Chapter 2 Problem's

(2.1) Let $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$

$h[n] = 2\delta[n+1] + 2\delta[n-1]$

Compute & plot each of the following convolution's

(a) $y_1[n] = x[n] * h[n]$

$$x[n] = x[0]\,\delta[n] + x[1]\,\delta[n-1] + x[3]\,\delta[n-3]$$

$x[0] = 1$

$x[1] = 2$

$x[3] = -1$

$$y[n] = x[0]\,h[n-0] + x[1]\,h[n-1] + x[3]\,h[n-3]$$

$$y[n] = 2\delta[n+1] + 2\delta[n-1] + 4\delta[n] + 4\delta[n-2] - 2\delta[n-2] - 2\delta[n-4]$$

$$\Rightarrow y[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-2] - 2\delta[n-4]$$

(b) $$y_2[n] = x[n-2] * h[n]$$

$$x[n+2] = \delta[n+2] + 2\delta[n+1] - \delta[n-1]$$

$$h[n] = 2\delta[n+1] + 2\delta[n-1]$$

$$x[n+2] * h[n] = 1\cdot(2\delta[n+3] + 2\delta[n+1]) + 2[2\delta[n+2] + 2\delta[n]) - (2\delta(n) + 2\delta(n-2))$$

$$= 2\delta[n+3] + 4\delta[n+2] + 2\delta[n+1]$$
$$+ 2\delta[n] - 2\delta[n-2]$$

2.2

$$h[n] = \left(\frac{1}{2}\right)^{n-1} \{u[n+3] - u[n-10]\}$$

$$h[n] = \left(\frac{1}{2}\right)^{n-1} \{u[n+3] - u[n-10]\}$$

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^{n-1}, & -3 \leq n \leq 9 \\ 0 & \text{o.w} \end{cases}$$

$$h[n-k] = \left(\frac{1}{2}\right)^{n-k-1} \{u[n+3-k] - u[n-k-10]\}$$

$$= \begin{cases} \left(\frac{1}{2}\right)^{n-k-1} & -3+k \leq n \leq 9+k \\ 0 & \text{o.w} \end{cases}$$

$$= \begin{cases} \left(\frac{1}{2}\right)^{n-k \cdot 1} & n-9 \le k \le n+3 \\ 0 & \text{o.w} \end{cases}$$

2.3

$$x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2]$$

$$h[n] = u[n+2]$$

$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n-2} u[k-2]\, u[n+2-k]$$

$u[k-2] = 0 \quad \forall k < 2 \implies u[k-2] = 1 \quad \forall k \geq 2$

$u[n+2-k]$

$= u[-(k-(n+2))] \implies u[n+2-k] = 1$

$\forall k \le n+2$

$= 0 \quad \forall k > n+2$

$$y[n] = \sum_{k=2}^{n+2} \left(\frac{1}{2}\right)^{n-2}$$

$\vdots$ (need to do again)

