Chapter 2 Problem's

(2.1) Let $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$

$h[n] = 2\delta[n+1] + 2\delta[n-1]$

Compute & Plot each of the following Convolution's

(a) $y_1[n] = x[n] * h[n]$

$$x[n] = x[0]\,\delta[n] + x[1]\,\delta[n-1] + x[3]\,\delta[n-3]$$

$x[0] = 1$

$x[1] = 2$

$x[3] = -1$

$$y[n] = x[0]\,h[n-0] + x[1]\,h[n-1] + x[3]\,h[n-3]$$

$$y[n] = 2\delta[n+1] + 2\delta[n-1] + 4\delta[n] + 4\delta[n-2] - 2\delta[n-2] - 2\delta[n-4]$$

$$\Rightarrow y[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-2] - 2\delta[n-4]$$

(b) $y_2[n] = x[n-2] * h[n]$

$$x[n+2] = \delta[n+2] + 2\delta[n+1] - \delta[n-1]$$

$$h[n] = 2\delta[n+1] + 2\delta[n-1]$$

$$x[n+2] * h[n] = 1 \cdot \left(2\delta[n+3] + 2\delta[n+1]\right) + 2\left[2\delta[n+2] + 2\delta[n]\right) - \left(2\delta(n) + 2\delta(n-2)\right)$$

$$= 2\delta[n+3] + 4\delta[n+2] + 2\delta[n+1]$$

$$+ 2\delta[n] - 2\delta[n-2]$$

2.2

$$h[n] = \left(\frac{1}{2}\right)^{n-1} \{u[n+3] - u[n-10]\}$$

$$h[n] = \left(\frac{1}{2}\right)^{n-1} \{u[n+3] - u[n-10]\}$$

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^{n-1}, & -3 \le n \le 9 \\ 0 & \text{o.w} \end{cases}$$

$$h[n-k] = \left(\frac{1}{2}\right)^{n-k-1} \{u[n+3-k] - u[n-k-10]\}$$

$$= \begin{cases} \left(\frac{1}{2}\right)^{n-k-1} & -3+k \le n \le 9+k \\ 0 & \text{o.w} \end{cases}$$

$$= \begin{cases} \left(\frac{1}{2}\right)^{n-k \cdot 1} & n-9 \leq k \leq n+3 \\ 0 & \text{o.w} \end{cases}$$

2.3

$$x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2]$$

$$h[n] = u[n+2]$$

$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n-2} u[k-2]\, u[n+2-k]$$

$u[k-2] = 0 \quad \forall k < 2 \quad \Rightarrow \quad u[k-2] = 1 \quad \forall k \geq 2$

$u[n+2-k]$

$= u[-(k-(n+2))] \quad \Rightarrow \quad u[n+2-k] = 1$

$\forall k \leq n+2$

$= 0 \quad \forall k > n+2$

$$y[n] = \sum_{k=2}^{n+2} \left(\frac{1}{2}\right)^{n-2}$$

$\vdots$ (need to do again)

