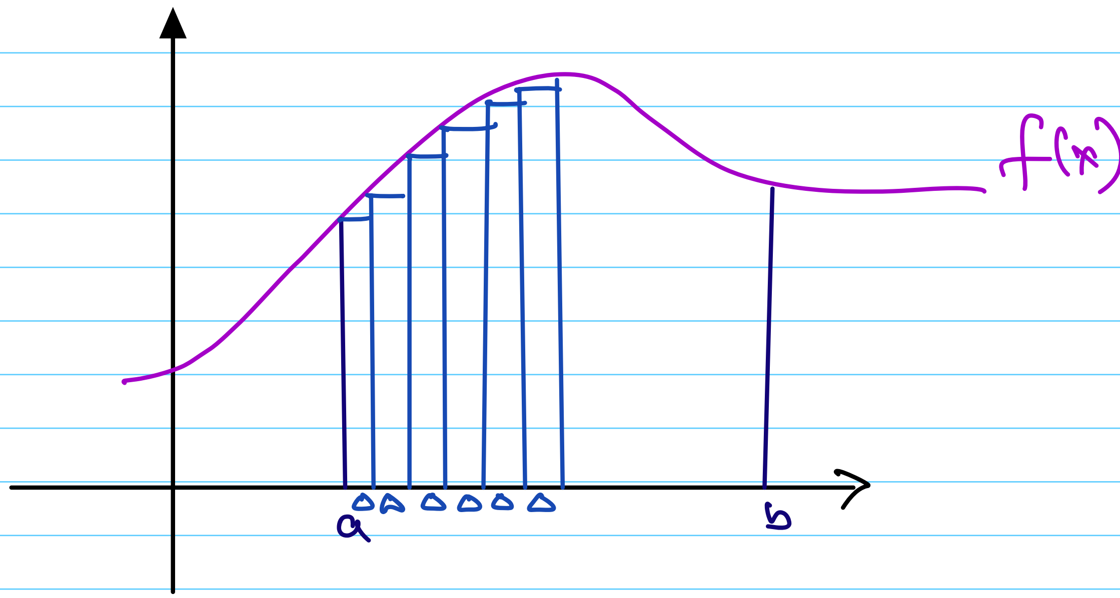
Lec11 :- Singularity Funtion's

Unit step, Impulse, Dirac Delta.

Fundamental theorem of calculus:



$$\int_a^b f(x)\,dx = F(a) - F(b)$$

$$\text{Area} \approx f(a)\Delta + f(a+\Delta)\Delta + \dots + f(b-\Delta)\Delta$$

$$f(x) = \frac{dF(x)}{dx}$$

$$\text{Area} \approx \frac{F(\cancel{a+\Delta}) - F(a)}{\cancel{\Delta}}\,\cancel{\Delta} + \frac{F(a+\cancel{2\Delta}) - F(\cancel{a+\Delta})}{\cancel{\Delta}} \cdot \cancel{\Delta}$$

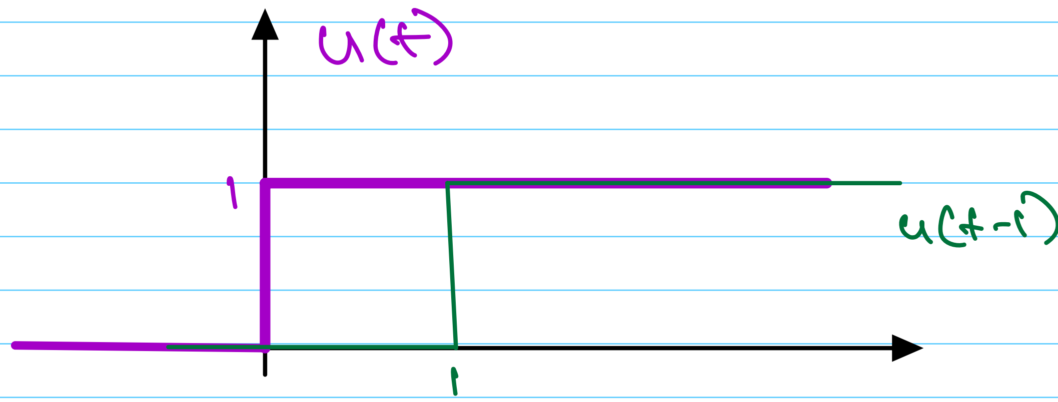
$$+ \cdots + \frac{F(b) - F(\cancel{b-\Delta})}{\cancel{\Delta}} \cdot \cancel{\Delta}$$