2.4 $\quad y[n] = x[n] * h[n]$

$$x[n] = \begin{cases} 1 & 3 \le n \le 8 \\ 0 & \text{o.w} \end{cases}$$

$$h[n] = \begin{cases} 1 & 4 \le n \le 15 \\ 0 & \text{o.w} \end{cases}$$

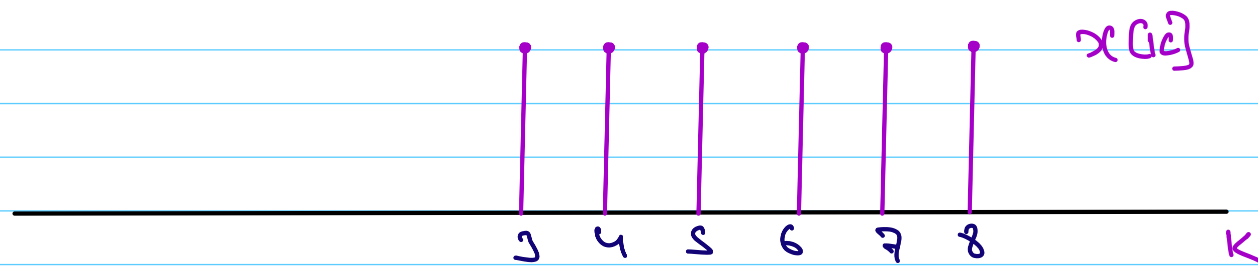
Sol^n

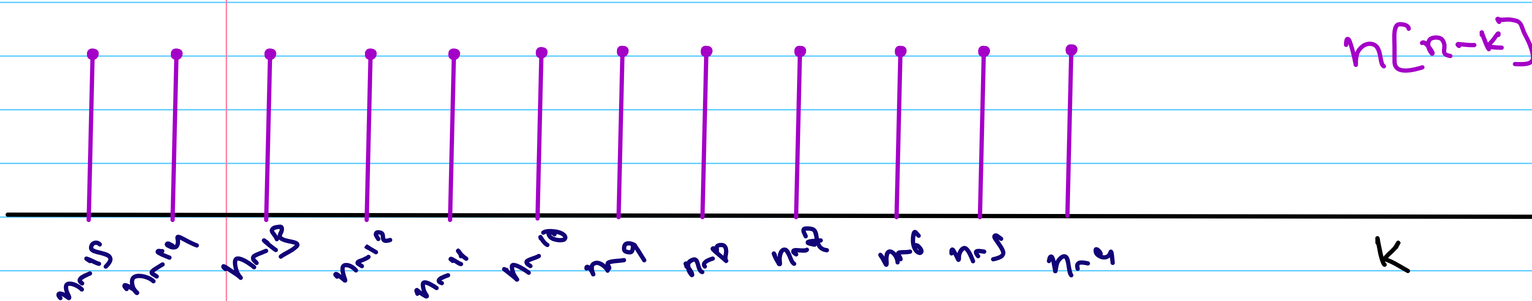
$$x[n] = u[n-3] - u[n-9]$$

$$h[n] = u[n-4] - u[n-16]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} \left(u[k-3] - u[k-9]\right)\left(u[n-k-4] - u[n-k-16]\right)$$





$$\Rightarrow \quad y[n] = \begin{cases} 0 & n \leq 6 \\ n-6 & 7 \leq n \leq 11 \\ 6 & 12 \leq n \leq 18 \\ 24-n & 19 \leq n \leq 23 \\ 0 & \text{o.w} \end{cases}$$

$n=7 \Rightarrow 1$

$n=8 \Rightarrow 2$

$n-6$

2.5

$$x[n] = \begin{cases} 1 & 0 \le n \le 9 \\ 0 & \text{o.w} \end{cases}$$

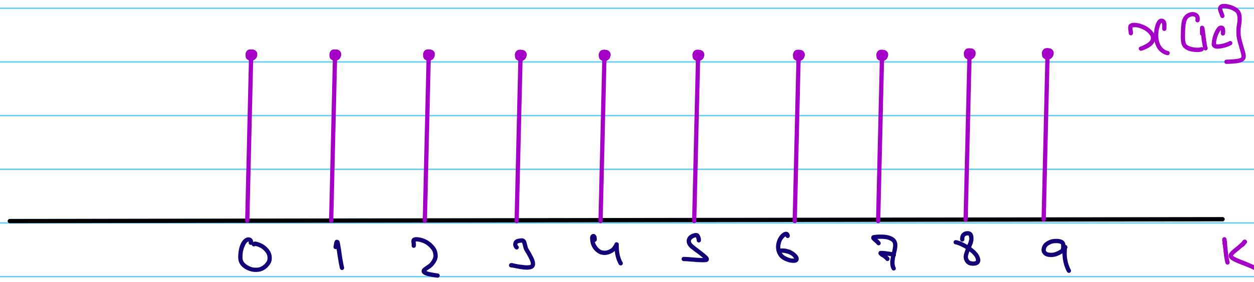
$$h[n] = \begin{cases} 1 & 0 \le n \le N \\ 0 & 0 \end{cases}$$