2.4 $\quad y[n] = x[n] * h[n]$

$$x[n] = \begin{cases} 1 & 3 \le n \le 8 \\ 0 & \text{o.w} \end{cases}$$

$$h[n] = \begin{cases} 1 & 4 \le n \le 15 \\ 0 & \text{o.w} \end{cases}$$

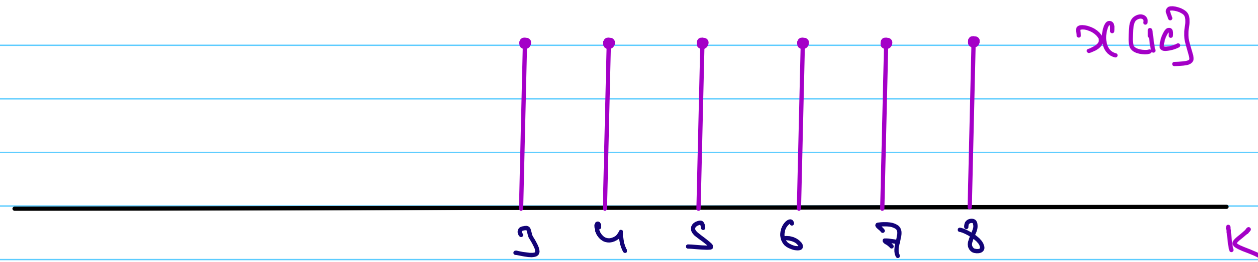
<u>sol^n</u>

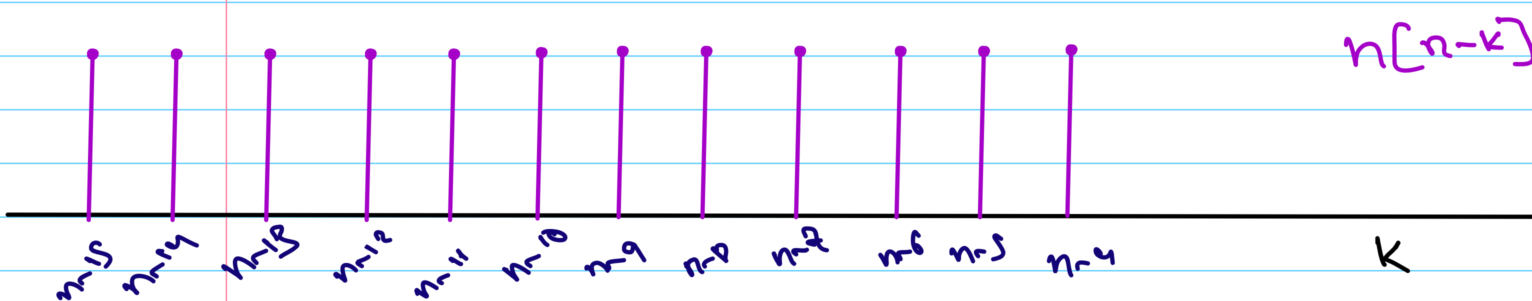
$$x[n] = u[n-3] - u[n-9]$$

$$h[n] = u[n-4] - u[n-16]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} \left(u[k-3] - u[k-9]\right)\left(u[n-k-4] - u[n-k-16]\right)$$





$n=7 \Rightarrow 1$

$n=8 \Rightarrow 2$

$n-6$

$$\Rightarrow \quad y[n] = \begin{cases} 0 & n \leq 6 \\ n-6 & 7 \leq n \leq 11 \\ 6 & 12 \leq n \leq 18 \\ 24-n & 19 \leq n \leq 23 \\ 0 & \text{o.w.} \end{cases}$$

2.5

$$x[n] = \begin{cases} 1 & 0 \le n \le 9 \\ 0 & \text{o.w} \end{cases}$$

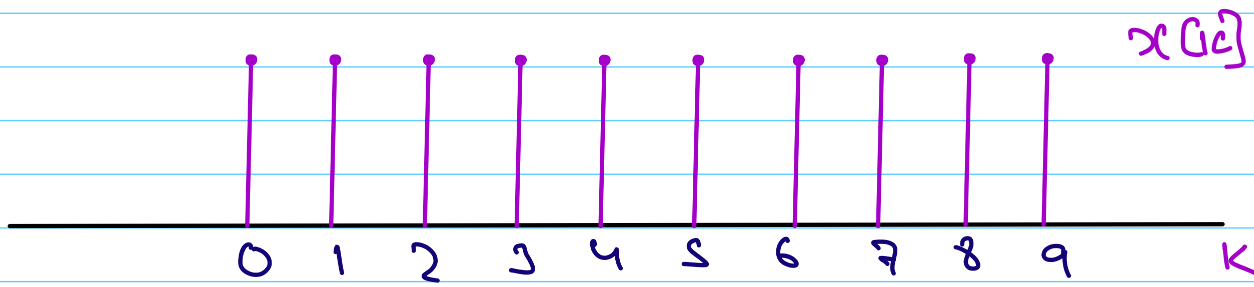
$$h[n] = \begin{cases} 1 & 0 \le n \le N \\ 0 & 0 \end{cases}$$

