3 Signals and Systems: Part II

Recommended Problems

Solution's are at the end

P3.1

Sketch each of the following signals.

(a) $x[n] = \delta[n] + \delta[n-3]$

(b) $x[n] = u[n] - u[n-5]$

(c) $x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + (\frac{1}{2})^2\delta[n-2] + (\frac{1}{2})^3\delta[n-3]$

(d) $x(t) = u(t+3) - u(t-3)$

(e) $x(t) = \delta(t+2)$

(f) $x(t) = e^{-t}u(t)$

P3.2

Below are two columns of signals expressed analytically. For each signal in column A, find the signal or signals in column B that are identical.

A	B
(1) $\delta[n+1]$	**(a)** $\sum_{k=-\infty}^{n} \delta[k]$
(2) $(\frac{1}{2})^n u[n]$	**(b)** $\frac{du(t)}{dt}$
(3) $\delta(t)$	**(c)** $\sum_{k=0}^{n} \delta[k]$
(4) $u(t)$	**(d)** $\sum_{k=0}^{\infty} (\frac{1}{2})^k \delta[n-k]$
(5) $u[n]$	**(e)** $\int_{-\infty}^{t} \delta(\tau)\, d\tau$
(6) $\delta[n+1]u[n]$	**(f)** $u[n]$
	(g) $\sum_{k=-\infty}^{\infty} (\frac{1}{2})^k \delta[n-k]$
	(h) $\delta[n+1]$
	(i) ϕ

P3.3

(a) Express the following as sums of weighted delayed impulses, i.e., in the form

$$x[n] = \sum_{k=-\infty}^{\infty} a_k \delta[n-k]$$

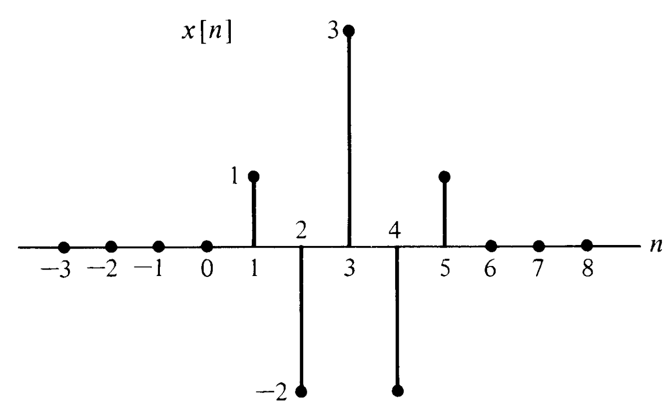


Figure P3.3-1

(b) Express the following sequence as a sum of step functions, i.e., in the form

$$s[n] = \sum_{k=-\infty}^{\infty} a_k u[n-k]$$

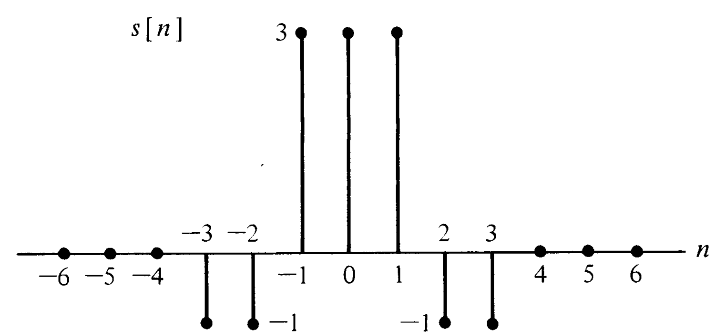


Figure P3.3-2

P3.4

For $x(t)$ indicated in Figure P3.4, sketch the following:

(a) $x(1-t)[u(t+1)-u(t-2)]$

(b) $x(1-t)[u(t+1)-u(2-3t)]$

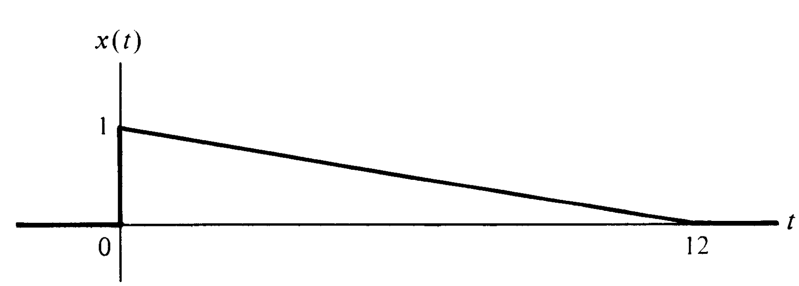


Figure P3.4

P3.5

Consider the three systems H, G, and F defined in Figure P3.5-1.