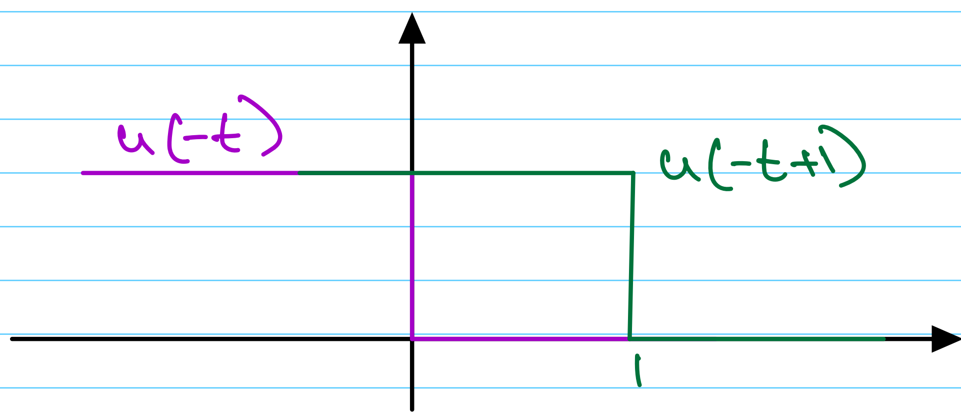
$$\text{Area} \approx \frac{F(b) - F(a)}{\Delta} \cdot \Delta$$

in Limit $\quad \text{Area} = \lim\limits_{\Delta \to 0} \dfrac{F(b) - F(a)}{\Delta} \cdot \Delta$

$$= F(b) - F(a)$$

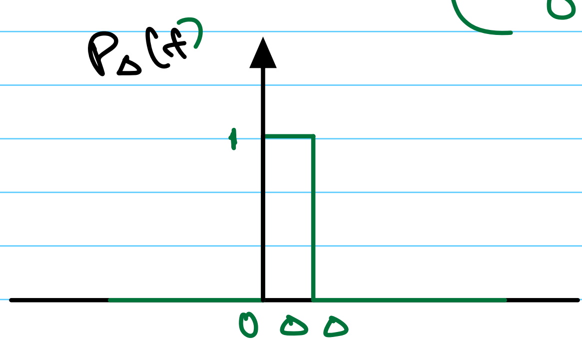
Unit Step Function:





Pulse function:

$$P_\Delta(t) = \begin{cases} 0 & t \leq 0 \\ 1 & 0 \leq t < \Delta \\ 0 & \Delta < t \end{cases}$$



$P_\Delta(t) = u(t) - u(t-\Delta)$

Let's scale Pulse function with $\frac{1}{\Delta}$

$\Rightarrow \quad \frac{1}{\Delta} P_\Delta(t) \quad$ area = 1 always

Dirac delta:

$$\delta(t) = \lim_{\Delta \to 0} \frac{1}{\Delta} P_\Delta(t)$$

$$\delta(t) = \begin{cases} \infty & t = 0 \\ & t \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \quad \text{(area)}$$

$$\Rightarrow \quad 1 = \int_{-\infty}^{+\infty} \delta(t) dt = \int_{0^-}^{0^+} \delta(t)\, dt$$

$$\Rightarrow \quad \int_{-\infty}^{t} \delta(t') dt' = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

$$\Rightarrow \int_{-\infty}^{t} \delta(t')\,dt' = u(t)$$

integral of Dirac Delta is unit step

$$\Rightarrow \frac{d}{dt} u(t) = \delta(t)$$

Similarly $\quad \frac{d}{dt} u(t-a) = \delta(t-a)$

let's P is a differentiator operator.