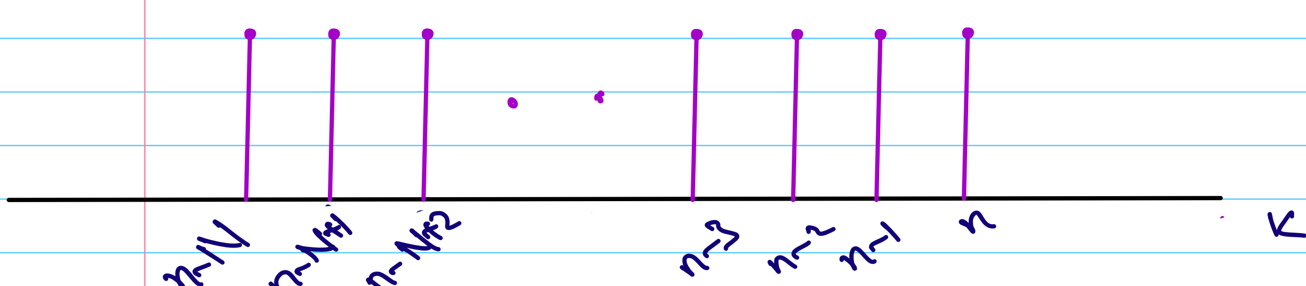
Sol^n

$$x[n] = \sum_{k=0}^{9} \delta[n-k]$$

$$h[n] = \sum_{m=0}^{N} \delta[n-m] \quad (n\text{-axis})$$

$$h[-(k-n)] = \sum_{k=n-N}^{n} \delta[n-k]$$





we know that $y[4] = 5 \Rightarrow 4 - N \leq 0$

$\Rightarrow N \geq 4$

$y[14] = 0 \Rightarrow 14 - N \geq 10$

$\Rightarrow N \leq 4$

$\Rightarrow N = 4$

2.6 $y[n] = x[n] * h[n]$

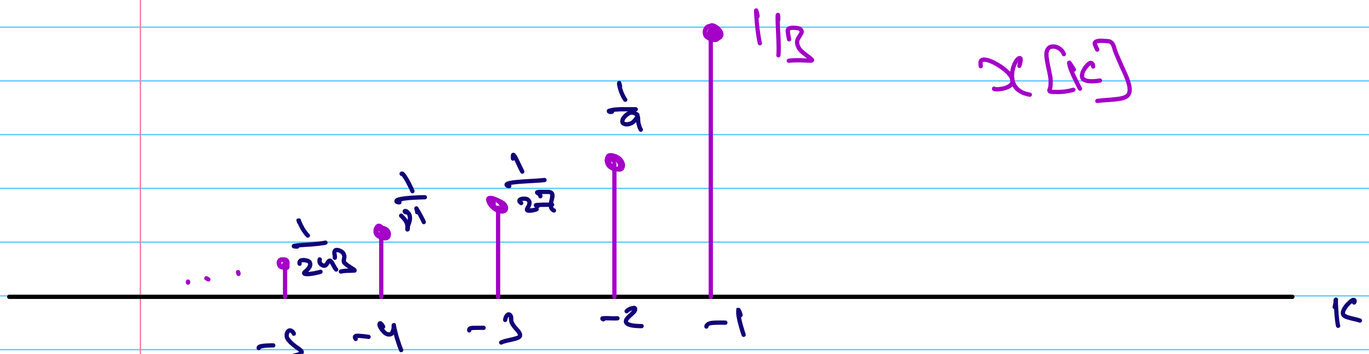
where $x[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1]$