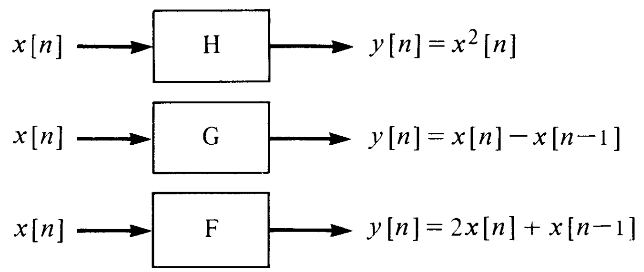


Figure P3.5-1

The systems in Figures P3.5-2 to P3.5-7 are formed by parallel and cascade combination of H, G, and F. By expressing the output $y[n]$ in terms of the input $x[n]$, determine which of the systems are equivalent.

(a)

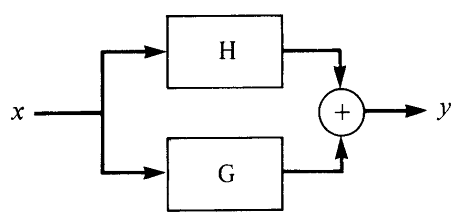


Figure P3.5-2

(b)

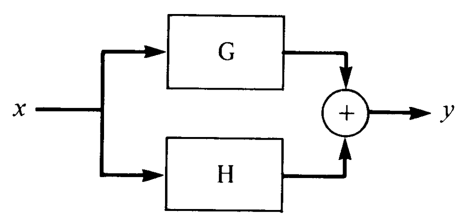


Figure P3.5-3

(c)

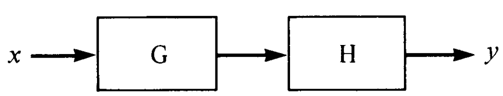


Figure P3.5-4

(d)

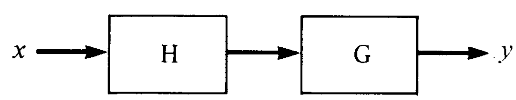


Figure P3.5-5

(e)

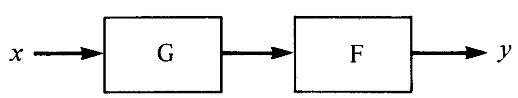


Figure P3.5-6

(f)

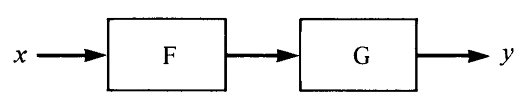


Figure P3.5-7

P3.6

Table P3.6 contains the input-output relations for several continuous-time and discrete-time systems, where $x(t)$ or $x[n]$ is the input. Indicate whether the property along the top row applies to each system by answering yes or no in the appropriate boxes. Do not mark the shaded boxes.

	Properties					
$y(t)$, $y[n]$	**Memoryless**	**Linear**	**Time-Invariant**	**Causal**	**Invertible**	**Stable**
(a) $(2 + \sin t)x(t)$						
(b) $x(2t)$						
(c) $\sum_{k=-\infty}^{\infty} x[k]$						
(d) $\sum_{k=-\infty}^{n} x[k]$						
(e) $\dfrac{dx(t)}{dt}$						
(f) $\max\{x[n], x[n-1], \ldots, x[-\infty]\}$						

Table P3.6

P3.7

Consider the following systems

$$\text{H:} \qquad y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \qquad \text{(an integrator)},$$

$$\text{G:} \qquad y(t) = x(2t),$$

where the input is $x(t)$ and the output is $y(t)$.

(a) What is H^{-1}, the inverse of H? What is G^{-1}?

(b) Consider the system in Figure P3.7. Find the inverse F^{-1} and draw it in block diagram form in terms of H^{-1} and G^{-1}.

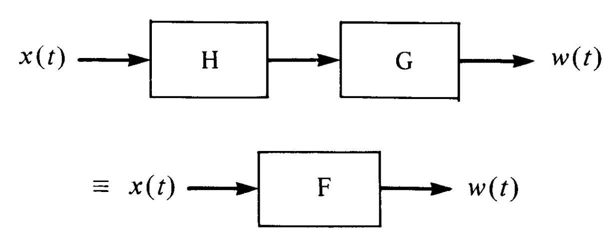


Figure P3.7

Optional Problems

P3.8

In this problem we illustrate one of the most important consequences of the properties of linearity and time invariance. Specifically, once we know the response of a linear system or of a linear, time-invariant (LTI) system to a single input or the responses to several inputs, we can directly compute the responses to many other input signals.

