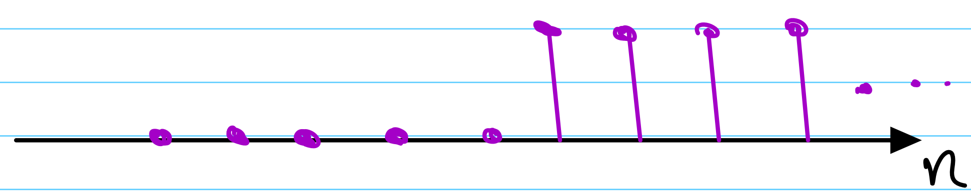
Lec03:- Signal's & System's PART II

in addition to the sinusoidal and exponential signal's, (Real & complex) both for continuous & discrete time will form very important Building Blocks of Fourier analysis.

Today: unit step & unit impulse

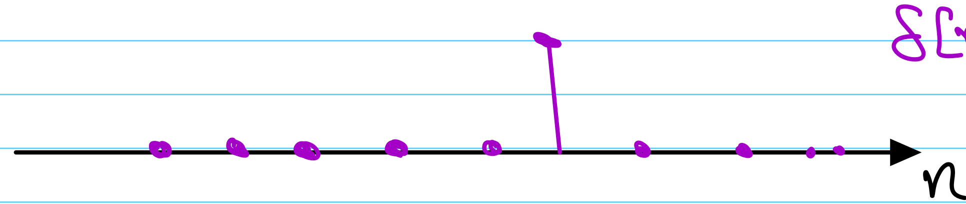
Unit step Function: Discrete-Time

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



Unit Impulse Function: Discrete-Time

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



* $\delta[n] = u[n] - u[n-1]$

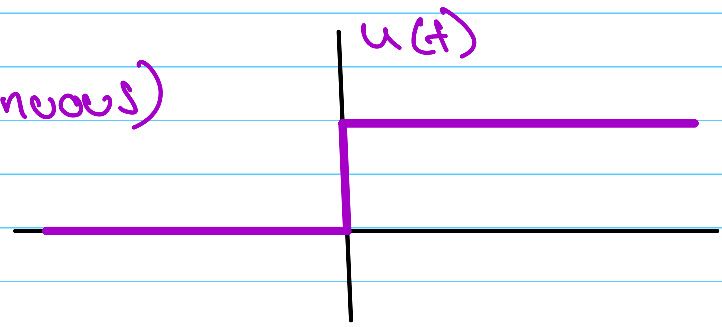
$u[n-1]$: delayed by 1

* $u[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

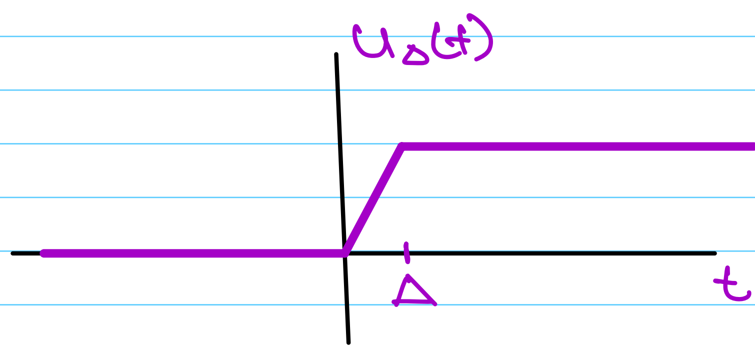
$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

Unit step function - Continuous time

(discontinuous)