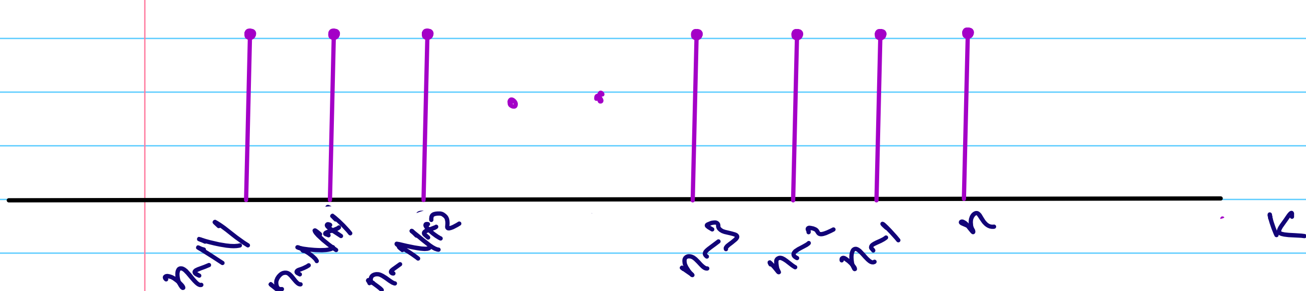
Soln

$$x[n] = \sum_{k=0}^{9} \delta[n-k]$$

$$h[n] = \sum_{m=0}^{N} \delta[n-m] \quad (n\text{-axis})$$

$$h[-(k-n)] = \sum_{k=n-N}^{n} \delta[n-k]$$





we know that $y[4]=5 \Rightarrow 4-N \le 0$

$\Rightarrow N \ge 4$

$y[14]=0 \Rightarrow 14-N \ge 10$

$\Rightarrow N \le 4$

$\Rightarrow N=4$

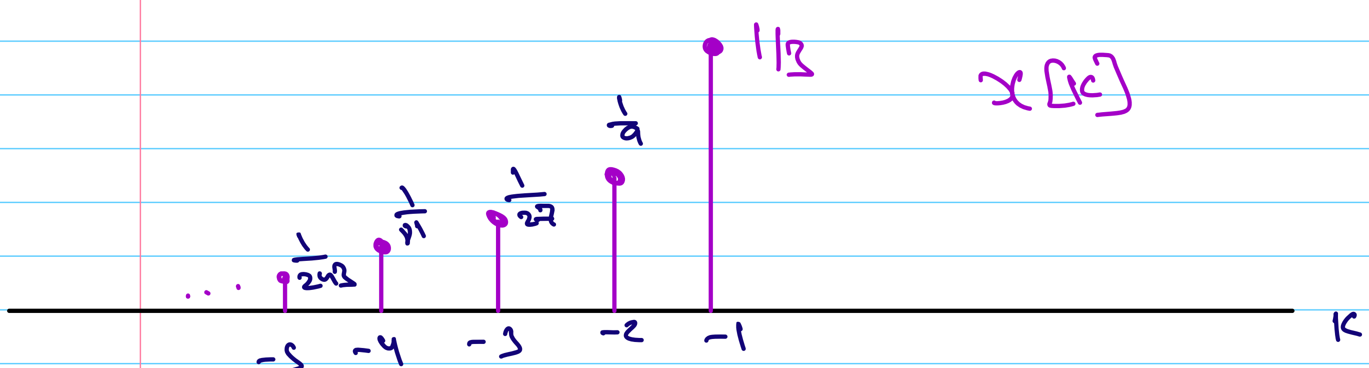
2.6 $y[n] = x[n] * h[n]$

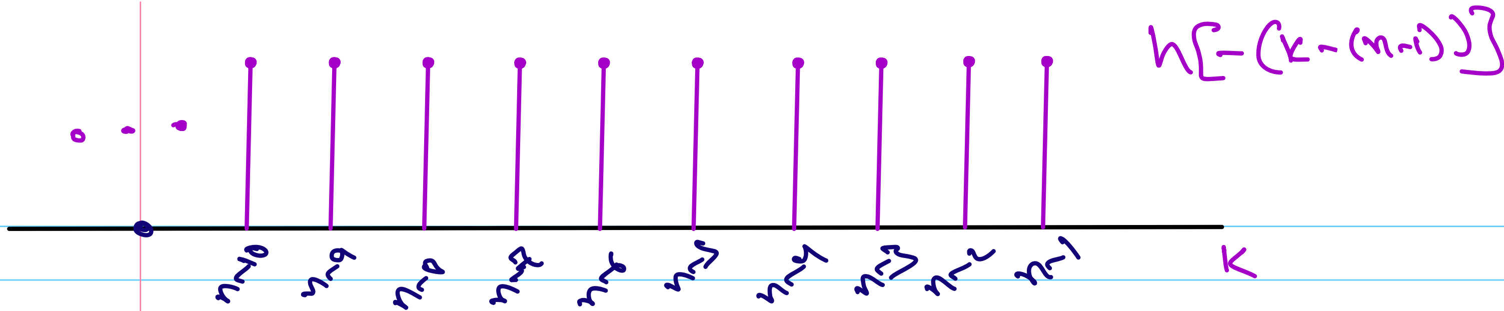
where $x[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1]$

$h[n] = u[n-1]$

sol $x[k] \left(\frac{1}{3}\right)^{-k} u[-(k+1)]$

$h[n-k] = u[n-1-k] = u[-(k-(n-1))]$





for $n \geq 0$ $\quad y[n] = \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^k$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k - 1$$

$$= \frac{1}{1-\frac{1}{3}} - 1 = \frac{3}{2} - 1 = \frac{1}{2}$$

for $n < 0$

$$y(n) = \sum_{-\infty}^{n} \left(\frac{1}{3}\right)^{-(n-1)}$$

2.7 $\quad y[n] = \sum_{k=-\infty}^{\infty} x[k]\, g[n-2k]$

between its input $x[n]$ and its output $y(n)$, where $g[n] = u[n] - u[n-4]$

(a) Determine $y[n]$ when $x[n] = \delta[n-1]$

soln

$$g[n] = u[n] - u[n-4]$$

$$g[n-2k] = u[n-2k] - u[n-2k-4]$$

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k-1]\, g[n-2k]$$

$$= g[n-2]$$

$$= u[n-2] - u[n-6]$$

(b) Determine $y[n]$ when $x[n] = \delta[n-2]$

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k-2]\, g[n-2k]$$

$$= g[n-4] = u[n-4] - u[n-8]$$

(c) is S LTI?