(a) Consider an LTI system whose response to the signal $x_1(t)$ in Figure P3.8-1(a) is the signal $y_1(t)$ illustrated in Figure P8-1(b). Determine and sketch carefully the response of the system to the input $x_2(t)$ depicted in Figure P3.8-1(c).

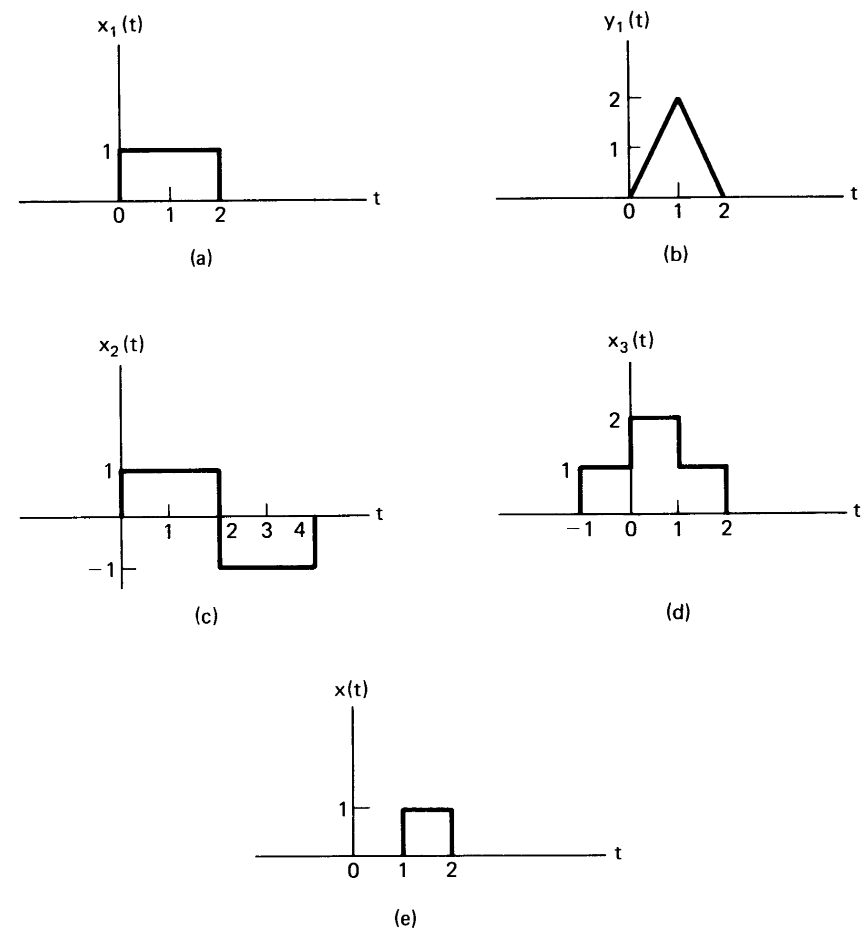


Figure P3.8-1

(b) Determine and sketch the response of the system considered in part (a) to the input $x_3(t)$ shown in Figure P3.8-1(d).

(c) Suppose that a second LTI system has the following output $y(t)$ when the input is the unit step $x(t) = u(t)$:

$$y(t) = e^{-t}u(t) + u(-1 - t)$$

Determine and sketch the response of this system to the input $x(t)$ shown in Figure P3.8-1(e).

(d) Suppose that a particular discrete-time linear (but possibly not time-invariant) system has the responses $y_1[n]$, $y_2[n]$, and $y_3[n]$ to the input signals $x_1[n]$, $x_2[n]$, and $x_3[n]$, respectively, as illustrated in Figure P3.8-2(a). If the input to this system is $x[n]$ as illustrated in Figure P3.8-2(b), what is the output $y[n]$?