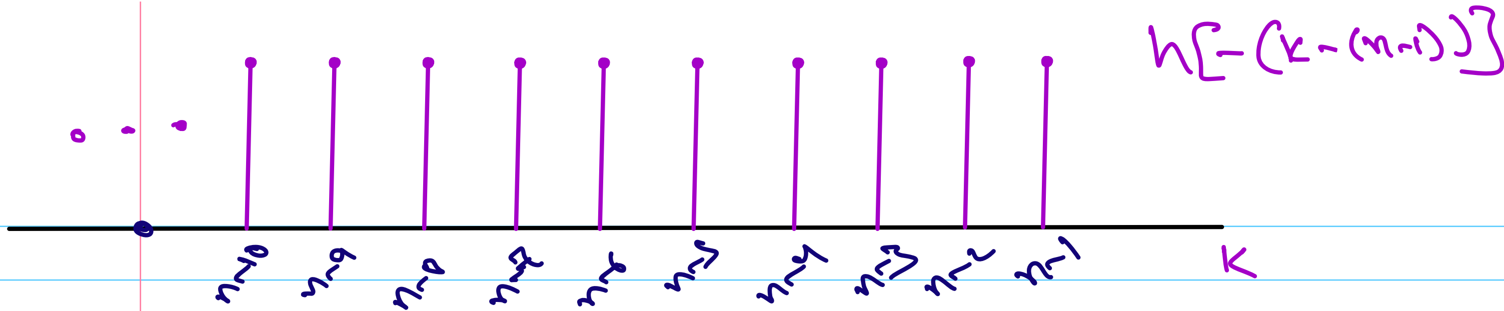
$h[n] = u[n-1]$

Sol

$x[k] \quad \left(\frac{1}{3}\right)^{-k} u[-(k+1)]$

$h[n-k] = u[n-1-k] = u[-(k-(n-1))]$





for $n \geq 0$ $\quad y[n] = \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^k$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k - 1$$

$$= \frac{1}{1-\frac{1}{3}} - 1 = \frac{3}{2} - 1 = \frac{1}{2}$$

for $n < 0$

$$y(n) = \sum_{-\infty}^{n} \left(\frac{1}{3}\right)^{-(n-1)}$$

2.7

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, g[n-2k]$$

between its input $x[n]$ and its output $y(n)$, where $g[n] = u[n] - u[n-4]$

(a) Determine $y[n]$ when $x[n] = \delta[n-1]$

soln

$$g[n] = u[n] - u[n-4]$$

$$g[n-2k] = u[n-2k] - u[n-2k-4]$$

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k-1]\, g[n-2k]$$

$$= g[n-2]$$

$$= u[n-2] - u[n-6]$$

(b)

Determine $y[n]$ when $x[n] = \delta[n-2]$

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k-2]\, g[n-2k]$$

$$= g[n-4] = u[n-4] - u[n-8]$$

(c) is S LTI?

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, g[n-2k]$$

NO

(d)

Determine $y[n]$ when $x[n] = u[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} u[k]\, g[n-2k]$$

$$= \sum_{k=0}^{\infty} g[n-2k]$$

$$= \sum_{k=0}^{\infty} u[n-2k] - u[n-2k-4]$$

2.8

$$x(t) = \begin{cases} t+1 & 0 \le t \le 1 \\ 2-t & 1 < t \le 2 \\ 0 & \text{elsewher.} \end{cases}$$

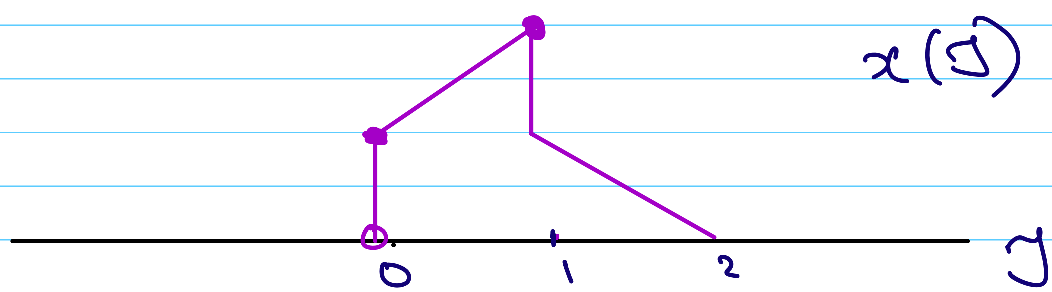
$$h(t) = \delta(t+2) + 2\delta(t+1)$$

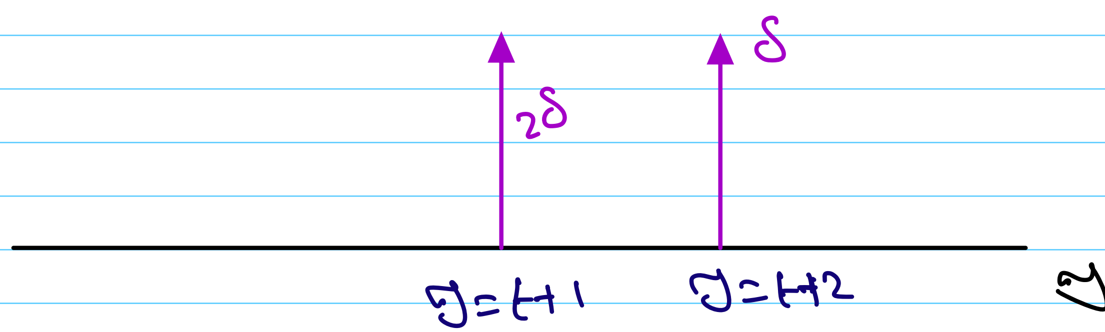
$$h(t-\tau) = \delta(t+2-\tau) + 2\delta(t+1-\tau)$$