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, g[n-2k]$$

NO

(d) Determine $y[n]$ when $x[n] = u[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} u[k]\, g[n-2k]$$

$$= \sum_{k=0}^{\infty} g[n-2k]$$

$$= \sum_{k=0}^{\infty} u[n-2k] - u[n-2k-4]$$

(2.8)

$$x(t) = \begin{cases} t+1 & 0 \le t \le 1 \\ 2-t & 1 < t \le 2 \\ 0 & \text{elsewher.} \end{cases}$$

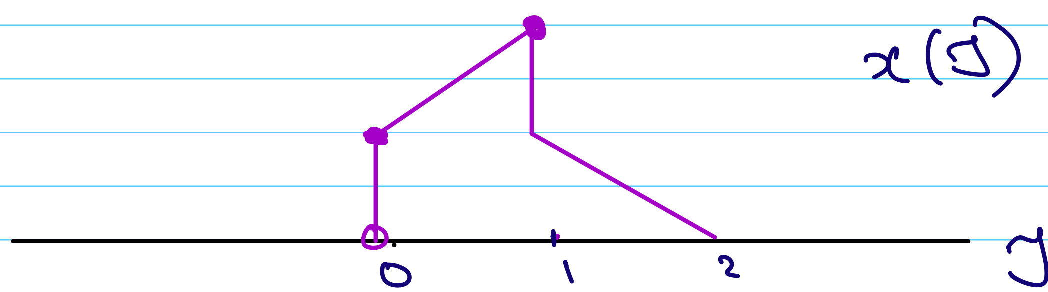
$$h(t) = \delta(t+2) + 2\delta(t+1)$$

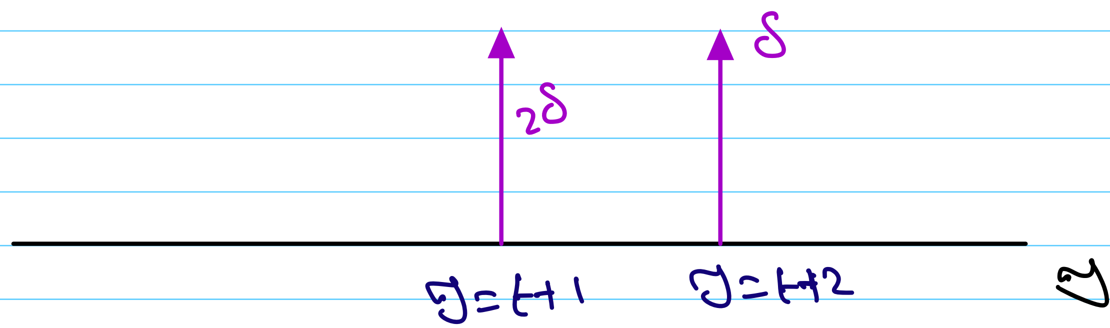
$$h(t-\tau) = \delta(t+2-\tau) + 2\delta(t+1-\tau)$$

$$= \delta\big(-(\tau-(t+2))\big) + 2\delta\big(-(\tau-(t+1))\big)$$

= Impulse function at

$\tau = t+2$ & $\tau = t+1$





$$y(t) = \begin{cases} 0 & t < -2 \\ t+3 & -2 \le t \le -1 \\ 2-2t & -1 \le t \le 0 \\ 0 & \text{o.w} \end{cases}$$

2.9 $h(t)= e^{2t}\,u(-t+4)+e^{-2t}\,u(t-5)$

Determine A and B s.t

$$h(t-\tau)=\begin{cases} e^{-2(t-\tau)} & \tau < A \\ 0 & A < \tau < B \\ e^{2(t-\tau)} & B < \tau \end{cases}$$

Sol^n

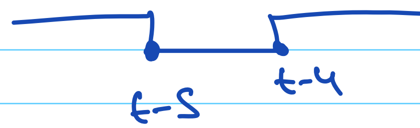
$$h(t)= e^{2t}\,u(-t+4)+e^{-2t}\,u(t-5)$$

$$h(t-\tau)= e^{2t-2\tau}\,u(-t+\tau+4)$$
$$+e^{-2t+2\tau}\,u(t-\tau-5)$$

$$h(t-\tau) = e^{2t}\cdot e^{-2\tau}\,u(\tau-(t-4))$$
$$+ e^{-2t}\cdot e^{2\tau}\,u(-(\tau-(t-5)))$$

$$= \underbrace{e^{2t} \cdot e^{-2y} u(y-(t-4))}_{=0 \ \forall y < t-4} + \underbrace{e^{-2t} \cdot e^{2y} u(-(y-(t-5))}_{=0 \ \forall y > t-5} \Big]$$

