes along.

(1) Sinosiodial signal is Periodic

$$x(t) = x(t + T_0)$$

$$A\cos(\omega_0 t + \phi) = A\cos(\omega_0 t + \underbrace{\omega_0 T_0}_{2\pi m} + \phi)$$

$$\Rightarrow T_0 = \frac{2\pi m}{\omega_0} \Rightarrow \boxed{\text{Period} = T = \frac{2\pi}{\omega_0}}$$

Period = smallest value of T_0

* Time shift of a sinusodial is equivalent to phase change.

$$A\cos(\omega_0 \underbrace{(t+t_0)}_{\substack{\text{Change in} \\ \text{time} \\ \text{(time-shift)}}}) = A\cos(\omega_0 t + \underbrace{\omega_0 t_0}_{\substack{\text{change} \\ \text{in Phase}}})$$

Time shift $\Longleftrightarrow$ Phase change

* time shift produces phase change & phase change produces time shift.

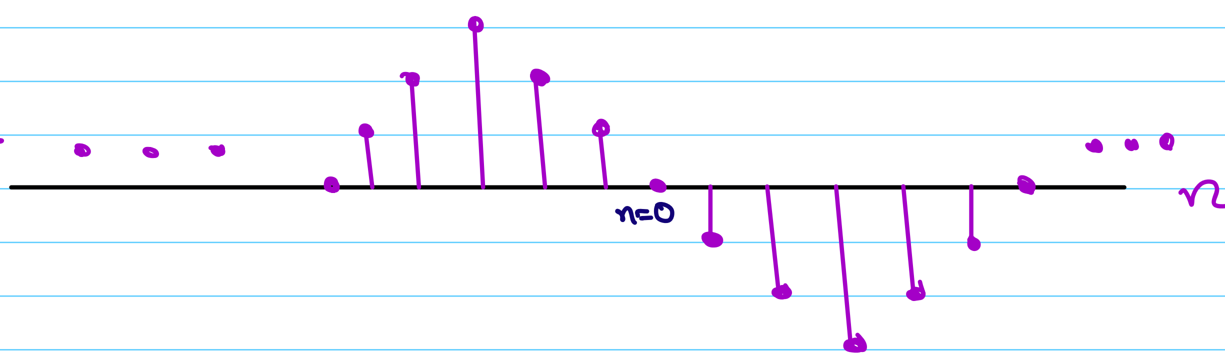
in effect, changing the phase corresponding to move this signal in time.

Discrete-Time Sinusoidal signal

$$x[n] = \underbrace{A}_{\text{Amplitude}} \cos(\underbrace{\Omega_0}_{\text{freq}} n + \underbrace{\phi}_{\text{phase}})$$

$\Omega_0 = \frac{2\pi}{12}$

$\phi = 0$



Relation b/w time-shift & phase change

* Time shift $\Longrightarrow$ Phase change

$$A \cos[\Omega_0(n+n_0)] = A\cos[\Omega_0 n + \underbrace{\Omega_0 n_0}_{\text{change in phase}}]$$

Phase change $\overset{?}{\Longrightarrow}$ time-shift

$$A\cos(\Omega_0 n + \phi) = A\cos(\Omega_0(n+n_0))$$

$$\phi_0 = \Omega_0 n_0$$

* n_0 may or may not be integer

Periodic?

$$x[n] = x[n+N]$$

Smallest integer $\triangleq$ Period

$$A\cos(\Omega_0(n+N)+\phi)$$

$$= A\cos(\Omega_0 n + \underbrace{\Omega_0 N}_{2\pi m} + \phi)$$

integer multiple of 2π?

Periodic $\Rightarrow$ $\Omega_0 N = 2\pi m$

$$N = \frac{2\pi m}{\Omega_0}$$

N, m b integer's

Smallest N (if any) = Period.