$$= \delta\Big(-\big(\tau-(t+2)\big)\Big) + 2\delta\Big(-\big(\tau-(t+1)\big)\Big)$$

= Impulse function at

$\tau = t+2$ & $\tau = t+1$





$$y(t) = \begin{cases} 0 & t < -2 \\ t+3 & -2 \le t \le -1 \\ 2-2t & -1 \le t \le 0 \\ 0 & \text{o.w} \end{cases}$$

(2.9) $h(t) = e^{2t}\, u(-t+4) + e^{-2t}\, u(t-5)$

Determine A and B s.t

$$h(t-\tau) = \begin{cases} e^{-2(t-\tau)} & \tau < A \\ 0 & A < \tau < B \\ e^{2(t-\tau)} & B < \tau \end{cases}$$

Sol^n

$$h(t) = e^{2t}\, u(-t+4) + e^{-2t}\, u(t-5)$$

$$h(t-\tau) = e^{2t-2\tau}\, u(-t+\tau+4)$$

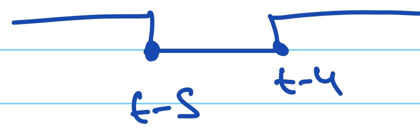
$$+ e^{-2t+2\tau}\, u(t-\tau-5)$$

$$h(t-\tau) = e^{2t} \cdot e^{-2\tau}\, u(\tau-(t-4))$$

$$+ e^{-2t} \cdot e^{2\tau}\, u(-(\tau-(t-5)))$$

$$= \underbrace{e^{2t}\cdot e^{-2y}\, u(y-(t-4))}_{=0 \ \forall\, y<t-4} + \underbrace{e^{-2t}\cdot e^{2y}\, u(-(y-(t-5)))}_{=0 \ \forall\, y>t-5}\Big]$$

$=$



$$\Rightarrow \quad h(t-y) = \begin{cases} e^{-2(t-y)} & y < t-5 \\ 0 & t-5 \leq y \leq t-4 \\ e^{2(t-y)} & t-4 < y \end{cases}$$