$$u(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}$$



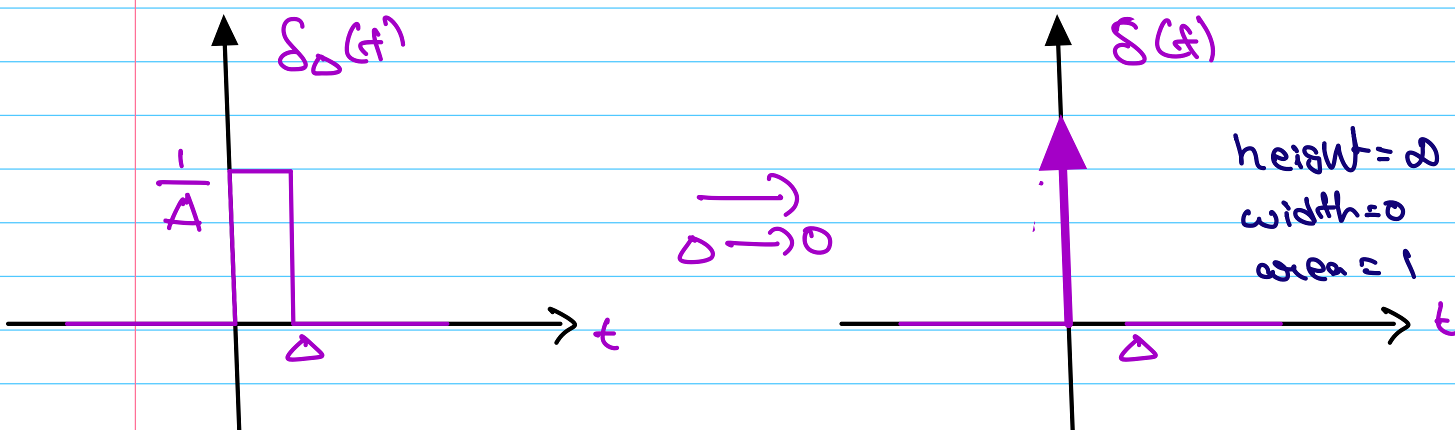
$u(t) = u_\Delta(t)$ as $\Delta \to 0$

Unit Impulse Function

$$\delta(t) = \frac{du(t)}{dt}$$

$$\delta_\Delta(t) = \frac{du_\Delta(t)}{dt}$$

$$\delta(t) = \delta_\Delta(t) \quad \text{as} \quad \Delta \to 0$$



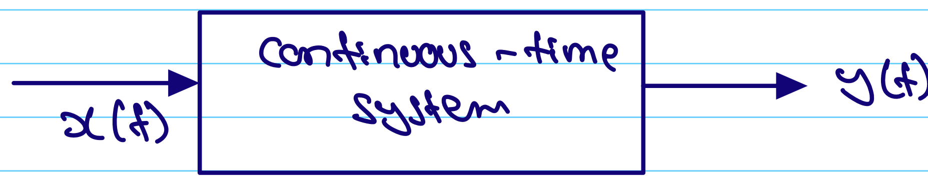
$$\delta(t) = \frac{du(t)}{dt}$$

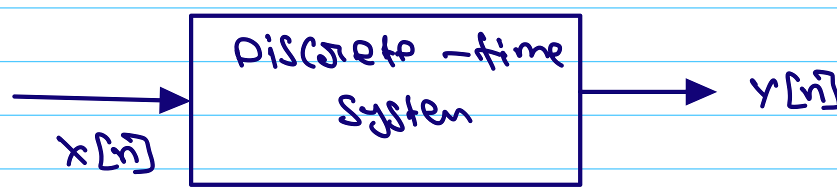
$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

Linear-time invariant system's

System's :- (in general)

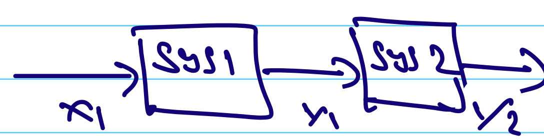
A system in general is an transformation from an input signal to output signal.





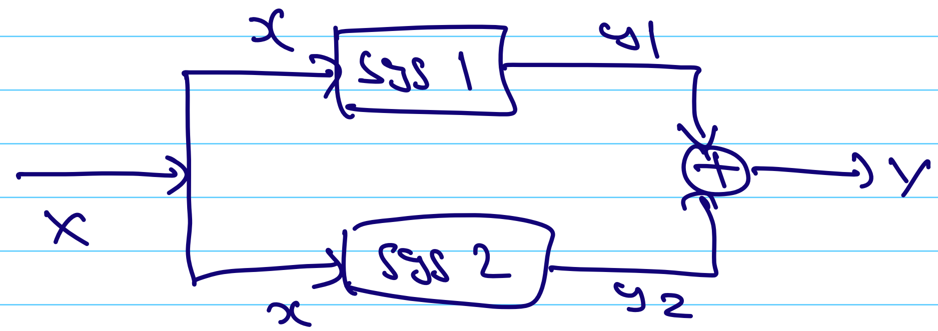
* few basic & Important interconnection's of system

① Cascade (Series)