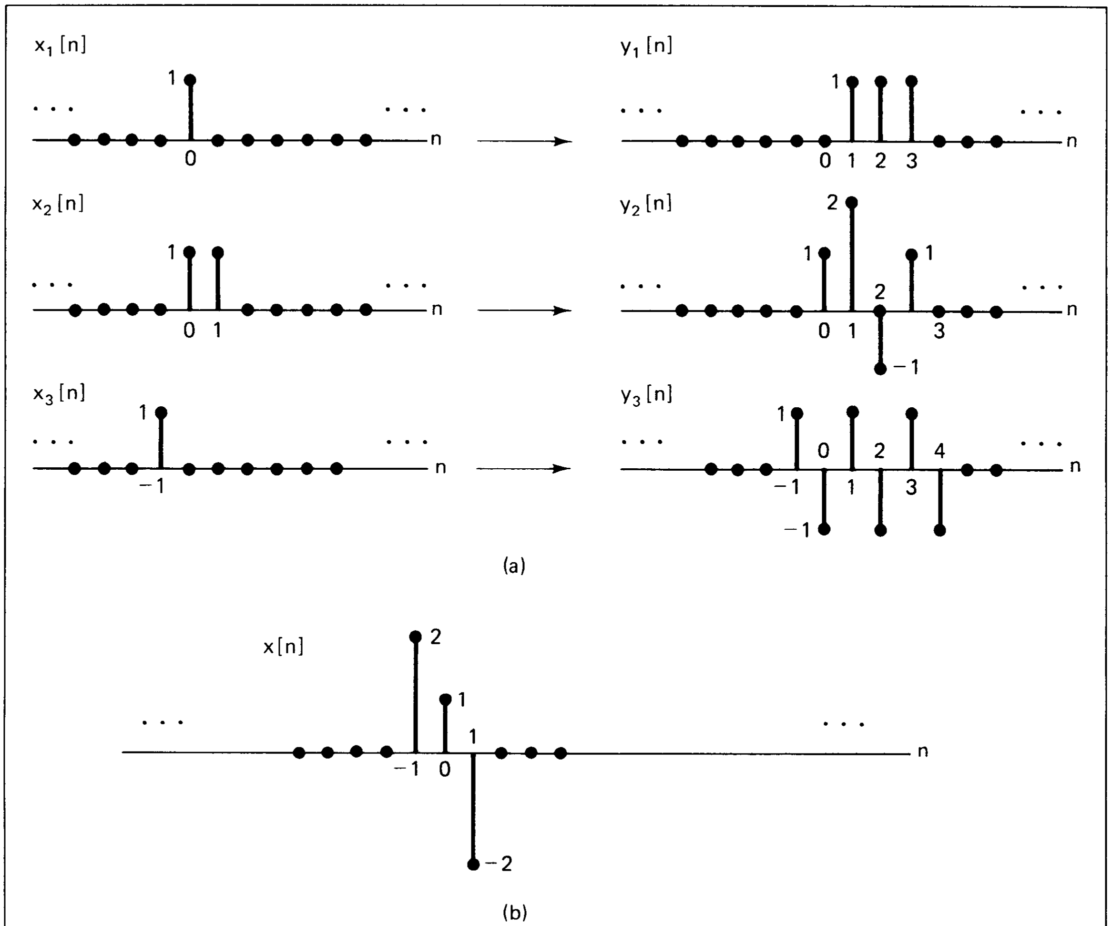


Figure P3.8-2

(e) If an LTI system has the response $y_1[n]$ to the input $x_1[n]$ as in Figure P3.8-2(a), what would its responses be to $x_2[n]$ and $x_3[n]$?

(f) A particular linear system has the property that the response to t^k is $\cos kt$. What is the response of this system to the inputs

$$x_1(t) = \pi + 6t^2 - 47t^5 + \sqrt{e}t^6$$

$$x_2(t) = \frac{1 + t^{10}}{1 + t^2}$$

P3.9

(a) Consider a system with input $x(t)$ and with output $y(t)$ given by

$$y(t) = \sum_{n=-\infty}^{+\infty} x(t)\delta(t - nT)$$

(i) Is this system linear?

(ii) Is this system time-invariant?

For each part, if your answer is yes, show why this is so. If your answer is no, produce a counterexample.

(b) Suppose that the input to this system is $x(t) = \cos 2\pi t$. Sketch and label carefully the output $y(t)$ for each of the following values of T: $T = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{12}$. All of your sketches should have the same horizontal and vertical scales.

(c) Repeat part (b) for $x(t) = e^t \cos 2\pi t$.

P3.10

(a) Is the following statement true or false?

The series interconnection of two linear, time-invariant systems is itself a linear, time-invariant system.

Justify your answer.

(b) Is the following statement true or false?

The series connection of two nonlinear systems is itself nonlinear.

Justify your answer.

(c) Consider three systems with the following input-output relations:

System 1: $y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

System 2: $y[n] = x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2]$

System 3: $y[n] = x[2n]$

Suppose that these systems are connected in series as depicted in Figure P3.10. Find the input-output relation for the overall interconnected system. Is this system linear? Is it time-invariant?