$A = t-5 \qquad B = t-4$

2.10

$$x(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{o.w} \end{cases}$$

and $h(t) = x(t/\alpha) \qquad 0 < \alpha \leq 1$

(a) Determine and sketch $y(t) = x(t) * h(t)$

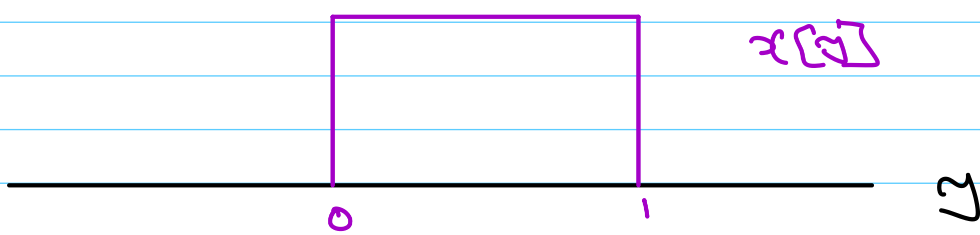
Sol^n

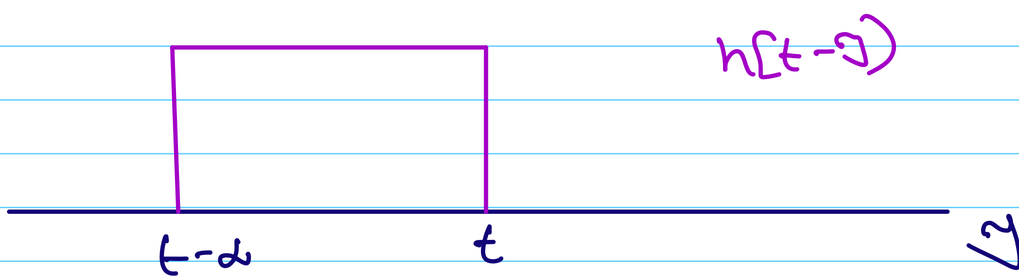
$$x(t) = u(t) - u(t-1)$$

$$h(t) = u(t/\alpha) - u\left(\frac{t-1}{\alpha}\right)$$

$$h(t) = \begin{cases} 1 & 0 \le t < \alpha \le 1 \\ 0 & \text{o.w} \end{cases}$$

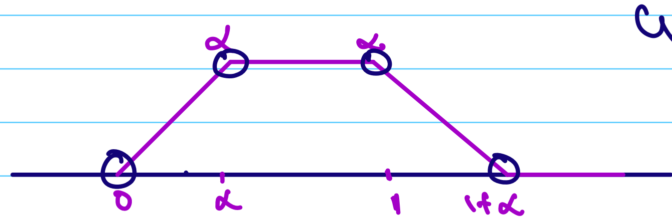
$$h(t-\tau) = h(-(\tau-t)) = \begin{cases} 1 & t-\alpha < \tau \le t \\ 0 & \text{o.w} \end{cases}$$





$$y(t) = \begin{cases} t & 0 < t \le \alpha \\ \alpha & \alpha < t \le 1 \\ 1+\alpha-t & 1 < t < 1+\alpha \\ 0 & \text{o.w} \end{cases}$$

(b)



4 discontinuous of $\frac{dy}{dt}$

if $\alpha = 1 \Rightarrow$ 3 discontinuous

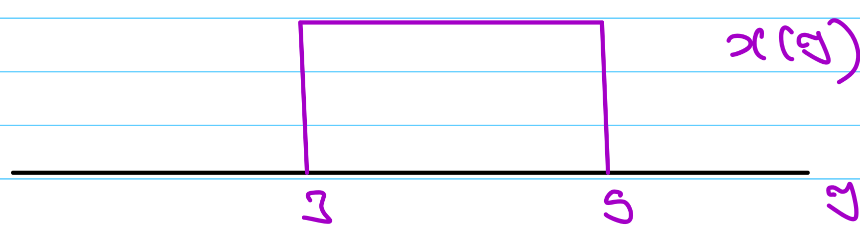
(2.11) $x(t) = u(t-3) - u(t-5)$

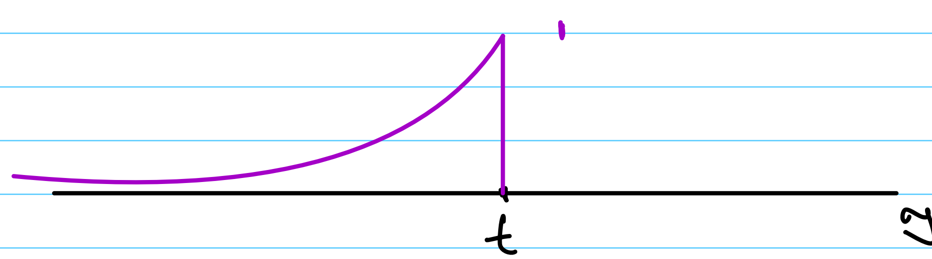
$$h(t) = e^{-3t} u(t)$$

(a) Compute $y(t) = x(t) * h(t)$

$$h(t-\tau) = e^{-3(t-\tau)} u(t-\tau)$$

$$= e^{3(\tau-t)} u(-(\tau-t))$$





$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{3}^{t} \left[u(\tau-3) - u(\tau-5)\right] e^{-3(t-\tau)}\, u(t-\tau)\, d\tau$$

$$= \int_{3}^{t} e^{-3(t-\tau)}\, d\tau = e^{-3t}\left[\frac{e^{3\tau}}{3}\right]_3^t$$

$$= e^{-3t}\left[\frac{e^{3t} - e^{9}}{3}\right]$$

$$= \frac{1 - e^{-3(t-3)}}{3}$$

$3 \le t \le 5$

for $t > 5$