<u>Operator</u> : function in t $\longrightarrow$ function in t

Operator: $f \longrightarrow g$

Transform : $f(t) \longrightarrow F(f)$

function : $t \longrightarrow f(t)$

$$P : \frac{d}{dt} \Rightarrow P(x) = \frac{dx}{dt}$$

$\frac{1}{p} : \int_0^t \Rightarrow \frac{1}{p}[f(x)] = \int_0^t f(x)\,dx$

Shifting Property:

$$\int_{-\infty}^{+\infty} f(t')\,\delta(t'-t)\,dt'$$

$\Rightarrow$ $\delta(t'-t)$ in t' axis is

0 at all time & ∞ at $t'=t$

$$\Rightarrow \quad \delta(t'-t) = \begin{cases} 0 & t' \neq t \\ \infty & t' = t \end{cases}$$

$$\Rightarrow \int_{-\infty}^{+\infty} f(t')\,\delta(t'-t)\,dt = \int_{t^-}^{t^+} f(t)\,\delta(t'-t)\,dt$$

$$= f(t) \int_{t'}^{t^+} \delta(t'-t)\, dt$$

$$= f(t) \cdot 1$$

$$\Rightarrow \boxed{\int_{-\infty}^{+\infty} f(t')\, \delta(t'-t)\, dt = f(t)}$$

Shifting Property

Basically The impulse function goes and takes the value of that function $f(t')$ at that point t.

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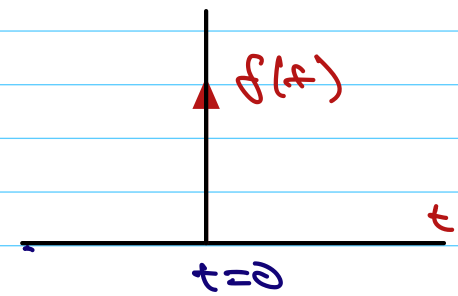
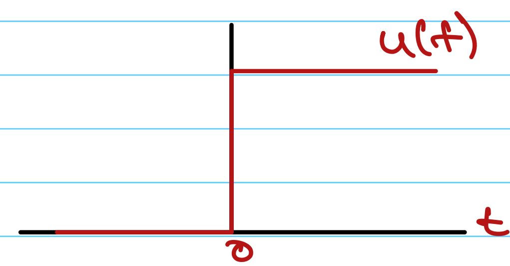
① $\delta(t) = \begin{cases} +\infty & t = 0 \\ 0 & t \neq 0 \end{cases}$



② $\frac{1}{P}\left[\delta(t)\right] = u(t)$

$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

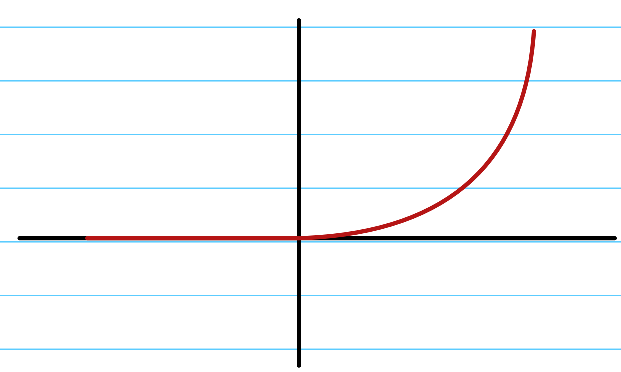


③ $\frac{1}{P}\left(u(t)\right) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$