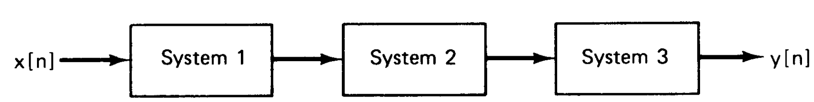


Figure P3.10

(d) Consider a second series interconnection of the form of Figure P3.10 where the three systems are specified by the following equations, with a, b, and c real numbers:

System 1: $y[n] = x[-n]$

System 2: $y[n] = ax[n-1] + bx[n] + cx[n+1]$

System 3: $y[n] = x[-n]$

Find the input-output relation for the overall interconnected system. Under what conditions on the numbers a, b, and c does the overall system have each of the following properties?

(i) The overall system is linear and time-invariant.

(ii) The input-output relation of the overall system is identical to that of system 2.

(iii) The overall system is causal.

P3.11

Determine whether each of the following systems is linear and/or time-invariant. In each case, $x[n]$ denotes the input and $y[n]$ denotes the output. Assume that $x[0] > 0$.

(a) $y[n] = x[n] + x[n - 1]$

(b) $y[n] = x[n] + x[n - 1] + x[0]$

P3.12

(a) Show that causality for a continuous-time linear system implies the following statement:

For any time t_0 and any input $x(t)$ such that $x(t) = 0$ for $t < t_0$, the corresponding output $y(t)$ must also be zero for $t < t_0$.

The analogous statement can be made for discrete-time linear systems.

(b) Find a nonlinear system that satisfies this condition but is not causal.

(c) Find a nonlinear system that is causal but does not satisfy this condition.

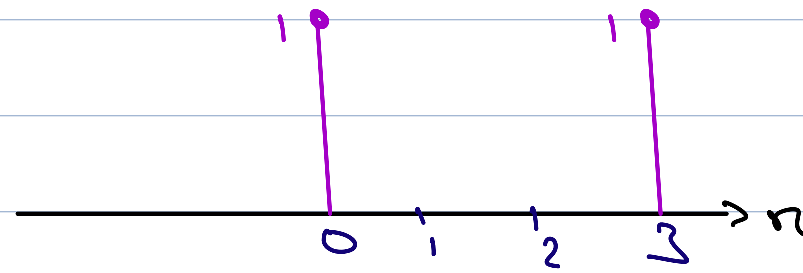
(d) Show that invertibility for a discrete-time linear system is equivalent to the following statement:

The only input that produces the output $y[n] = 0$ for all n is $x[n] = 0$ for all n.

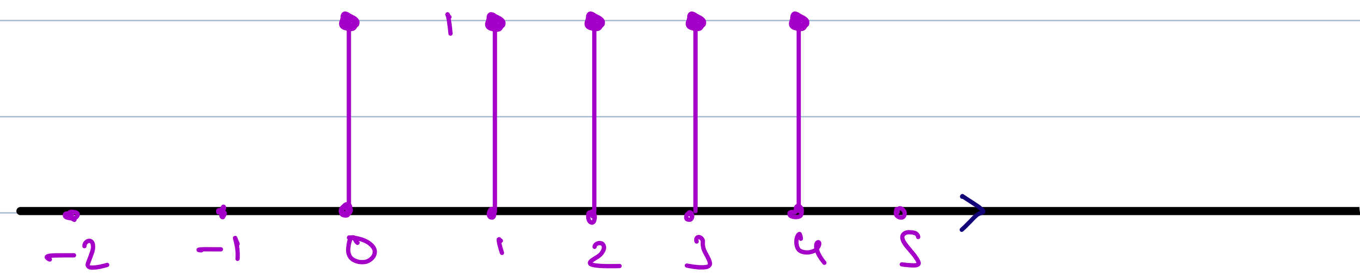
The analogous statement is also true for continuous-time linear systems.

(e) Find a nonlinear system that satisfies the condition of part (d) but is not invertible.

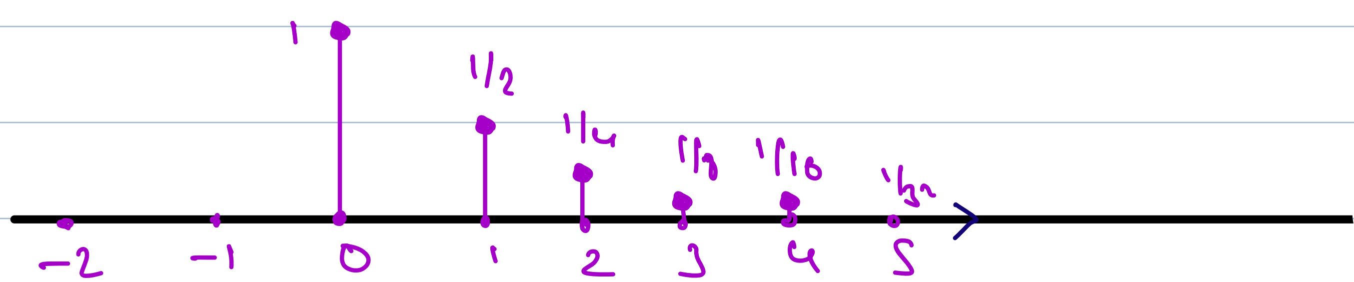
P3.1 (a) $x[n] = \delta[n] + \delta[n-3]$