$=$



$$\Rightarrow \quad h(t-y) = \begin{cases} e^{-2(t-y)} & y < t-5 \\ 0 & t-5 \leq y \leq t-4 \\ e^{2(t-y)} & t-4 < y \end{cases}$$

$$A = t-5 \qquad B = t-4$$

2.10

$$x(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{o.w} \end{cases}$$

and $h(t) = x(t/\alpha)$ $\quad 0 < \alpha \leq 1$

(a) Determine and sketch $y(t) = x(t) * h(t)$

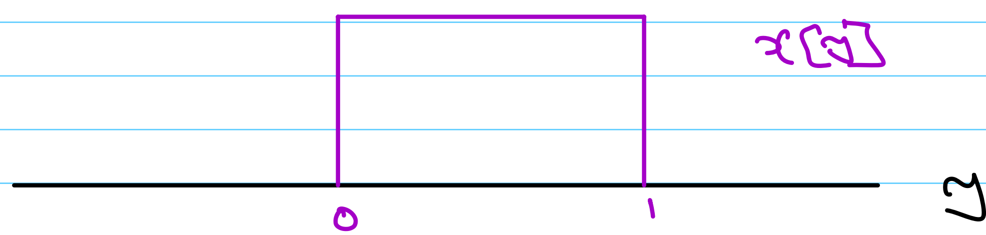
$\underline{\underline{sol^n}}$

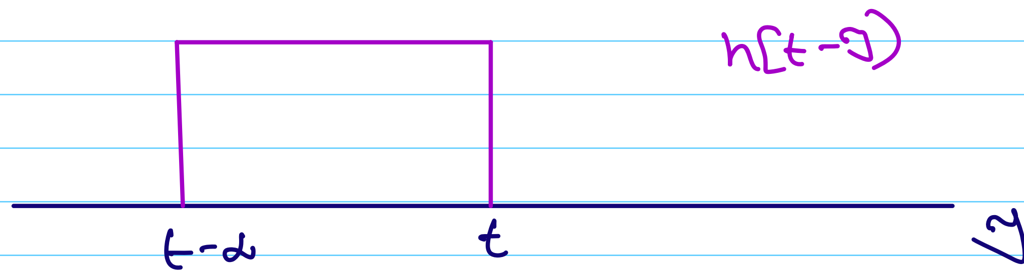
$$x(t) = u(t) - u(t-1)$$

$$h(t) = u(t/\alpha) - u\left(\frac{t-1}{\alpha}\right)$$

$$h(t) = \begin{cases} 1 & 0 \le t < \alpha \le 1 \\ 0 & o.w \end{cases}$$

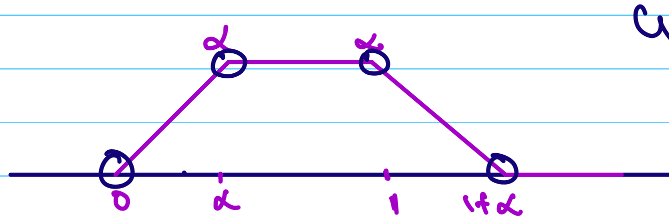
$$h(t-y) = h(-(y-t)) = \begin{cases} 1 & t-\alpha < y \le t \\ 0 & o.w \end{cases}$$





$$y(t) = \begin{cases} t & 0 < t \le \alpha \\ \alpha & \alpha < t \le 1 \\ 1+\alpha-t & 1 < t < 1+\alpha \\ 0 & o.w \end{cases}$$

(b)



4 discontinuous of $\frac{dy}{dt}$

if $\alpha = 1 \Rightarrow$ 3 discontinuous

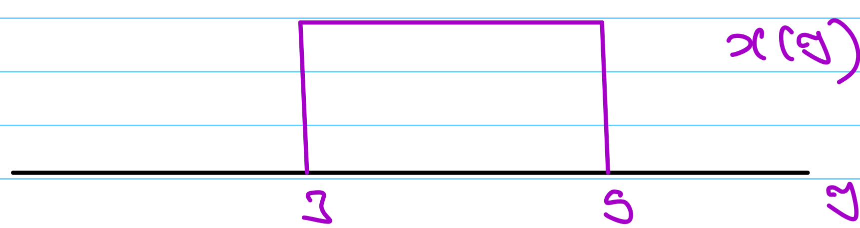
(2.11) $x(t) = u(t-3) - u(t-5)$

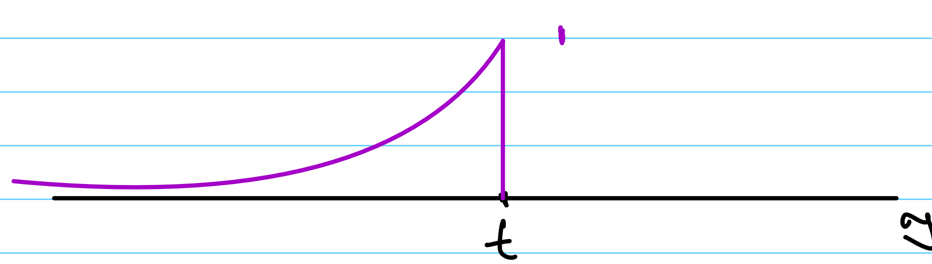
$h(t) = e^{-3t} u(t)$

(a) Compute $y(t) = x(t) * h(t)$

$h(t-\tau) = e^{-3(t-\tau)} u(t-\tau)$

$= e^{3(\tau-t)} u(-(\tau-t))$





$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{3}^{t} \left[u(\tau-3) - u(\tau-5)\right] e^{-3(t-\tau)}\, u(t-\tau)\, d\tau$$