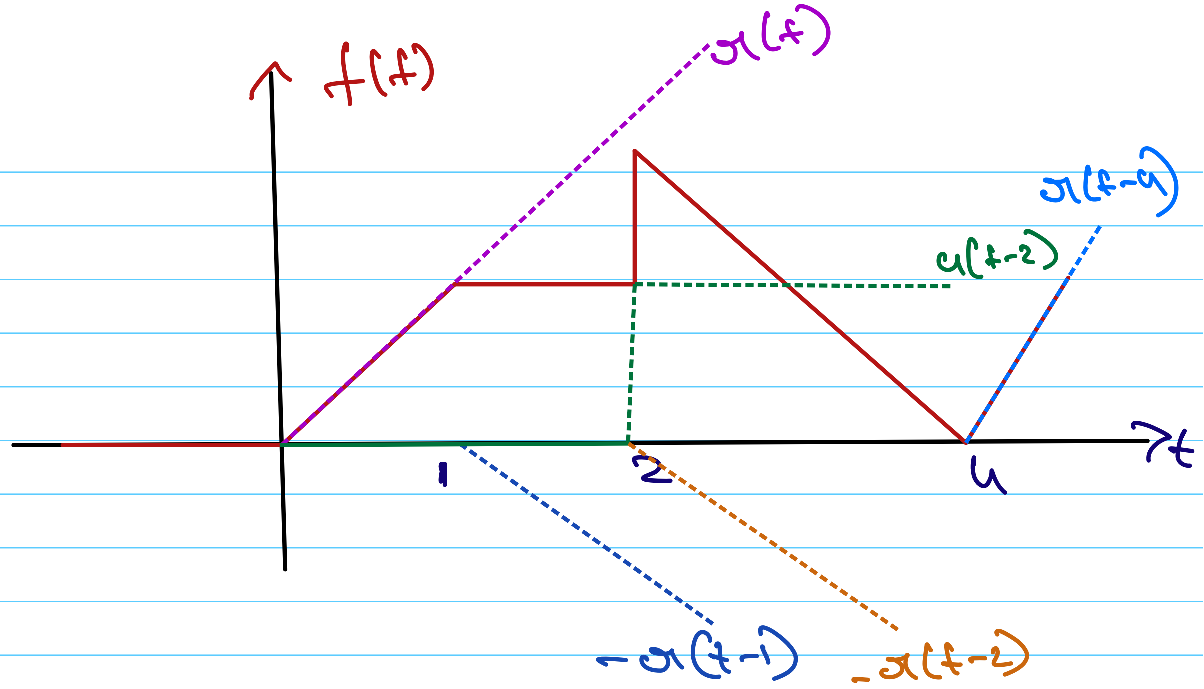
$r(t) = t \cdot u(t)$

④ $\frac{1}{P}\left(r(t)\right)$

$= \begin{cases} t^2 & t \geq 0 \\ 0 & t < 0 \end{cases}$



Ex:



would I be able to discribe this function in term's of singularity function?

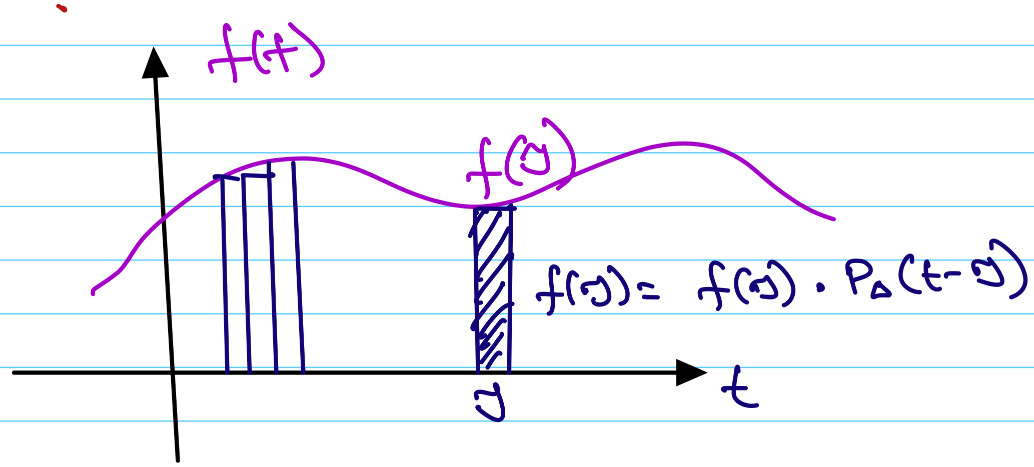
$$f(t) = r(t) - r(t-1) + u(t-2) - r(t-2) + r(t-4)$$

why are we writing in this way?
(because of superposition)

$\Rightarrow$ if we have linear system, then we can reduce the input signal into smaller function(summation of smaller function's) to the response that we know.

* it is easier to determine & then we can write overall response to the input as sum of the response.

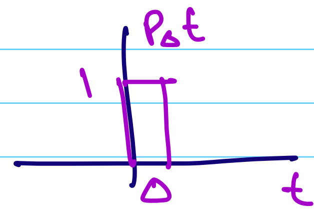
What if we have a arbitrary function $f(t)$?