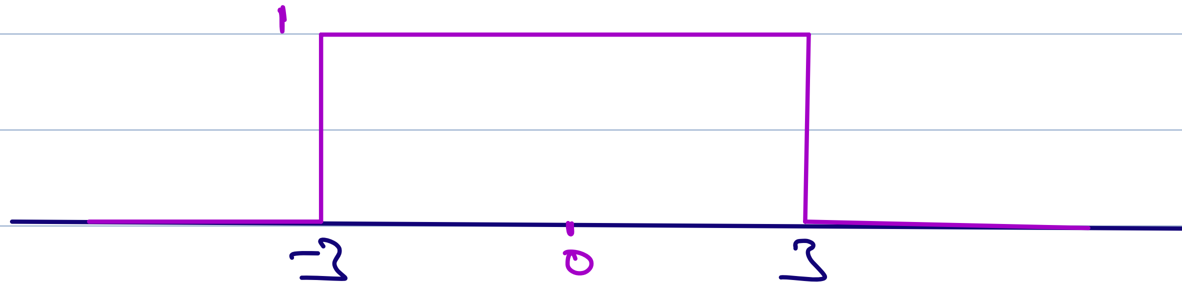
(b) $x[n] = u[n] - u[n-5]$



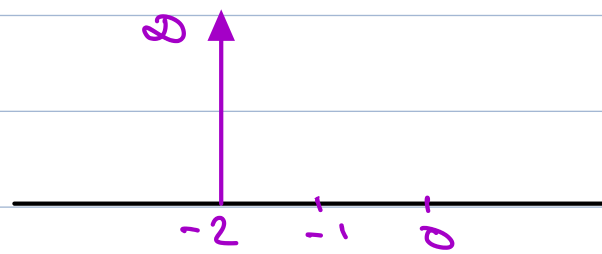
(c) $x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + \left(\frac{1}{2}\right)^2 \delta[n-2] + \left(\frac{1}{2}\right)^3 \delta[n-3]$



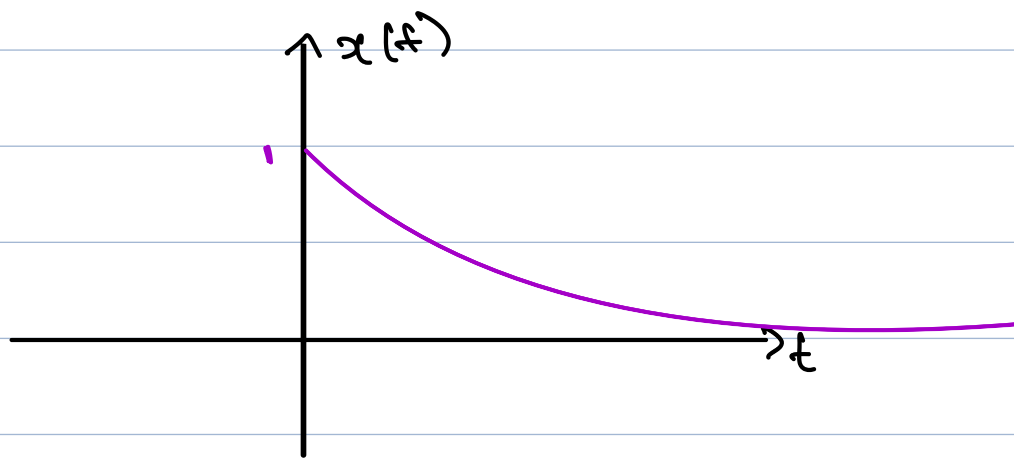
(d) $x(t) = u(t+3) - u(t-3)$



(e) $x(t) = \delta(t+2)$



(f) $x(t) = e^{-t}\, u(t)$



P3.2

(1) $\delta[n+1] = \delta(n+1)$ (h)

(2) $\left(\frac{1}{2}\right)^n u[n] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \delta(n-k)$ (d)

