Matgeo-1.7.5

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Question

Find the value of p for which the points (-5,1),(1,p),(4,-2) are collinear

Solution

Let the points be

Point	Name
$\begin{pmatrix} -5 \\ 1 \end{pmatrix}$	Point A
$\begin{pmatrix} 1 \\ p \end{pmatrix}$	Point B
$\begin{pmatrix} 4 \\ -2 \end{pmatrix}$	Point C

Table: Variables Used

The difference vectors are

$$(B - A) = \begin{pmatrix} 6 \\ p - 1 \end{pmatrix}, \tag{1}$$

$$(C - A) = \begin{pmatrix} 9 \\ -3 \end{pmatrix}. \tag{2}$$

Solution(Continuation)

Thus, $M^T = (B - A \;\; C - A)^T = \begin{pmatrix} 6 & p-1 \\ 9 & -3 \end{pmatrix}$.

Apply row operations to convert M^T into upper triangular form.

$$\begin{pmatrix} 6 & p-1 \\ 9 & -3 \end{pmatrix} \xrightarrow{R_2 \to R_2 - \frac{3}{2}R_1} \begin{pmatrix} 6 & p-1 \\ 0 & -\frac{3}{2}(p+1) \end{pmatrix}. \tag{3}$$

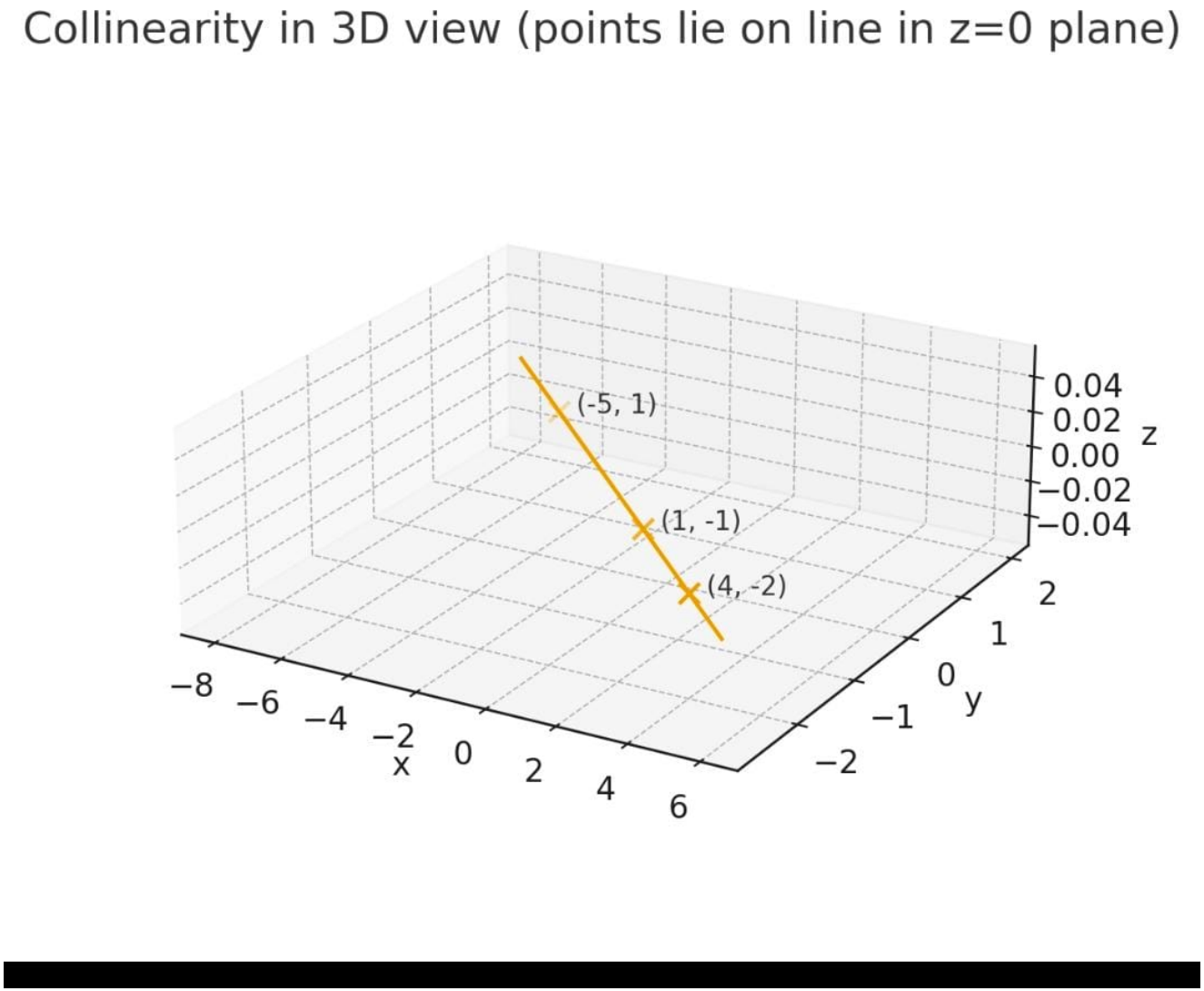
For collinearity, $\text{rank}(M^T) = 1$. This happens when the second row is zero: $-\frac{3}{2}(p+1) = 0$.

Conclusion

$p = -1$

Hence, the three points A, B, C are collinear when $p = -1$.

Graphical Representation



Figure