Can we approximate this function with a summation of singularity function.
(may be with dirac-delta function)

Let's break it down to summation of Pulse function's

$P_\Delta(t) =$



$$f(y) = f(y) \cdot P_\Delta(t-y)$$

(this Basically describes one Box)

* we can Put these Box Next to each other and approximate this function.

$$f(t) \approx \sum_{n=-\infty}^{+\infty} f(n\Delta) \cdot P_\Delta(t - n\Delta)$$

$f(n\Delta)$ is like a look up table

$n\Delta$	$f(n\Delta)$
$n=0$	$f(0)=1$
$n=1$	$f(n\Delta)=3.5$
$n=2$	$f(2\Delta)=4.37$
$\vdots$	$\vdots$

Look up table of function values

$$f(t) \approx \sum_{n=-\infty}^{+\infty} f(n\Delta) \cdot \frac{1}{\Delta} P_\Delta(t-n\Delta)\Delta$$

$$f(t) = \lim_{\Delta \to 0} \sum_{n=-\infty}^{+\infty} f(n\Delta) \cdot \frac{1}{\Delta} P_\Delta(t-n\Delta) \cdot \Delta$$

$$f(t) = \int_{-\infty}^{+\infty} f(y)\, \delta(t-y)\, dy$$

(shifting property)

↑ we have actually decomposed into continuous sample points

$$f(t) = \int_{-\infty}^{+\infty} f(y)\, \delta(t-y)\, dy$$

↑

this is nothing but a look up table (continuous look-up table)

(this is not a function, this is the value of function at time t)

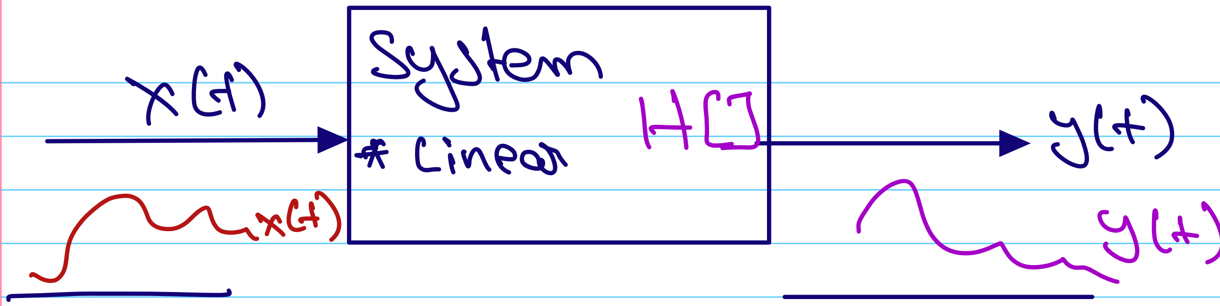
⇒ the delta's at different time formed a BASIS.

⇒ delayed impules are the BASIS function's (Trivial BASIS)

⇒ so, we can write any function $f(t)$ as this sum of all the delayed impulses.

⇒ which mean's, we can show any function by knowing exactly the value of any point in time.

arbitrary function $= x(t) =$ Linear combination of delayed impulses $\delta(t-t_0)$



Operator H[ ], which defines system

$$y(t) = H[x(t)]$$

Where we can decompose $x(t)$ into linear combination of delayed impulses

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau$$

Continuous loop up table

$$y(t) = H[x(t)]$$

$H[\ ]$:- Linear operator
(Linear system)

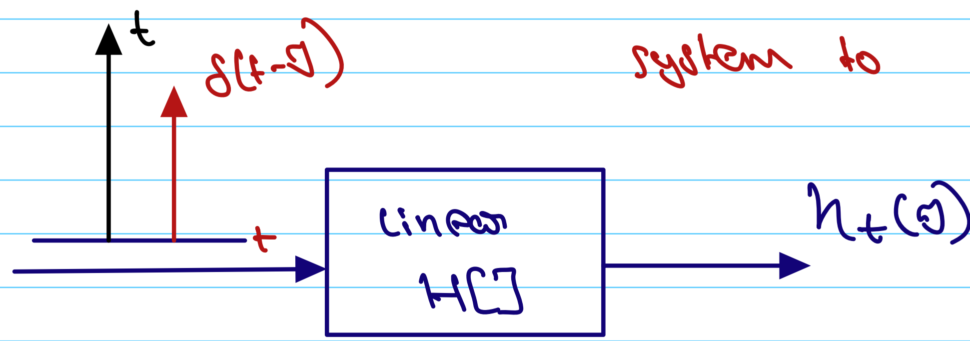
⇒ if we have linear system, then we can use superposition.

