

1.7.5

AI25BTECH11019 - MENAVATH SAI SANJANA

Question:

Find the value of p for which the points $(-5,1), (1,p), (4,-2)$ are collinear

Solution:

Let the points be

Point	Name
$\begin{pmatrix} -5 \\ 1 \end{pmatrix}$	Point A
$\begin{pmatrix} 1 \\ p \end{pmatrix}$	Point B
$\begin{pmatrix} 4 \\ -2 \end{pmatrix}$	Point C

TABLE 0: Variables Used

The difference vectors are

$$(B - A) = \begin{pmatrix} 6 \\ p - 1 \end{pmatrix}, \quad (0.1)$$

$$(C - A) = \begin{pmatrix} 9 \\ -3 \end{pmatrix}. \quad (0.2)$$

$$\text{Thus, } M^T = (B - A \quad C - A)^T = \begin{pmatrix} 6 & p - 1 \\ 9 & -3 \end{pmatrix}.$$

Apply row operations to convert M^T into upper triangular form.

$$\begin{pmatrix} 6 & p - 1 \\ 9 & -3 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 - \frac{3}{2}R_1} \begin{pmatrix} 6 & p - 1 \\ 0 & -\frac{3}{2}(p + 1) \end{pmatrix}. \quad (0.3)$$

For collinearity, $\text{rank}(M^T) = 1$. This happens when the second row is zero: $-\frac{3}{2}(p + 1) = 0$.

$$p = -1$$

Hence, the three points A, B, C are collinear when $p = -1$.

Collinearity in 3D view (points lie on line in $z=0$ plane)

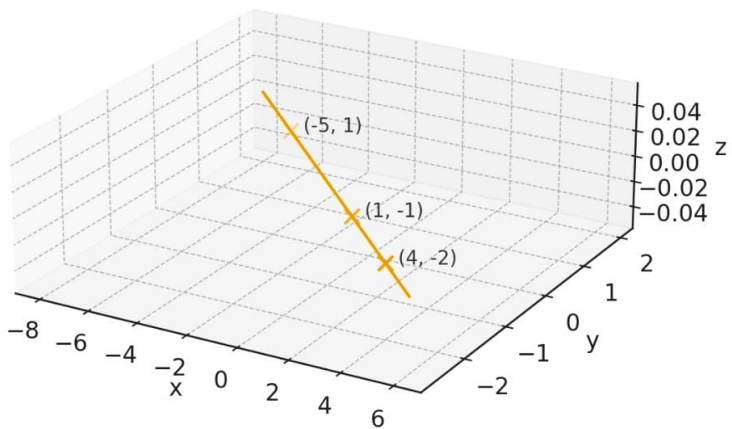


Fig. 0.1