Lec 05:-

transition from Fourier series to fourier transform.

* And that is the transition from Periodic Phenomenon to non-Periodic Phenomenon.

* The transition from Periodic to non periodic Phenomenon, the way we are gonna accomplish that is to view a non-Periodic Phenomenon as the limiting case of Periodic Phenomenon as the Period tends to infinity.

two Aspect's of Fourier Transform.
Analysis & Synthesis.

Fourier Analysis:- The analysis is if
$f(t)$ is Periodic, then we have fourier

co-efficient's $\hat{f}(k) = \int_0^1 e^{-2\pi i k t} f(t)\, dt$

That's analyzing the function the signal into its constituent component's, figuring out how much each complex exponential contributes to the whole $f(t)$

Fourier synthesis :- Synthesis is writing the series, recovering the function from its constituent component's.

$$f(t) = \sum_{k=-\infty}^{\infty} \hat{f}(k) e^{2\pi i k t}$$

Both of these thing's generalize's the fourier transform.

* The fourier transform is generalization of fourier co-efficient

* The inverse fourier transform is generalization of fourier series.

* we need to setup what $f(t)$ is Periodic of Period T and let $T \to \infty$ ultimately.

* The Building Block's for a Period of Signal T are Complex exponential's of Period T

$$e^{2\pi i k\left(\frac{t}{T}\right)} \Rightarrow \text{Periodic of Period } T$$

The fourier series are of form

$$f(t) = \sum_{k=-\infty}^{\infty} C_k \, e^{2\pi i \left(\frac{k}{T}\right)t}$$

$$C_k = \frac{1}{T} \int_0^T e^{-2\pi i \left(\frac{k}{T}\right)t} f(t)\, dt$$

when we have a signal $f(t)$ of Period 1 we have Building Block's of Complex exponential's of Period 1 i.e $e^{2\pi i k t}$

$e^{2\pi i k t} = \cos 2\pi k t + i \sin 2\pi k t$

Time Period $= \frac{2\pi}{\omega} = \frac{2\pi}{2\pi k} = \frac{1}{k}$

$\Rightarrow$ Time Period $= 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

Freq $= 1, 2, 3, 4, \ldots$

When we have $f(t)$ of Period T

The Building Blocks are $e^{2\pi i \frac{k}{T} t}$

Time Period $= \frac{2\pi}{\omega} = \frac{2\pi}{2\pi \frac{k}{T}} = \frac{T}{k}$

$T = T, \frac{T}{2}, \frac{T}{3}, \frac{3}{T}, \ldots$

freq $= \frac{\pm 1}{T}, \frac{\pm 2}{T}, \frac{\pm 3}{T}, \frac{\pm 4}{T}, \ldots$

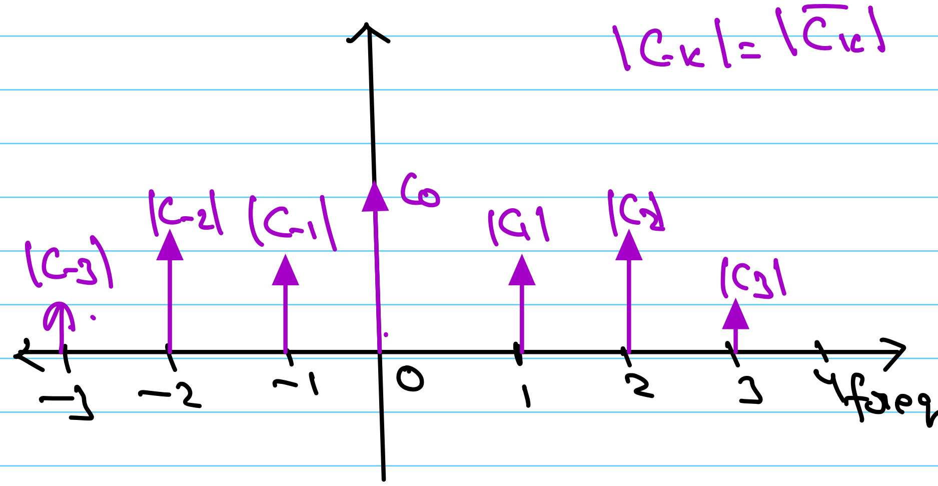
we can also write the formula of

$$C_k = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-2\pi i \frac{k}{T} t} f(t)\, dt$$

Picture of spectrum of freq

(1) Period $T = 1$

Spectrum:



Time period : $\infty, \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{4}, \ldots$