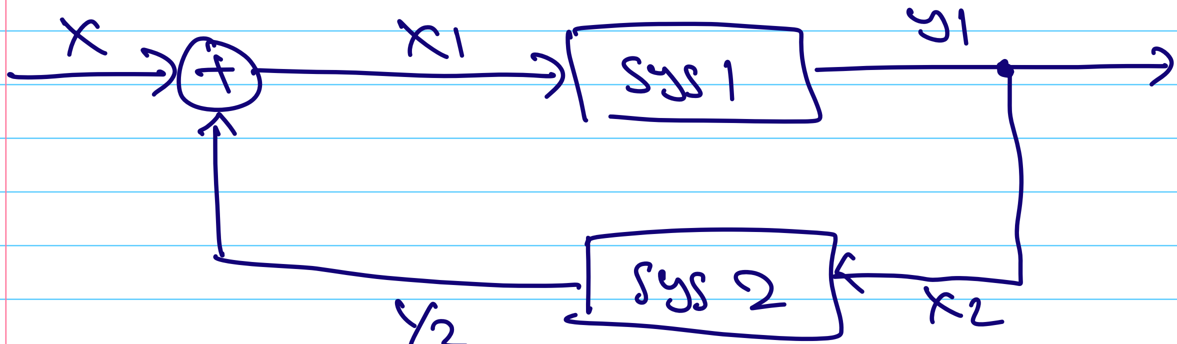
The order in which you cascade system's is very Important for LTI system, overall system transformation is independent of the order in which the system's are cascaded.

② Parallel



* Order doesnt matter (as we are simple adding output's)

③ Feed BACK :-



in feedback, we have one system, The output of sys 1 is fed back through sys 2 added to the overall input, that sum form's input to sys 1.

These are the system's in general, and we can't say much about system's when we try to treat them in most general form. It's useful and important to focus in on properties a system may or may not have.

System Properties:-

Some of them, we want to Impose on system, and some of them we don't want, But as thing's progress, we will tend to find it useful, whether a

System does or doesnt have certain Property

① **Memoryless:-**

The output at any given time $t = t_0$ depend's only on input at the same time

$\Rightarrow$ The output at a specific time depend's only on input at that time.

$x(t)$ @ $t = t_0 \longrightarrow y(t)$ @ $t = t_0$

$x[n]$ @ $n = n_0 \longrightarrow y[n]$ @ $n = n_0$

Ex:

① $y(x) = x^2(t)$ Yes

$y[n] = x[n]^2$

② $y(t) = \int_{-\infty}^{t} x^2(t)\,dt$ NO

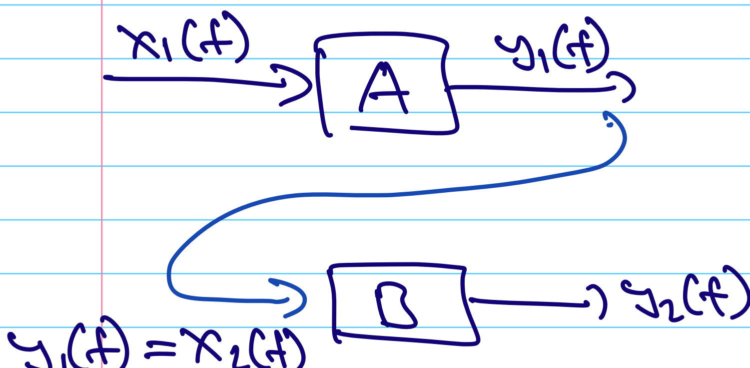
③ $y[n] = x[n-1]$ NO

unit delay.

② Invertibility

* Given the output of the system, you can figure out uniquely what the input is.

* Given the output, there is only one input could have caused it.



if $B =$ Inverse of A

then $y_2 = x_1$ (Identity)

Ex: $y_1(t) = \int_{-\infty}^{t} x_1(\tau)\, d\tau$ (integration)

$y_2(t) = \frac{dx_2(t)}{dt}$ (differentiation)

System A $y = x^2$

Invertible No

Memoryless yes

(3) Causality:-

Output at any time depend's only on input Prior or equal to that time

Or:- System can't anticipate "future" inputs.

$$x_1(t) \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow y_2(t)$$

if $x_1(t) = x_2(t)$ $t < t_0$

Then $y_1(t) = y_2(t)$ $t < t_0$

4) Stability:-

Bounded input - Bounded output

$\Rightarrow$ if the input never gets above some finite value, then stability requires the output also stay's within some finite value.

5) Time-invariance :-

System doesn't really care what you call origin.

$$x(t) \longrightarrow y(t)$$

$$x(t-t_0) \longrightarrow y(t-t_0)$$

6) Linearity

$$x_1(t) \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow y_2(t)$$

$$a_1 x_1(t) + b_2 x_2(t) \longrightarrow a y_1(t) + b y_2(t)$$

System's that are Time-invariant, and Linear, the use of Impulse function both continuous & Discrete time Provides an extraordinary Important & useful mechanism for characterizing those system's