$$y(t) = H[x(t)] = H\left[\int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau\right]$$

(Linearity)

$$= \int_{-\infty}^{\infty} x(\tau)\,\underbrace{H[\delta(t-\tau)]}\,d\tau$$

Response to a delayed impulse i.e, response of the system to $\delta(t-\tau)$



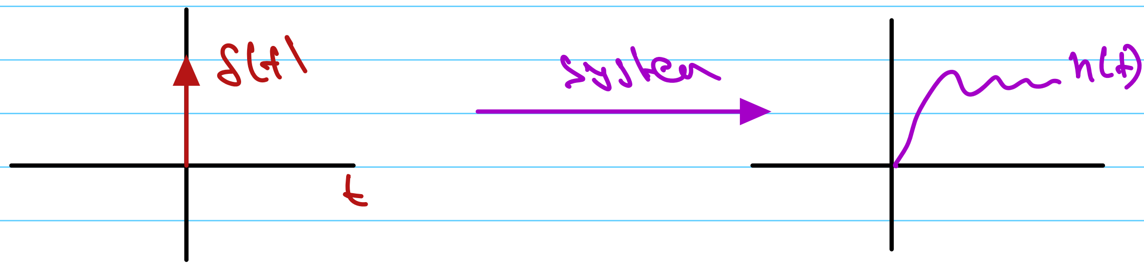
⇒ $h_t(\tau)$ is the impulse response of the system

superposition integral

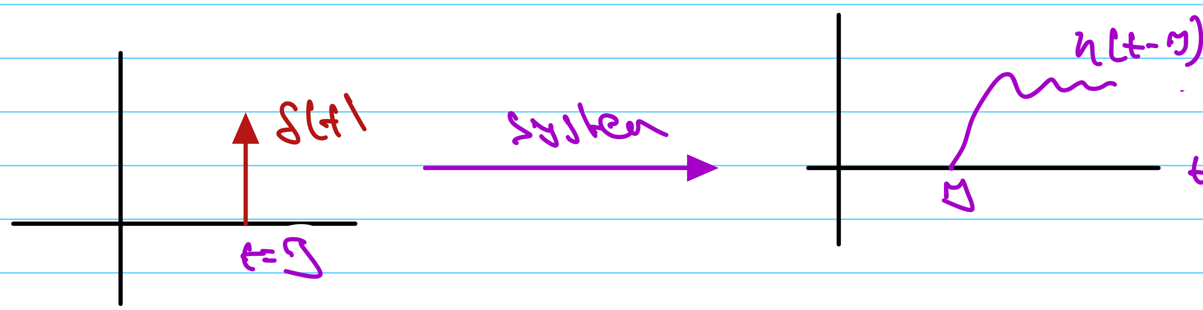
$$\Rightarrow \quad y(t) = \int_{-\infty}^{+\infty} x(\tau)\,\underbrace{h(t,\tau)}\,d\tau$$

Impulse response at $\tau = t$

if the system is time invariant



then



$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(\underbrace{t-\tau}_{\text{only depend's on } t-\tau \text{ (difference of time)}})\, d\tau$$

Shifting property $\Rightarrow$ $x(t) = \int_{-\infty}^{+\infty} x(\tau)\, \delta(t-\tau)\, d\tau$

Superposition integral $\Rightarrow$ $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t,\tau)\, d\tau$

when $H[\delta(t-\tau)] = h(t,\tau)$

Linear system ?

Linear Time invariant

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Convolution integral $\Rightarrow$

$$= (h * x)(t)$$

so for LTI system's, it's allow's us to find the response of a system to any arbitrary input $x(t)$ as long as we know its response to impulse

* if we know $h(t,\tau)$, Impulse response; we know everything about the system.