③ $\delta(t) = \frac{du(t)}{dt}$ ⓑ

④ $u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau = 1 \quad \forall t > 0$ ⓔ

⑤ $u[n] = u[n]$ ⓕ

⑥ $\delta[n+1]\,u[n] = 0$

P3.3

ⓐ $x[1]=1 \quad x[2]=-2 \quad x[3]=3$

$x[4]=-2$

$x(5)=1$

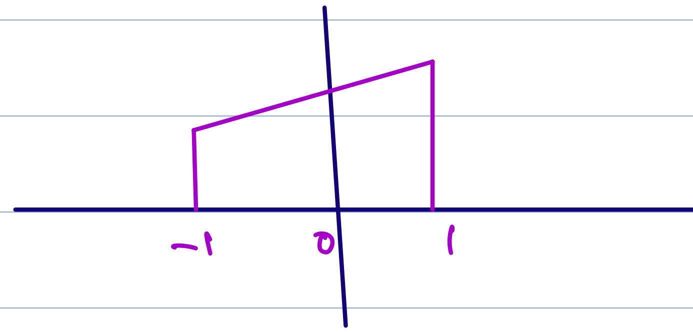
$$x[n] = 1\cdot\delta[n-1] - 2\,\delta[n-2] + 3\,\delta[n-3] - 2\,\delta[n-4] + \delta[n-5]$$

ⓑ

$$s[n] = -1\,\delta[n+3] - 1\,\delta[n+2] + 3\,\delta[n+1] + 3\,\delta[n] + 3\,\delta[n-1] - 1\,\delta[n-2] - 1\,\delta[n-3]$$

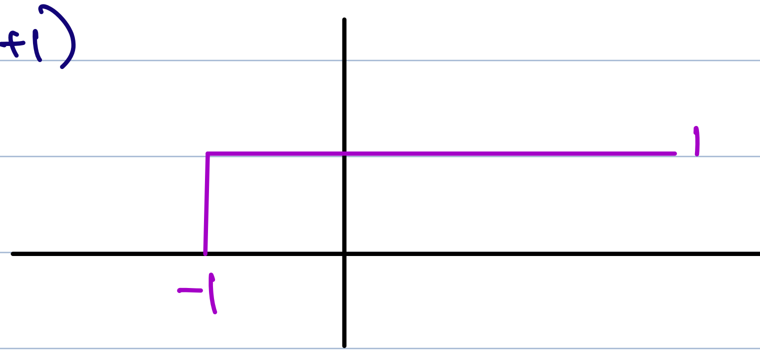
P3.4

(a) $x(1-t)\,[u(t+1) - u(t-2)]$



(b) $x(1-t)\,[u(t+1) - u(2-3t)]$

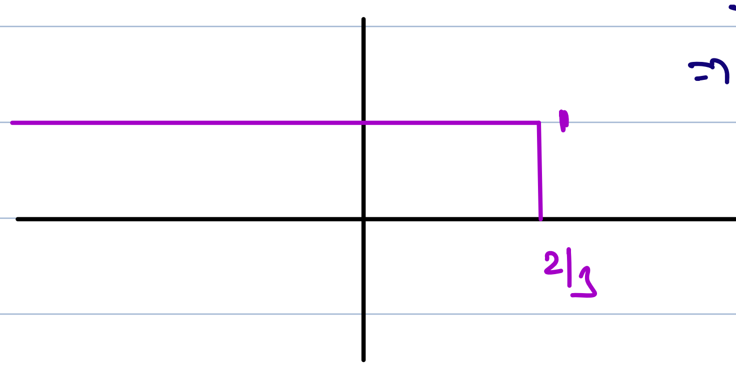
$u(t+1)$



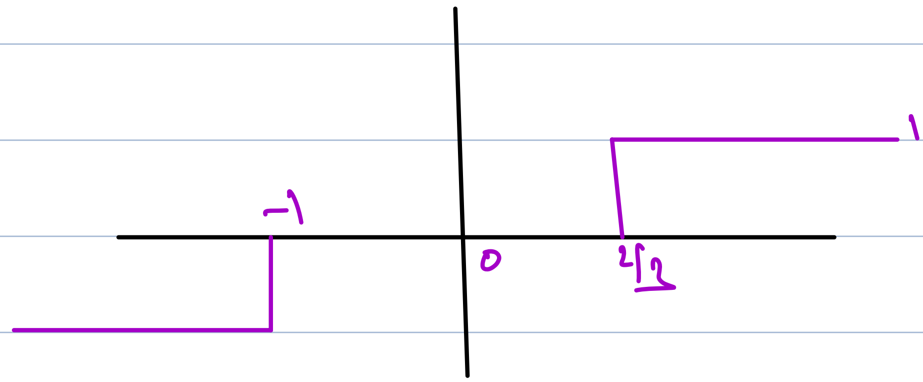
$u(2-3t)$: $2-3t > 0 \Rightarrow -3t > -2$