$$= \int_{3}^{t} e^{-3(t-\tau)}\, d\tau = e^{-3t} \left[\frac{e^{3\tau}}{3}\right]_{3}^{t}$$

$$= e^{-3t} \left[\frac{e^{3t} - e^{9}}{3}\right]$$

$$= \frac{\frac{1}{3} - e^{-3(t-3)}}{}$$

$3 \leq t \leq 5$

for $t > 5$

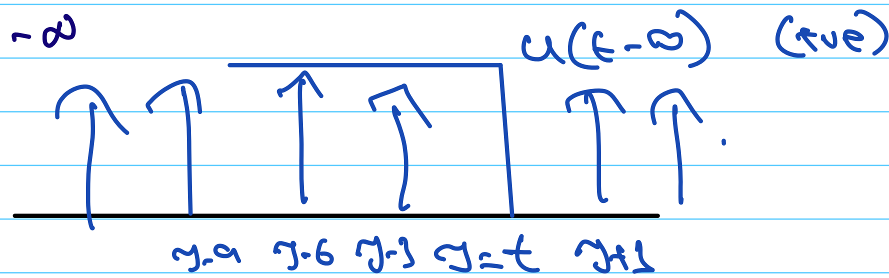
2.12 $$y(t) = e^{-t}u(t) * \sum_{k=-\infty}^{\infty} \delta(t-3k)$$

Show that $y(t) = Ae^{-t}$ for $0 \le t < 3$ and determine the value of A.

Sol^n

$$y(t) = \int_{\tau=-\infty}^{+\infty} \left(\sum_{k=-\infty}^{+\infty} \delta(t-3k) \right) e^{-(t-\tau)}\, u(t-\tau)\, d\tau$$

$$y(t) = \sum_{k=-\infty}^{+\infty} \int_{\tau=-\infty}^{+\infty} \delta(\tau-3k)\, e^{-(t-\tau)}\, u(-(\tau-t))\, d\tau$$



$$y(t) = e^{-t} \sum_{k=0}^{+\infty} \int_{\tau=-\infty}^{t} e^{\tau}\, \delta(\tau-3k)\, d\tau$$

$$y(t) = e^{-t} \sum_{k=0}^{+\infty} e^{-3k}$$

$$y(t) = e^{-t} \frac{1}{1-e^{-3}}$$

$$\Rightarrow y(t) = \frac{1}{1-e^{-3}} e^{-t}$$

where $A = \frac{1}{1-e^{-3}}$

2.13 D.T system with impulse response

$$h[n] = \left(\frac{1}{5}\right)^n u[n]$$

a

$$h[n] - A\,h[n-1] = \delta[n]$$

$$\left(\frac{1}{5}\right)^n u[n] - A\left(\frac{1}{5}\right)^{n-1} u[n-1]$$

$$= \delta[n]$$

$\Rightarrow$ for $n=1$ $\quad \frac{1}{5} - A\left(\frac{1}{5}\right)^0 \cdot 1 = 0$

$$A = \frac{1}{5}$$

(b)

$$h[n] = \left(\frac{1}{5}\right)^n u[n]$$

$$h[n] * g[n] = \delta[n]$$

$$\sum_{k=-\infty}^{\infty} h[k]\, g[n-k] = \delta[n]$$

Causal system

$$\sum_{k=0}^{\infty} h[k]\, g[n-k] = \delta[n]$$

$n=0$

$$\sum_{k=0}^{\infty} \left(\frac{1}{5}\right)^n u[k]\, g[-k] = 1$$

$$\Rightarrow \quad \sum_{k=0}^{\infty} \left(\frac{1}{5}\right)^0 g[-k] = 1$$

$$\Rightarrow \sum_{k=0}^{\infty} g[-k] = 1$$

$$g[0] + \underbrace{g[-1] + \cdots + g[-\infty]}_{0 \text{ (causal system)}} = 1$$