freq : $0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots$

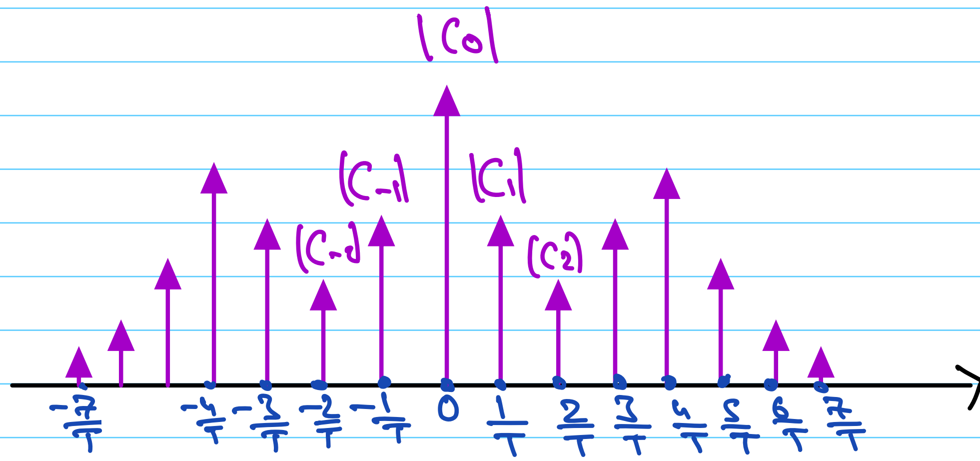
Spacing of the freq $= 1$

(2) Period $= T$

$$f(t) = \sum_{k=-\infty}^{\infty} C_k e^{2\pi i \frac{k}{T} t}$$

$$C_k = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-2\pi i \frac{k}{T} t} f(t)\, dt$$

Spectrum:



Time period: $\infty, \pm\frac{T}{1}, \pm\frac{T}{2}, \pm\frac{T}{3}, \dots$

freq : $0, \pm\frac{1}{T}, \pm\frac{2}{T}, \pm\frac{3}{T}, \dots$

so that the freq are $0, \pm\frac{1}{T}, \pm\frac{2}{T}, \pm\frac{3}{T}, \dots$ Point's in the spectrum are spaced $\frac{1}{T}$ apart and, indeed in the picture above the spectrum is getting more tightly packed as the period T increases.

* The reciprocal relationship b/w How the function appear's in time domain & How the function appear's

in freq domain.

$\Rightarrow$ if the Period is T, the spacing in Constituent Part's is $\frac{1}{T}$. There is a reciprocal relationship b/w Period and the frequencies.

* reciprocal relationship b/w two view's of the function.

$\Rightarrow$ if $T<1$, then $\frac{1}{T}>1$ (larger spacing)

The spectrum is spread out

if $T>1$, then $\frac{1}{T}<1$ (smaller spacing)

The spectrum is compressed.

* in Particular as $T \rightarrow \infty$, which is the case ultimately we want to deal with, the spectrum is getting more and more compressed, The spacing getting smaller & smaller.

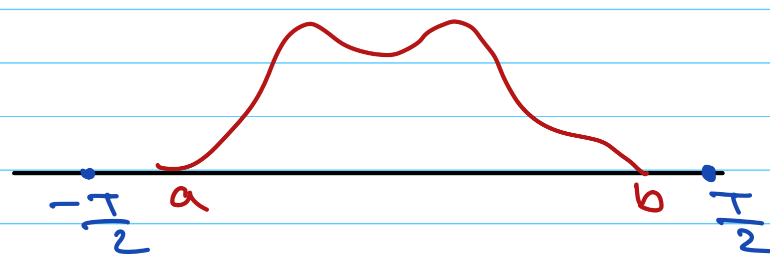
as $T \to \infty$, the spectrum becomes continuous.

$$C_k = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-2\pi i \frac{k}{T} t} f(t)\, dt$$

let $T \to \infty$, use this as a way of passing periodic to non-periodic phenomenon.

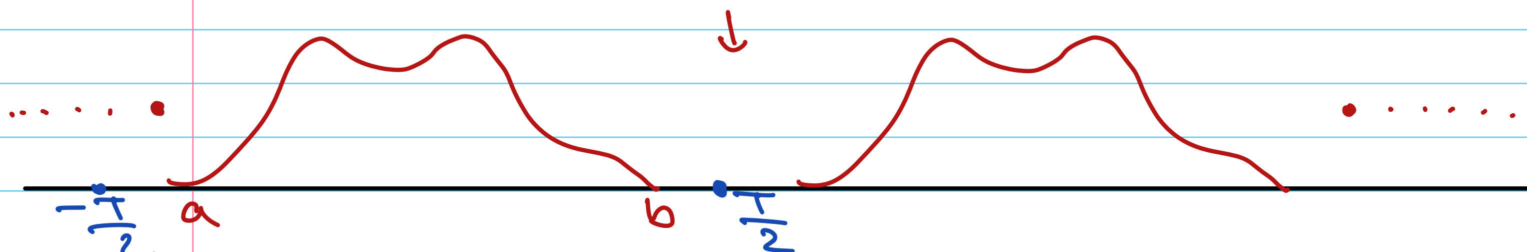
* we can't just let $T \to \infty$ and get fourier transform. we have to tickle a little bit.

* Suppose $f(t)$ looks like this



choose $\frac{T}{2}$ such that

& periodize with period T



fourier co-efficient

$$C_k = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-2\pi i \frac{k}{T} t} f(t)\, dt$$

$$\Rightarrow \quad C_k = \frac{1}{T} \int_a^b e^{-2\pi i \frac{k}{T} t} f(t)\, dt$$

$$\left| \int_a^b e^{-2\pi i \frac{k}{T} t} f(t)\, dt \right| \leq \int_a^b \left| e^{-2\pi i \frac{k}{T} t} f(t) \right| dt$$

$$\leq \int_a^b \left| e^{-2\pi i \frac{k}{T} t} \right| \left| f(t) \right| dt$$

$$= M \quad \text{(fixed Number)}$$

$\Rightarrow$ This M is a fixed Number, what does it say about kth fourier co-efficient,

$$|C_k| \leq \frac{1}{T} M$$

as $T \to \infty$ $\quad |C_k| \leq \frac{M}{T} \longrightarrow 0$

everything gonna die.

The Fourier coefficient's die.