$3t < 2$

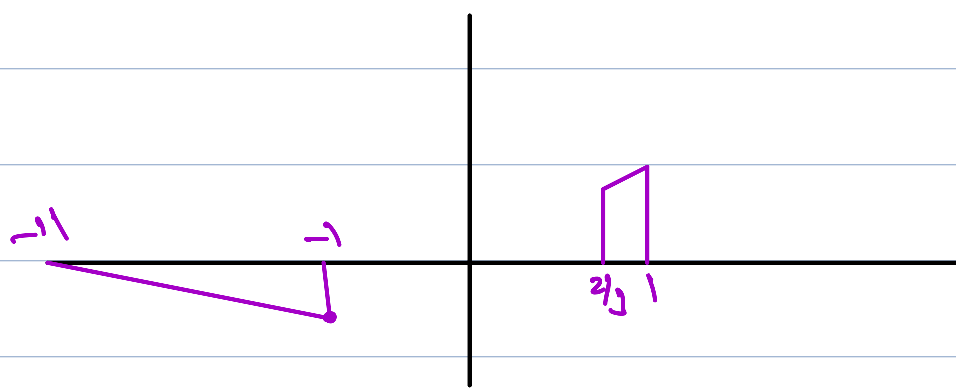
$\Rightarrow t < \frac{2}{3}$



$$u(t+1) - u(2-3t) =$$



$$x(1-t)\left[u(t+1) - u(2-3t)\right]$$



P3.5

(a) $$y[n] = x^2[n] + x[n] - x[n-1]$$

(b) $$y[n] = x^2[n] + x[n] - x[n-1]$$

(c) $$y[n] = \left(x[n] - x[n-1]\right)^2$$

$$= x^2[n] + x^2[n-1] - 2\,x[n]\,x[n-1]$$

(d) $$y[n] = x^2[n] - x^2[n-1]$$

ⓔ $y[n] = 2(x[n] - x[n-1]) + x[n-1] - x[n-2]$

$= 2x[n] - x[n-1] - x[n-2]$

ⓕ $y[n] = 2x[n] + x[n-1] - 2x[n-1] - x[n-2]$

$= 2x[n] - x[n-1] - x[n-2]$

$a = b$, $e = f$

P 3.6

ⓐ $y(t) = (2 + \sin t)\, x(t)$

(i) Memoryless ⟹ yes

(ii) Linear ⟹ yes

(iii) Time-invariant - NO

(iv) stable ⟶ yes

(b) $y(t) = x(2t)$

memoryless $\longrightarrow$ yes

$x_1(t) \xrightarrow{S} x_1(2t) = y_1(t)$

$x_2(t) \xrightarrow{S} x_2(2t) = y_2(t)$

$a x_1(t) + b x_2(t) \Rightarrow a x_1(2t) + b x_2(2t)$

Linear yes

Time-invariant $\longrightarrow$ yes

Causal $\longrightarrow$ yes

invertible $\longrightarrow$ yes $x(t) = y(t/2)$

stable $\longrightarrow$ yes

(c)

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]$$

P3.7

$$H: \; y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

$$G: \; y(t) = x(2t)$$

(a)

$$G: \; y(t) = x(2t)$$

$$G^{-1}: \; x(t) = y(t/2)$$

H^{-1} ?

$$\int_{-\infty}^{t} x(\tau)\, d\tau = y(t)$$

$$\frac{d}{dt} \int_{-\infty}^{t} x(\tau)\, d\tau = \frac{d}{dt} y(t)$$

$$\Rightarrow \quad x(t) = \frac{d}{dt} y(t) \; : \; H^{-1}$$

(b)

$$w(t) = G\left[H[x(t)] \right]$$

$$= G\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right]$$

$$w(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$$

$$\frac{d}{dt} w(t) = \frac{d}{dt}\int_{-\infty}^{2t} x(\tau)\, d\tau$$

$$= x(2t) \cdot \frac{d}{dt} 2t =$$

$$\frac{d}{dt} w(t) = 2x(2t)$$

$$\Rightarrow \quad x(2t) = \frac{1}{2}\frac{d}{dt} w(t)$$

$$\Rightarrow \quad x(t) = \frac{1}{2}\frac{d}{dt} w(t/2)$$