$$\Rightarrow g[0] = 1$$

2.14

$h_1(t) = e^{-(1-2j)t} u(t)$

$$\int_{-\infty}^{+\infty} |h_1(t)|$$

$$= \int_{-\infty}^{+\infty} |e^{-t}| \underbrace{|e^{2jt}|}_{=1} u(t)\, dt$$

$$= \int_{0}^{\infty} |e^{-t}|\, dt$$

$$= \left.\frac{e^{-t}}{-1}\right]_0^{\infty} = 1 - 0 < \infty$$

$\Rightarrow$ $h_1(t)$ is stable

(b) $h_2(t) = e^{-t} \cos(2t)\, u(t)$

$$\int_{0}^{\infty} e^{-t} |\cos(2t)|\, dt < \infty$$

$\Rightarrow$ stable

2.15

$h_1[n] = n\cos\left(\frac{\pi}{4}n\right)u[n]$

$$\sum_{k=-\infty}^{+\infty} |h_1[k]|$$

$$= \sum_{k=0}^{\infty} k\left|\cos\left(\frac{\pi}{4}k\right)\right|$$

this won't converge

$\Rightarrow$ unstable

(b) $h_2[n] = 3^n u[-n+10]$

$$\sum_{k=-\infty}^{+\infty} |3^k u[10-k]|$$

$$\Rightarrow \sum_{k=-\infty}^{10} 3^k \quad \text{finite} < \infty$$

$\Rightarrow$ Stable

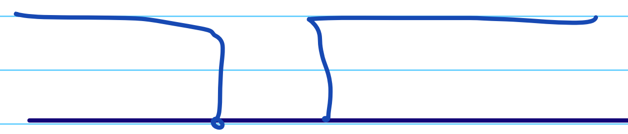
2.16 (a) if $x[n] = 0$ for $n < N_1$ and

$h[n] = 0$ for $n < N_2$, then $x[n] * h[n] = 0$

for $n < N_1 + N_2$

Sol^n

$$\sum_{k=-\infty}^{+\infty} x[k]\, h[n-k] = y[k]$$



$n - k > N_2$

$k < N_2 - n$

The convolution $= 0$

for $n - N_2 < N_1$

$\Rightarrow n < N_1 + N_2$

True.

(b) $y[n-1] = x[n-1] * h[n-1]$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

$$y[n-1] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-1-k]$$

$y[n-1] = \quad x[n] * h[n-1]$

or

$x[n-1] * h[n]$

Hence $\quad y[n-1] = x[n-1] * h[n-1]$ False.

(c)

if $\quad y(t) = x(t) * h(t)$

then $\quad y(-t) = x(-t) * h(-t)$

$$y(t) = \int_{-\infty}^{+\infty} x(\gamma)\, h(t-\gamma)\, d\gamma$$

$$y(-t) = \int_{-\infty}^{\infty} x(\gamma)\, h(-t-\gamma)\, d\gamma$$

$$y(-t) = \int_{-\infty}^{\infty} x(\gamma)\, h(-\gamma-t)\, d\gamma$$

$$= \int_{-\infty}^{\infty} x(-\gamma)\, h(\gamma-t)\, d\gamma$$

$$= x(-t) * h(-t)$$

2.17

$$\frac{d}{dt} y(t) + 4y(t) = x(t) \quad \text{(Initial rest)}$$

a) if $x(t) = e^{(-1+3j)t} u(t)$, what is $y(t)$

Soln

$$x(t) = e^{(-1+3j)t} u(t)$$

then

$$y_p(t) = K e^{(-1+3j)t} u(t)$$

$$\frac{dy_p}{dt} = K e^{(-1+3j)t} \cdot (-1+3j)$$

$$+4 y_p = 4K e^{(-1+3j)t}$$

$$\frac{d}{dt} y_p(t) + 4y(t) = x(t)$$

$$\Rightarrow \quad K(-1+3j) + 4K = 1$$

$$\Rightarrow \quad K = \frac{1}{3(1+j)} = \frac{1-j}{6}$$

$$y_p(t) = \frac{1-j}{6}$$

Now we need to find Homogenas sol^n

$$\frac{d}{dt} y(t) + 4y(t) = 0$$

$$r + 4 = 0 \qquad r = -4$$

$$\Rightarrow y_h(t) = ce^{-4t}$$

$$\Rightarrow y(t) = ce^{-4t} + \frac{1-j}{6} e^{(-1+3j)t}$$

$y(0) = 0$ (initial rest)

$$\Rightarrow 0 = C + \frac{1-j}{6}$$

$$\Rightarrow C = -\frac{1}{6}(1-j)$$

$$\boxed{y(t) = \frac{(1-j)}{6}\left(e^{(-1+3j)t} - e^{-4t}\right)}$$

ⓑ for $Re\{x(t)\}$ we get

$Re\{y(t)\}$

$$y(t) = \frac{(1-j)}{6}\left(e^{-t} \cdot e^{3jt} - e^{-4t}\right)$$

$$= \left(\frac{1-j}{6}\right)\left(e^{-t}(\cos 3t + j\sin 3t) - e^{-4t}\right)$$

$$= \frac{-e^{-4t}}{6} + \frac{j e^{-4t}}{6} + \frac{1}{6}e^{-t}\cos 3t + \frac{1}{6}e^{-t}\sin 3t + j\ldots$$

$$Re\{y(t)\} = \frac{1}{6}e^{-t}(\sin 3t + \cos 3t) - \frac{e^{-4t}}{6}$$

(2.18) Consider a causal LTI system's whose input $x[n]$ and output $y[n]$ are related by the difference eqn