$$x_1[n] = A\cos[\Omega_1 n + \phi]$$

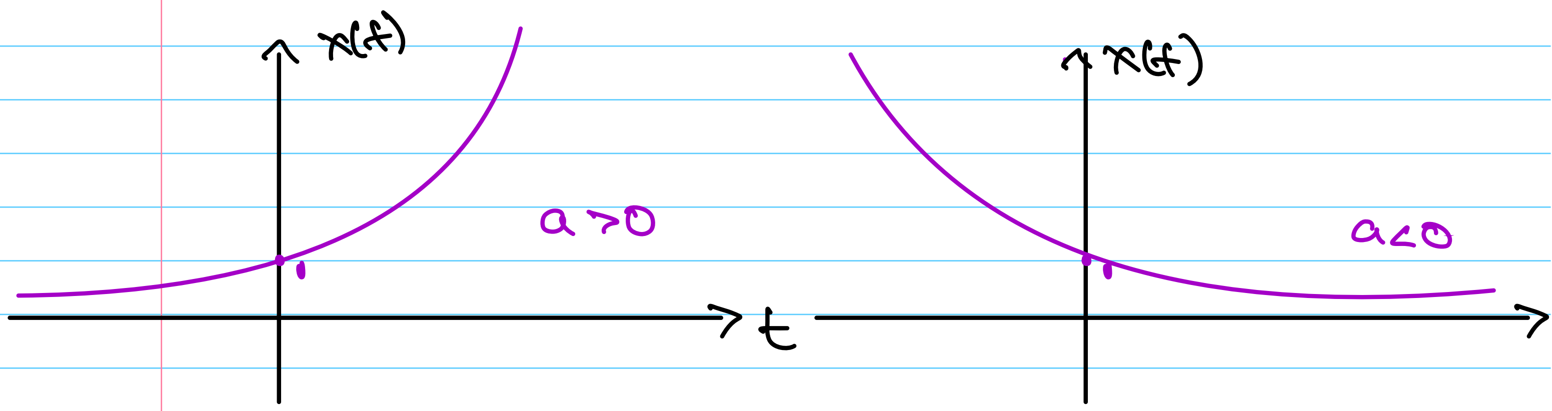
$$x_2[n] = A\cos[\Omega_2 n + \phi]$$

if $\Omega_2 = \Omega_1 + 2\pi m \Rightarrow x_2[n] = x_1[n]$

Class of Real & Complex exponential

(i) $x(t) = Ce^{at}$ (Real exponential)

C and a are real



Time shift $\Longleftrightarrow$ Scale change

$$Ce^{a(t+t_0)} = Ce^{at_0}\, e^{at}$$

Real exponential : Discrete-Time

$$x[n] = Ce^{\beta n} = C\alpha^n$$

(geometric series of progression)

What happen's if α is $-ve$?

$\Rightarrow$ if $\alpha > 0$, for $|\alpha|<1$, $|\alpha|>1$

there exis β such that $e^{\beta} = \alpha$

$\Rightarrow$ if $\alpha < 0$ $\Rightarrow$ there is no Real β such that $e^{\beta} = \alpha$

$\Rightarrow$ The only reason why in discrete - Time case its often most convienent to phrase real - expo $C\alpha^n$ rather than $Ce^{\beta n}$

Complex exponential

$$x(t) = Ce^{at}$$

C and a be complex number's

$$C = |C| e^{j\theta}$$

$$a = r + j\omega_0$$

$$x(t) = |c| e^{j\theta} e^{(r + j\omega_0)t}$$

$$= |c| e^{rt} \cdot e^{j(\omega_0 t + \theta)}$$

$$\Rightarrow x(t) = |c| e^{rt} \left(\cos(\omega_0 t + \theta) + j \sin(\omega_0 t + \theta) \right)$$

$$\Rightarrow \quad x(t) = |C|e^{rt}\cos(\omega_0 t+\theta) + j\,|C|e^{rt}\sin(\omega_0 t+\theta)$$

$\Rightarrow$ two sinusoidal signal's, 90° out of phase, (real part & imaginary part) and time varying amplitude factor.

Complex exponential : Discrete time

$$x[n] = C\alpha^n$$

C and α are complex number's

$$C = |C|\,e^{j\theta}$$

$$\alpha = |\alpha|\,e^{j\Omega_0}$$

$$x[n] = |C|e^{j\theta}\,(|\alpha|\,e^{j\Omega_0})^n$$

$$= |C||\alpha|^n\,e^{j(\Omega_0 n+\theta)}$$

$$x[n] = |C|\,|\alpha|^n\cos(\Omega_0 n+\theta) + j|C|\,|\alpha|^n\sin(\Omega_0 n+\theta)$$