$$y[n] = \frac{1}{4} y[n-1] + x[n]$$

Determine $y[n]$ if $x[n] = \delta[n-1]$

Solⁿ

$$y[n] = \frac{1}{4} y[n-1] + x[n]$$

Let $x[n] = \delta[n-1]$

$$y[n] = \frac{1}{4} y[n-1] + \delta[n-1]$$

$y[0] = 0$ Because it's causal

$y[1] = \frac{1}{4} y[0] + \delta[0] = 1$

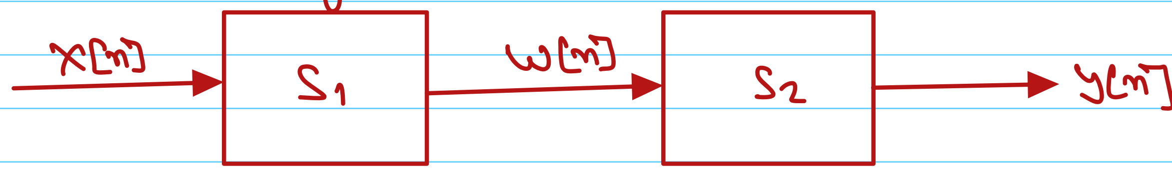
$y[2] = \frac{1}{4} y[1] + \delta[1] = \frac{1}{4}$

$y[3] = \frac{1}{4} y[2] = \frac{1}{4^2}$

:

$$y[n] = \frac{1}{4^{n-1}} u[n-1]$$

2.19 Cascade of system's



S_1 : causal LTI

$$w[n] = \frac{1}{2} w[n-1] + x[n]$$

S_2 : causal LTI

$$y[n] = \alpha y[n-1] + \beta w[n]$$

$$y[n] = -\frac{1}{8} y[n-2] + \frac{3}{4} y[n-1] + x[n]$$

(a) Determine α and β

Sol^n

$$y[n] = \alpha y[n-1] + \beta w[n]$$

$$\Rightarrow w[n] = \frac{y[n]}{\beta} - \frac{\alpha}{\beta} y[n-1]$$

$$w[n] = \frac{1}{2} w[n-1] + x[n]$$

$$\frac{y[n]}{\beta} - \frac{\alpha}{\beta} y[n-1] = \frac{y[n-1]}{2\beta} - \frac{\alpha}{2\beta} y[n-2] + x[n]$$

$$\Rightarrow \frac{y[n]}{\beta} = \frac{2\alpha+1}{2\beta} y[n-1] - \frac{\alpha}{2\beta} y[n-2] + x[n]$$

$$\Rightarrow y[n] = \frac{2\alpha+1}{2} y[n-1] - \frac{\alpha}{2} y[n-2] + x[n]\beta$$

$$\frac{-1}{8} = \frac{-\alpha}{2} \Rightarrow \alpha = \frac{1}{4}$$

$$\beta = 1$$

$$\alpha = \frac{1}{4}, \quad \beta = 1$$

(b) Show the impulse response of the cascade connection of S_1 and S_2

$$w[n] = \frac{1}{2} w[n-1] + x[n]$$

$$x[n] = \delta[n]$$

system is causal

$$h_1[0] = \frac{1}{2} w[\cancel{-1}]^{\to 0} + \delta[0] = 1$$

$$h_1[1] = \frac{1}{2} w[0] + \delta[1] = \frac{1}{2}$$

$$h_1[2] = \left(\frac{1}{2}\right)^2$$

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

Similarly

$$y[n] = \frac{1}{4} y[n-1] + w[n]$$

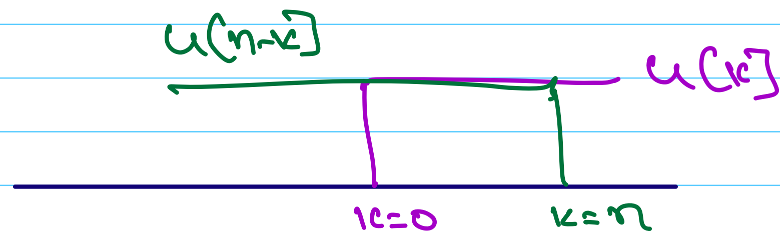
for $w[n] = \delta[n]$

$$h_2[n] = \left(\frac{1}{4}\right)^n u[n]$$

$$h[n] = h_1[n] * h_2[n]$$

$$h[n] = \sum_{k=-\infty}^{+\infty} h_1[k]\, h_2[n-k]$$

$$= \sum_{k=-\infty}^{+\infty} \left(\frac{1}{2}\right)^{k} u[k] \left(\frac{1}{4}\right)^{n-k} u[n-k]$$



$$h[n] = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k} \left(\frac{1}{4}\right)^{n-k}$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{2n-k}$$

$$= u[n] \left(\frac{1}{2}\right)^{2n} \sum_{k=0}^{n} 2^{k}$$

$$= u[n] \left(\frac{1}{2}\right)^{2n} \left(\frac{2^{n}-1}{2-1}\right)$$

$$h[n] = \left[\left(\frac{1}{2}\right)^{n} - \left(\frac{1}{2}\right)^{2n}\right] u[n]$$

2.20 Evaluate the following integrals

(a) $$\int_{-\infty}^{+\infty} u_0(t)\cos(t)\,dt =$$